Competition in successive markets: entry and mergers

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Abstract

This paper analyses successive markets where the intra-market linkage depends on the technology used to produce the final output. We investigate entry of new firms, when entry obtains by expanding the economy, as well as collusive agreements between firms. We highlight the differentiated effects of entry corresponding to a constant or decreasing returns technology. In particular, we show that, under decreasing returns, free entry in both markets does not entail the usual tendency for the input price to adjust to its marginal cost while it does under constant returns. Then, we analyse collusive agreements by stressing the role of upstream linkage on the profitability of horizontal mergers *à la* Salant, Switzer and Reynolds.

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1 Introduction

The analysis of collusion between downstream and upstream firms operating in successive markets traditionally relies on Cournot competition. In these markets, firms select non cooperatively the quantities of output of the good they produce, the output of the upstream firms serving as input in the production of the output in the downstream market. Collusion is represented as an agreement through which the insiders of the collusive agreement act in unison, reducing thereby the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them. Collusive outcomes are the Cournot equilibria corresponding to these reduced numbers of oligopolists, which are then compared with those arising when downstream and upstream firms act independently from each other in their respective markets. The link between the two markets follows from the fact that the downstream firms' unit cost appears as the unit revenue for the upstream ones : the price paid for a unit of input for the firms in the former constitutes the unit receipt for the firms in the latter. The papers by Salinger (1988), or Gaudet and Van Long (1996) are typical examples where this framework is used. In particular, both these papers adopt the assumption that downstream firms are price-takers when buying the input while upstream firms behave as Cournot competitors on the resulting demand function. However, both these papers use hidden assumptions when describing in the model how the input price depends on the decisions of the firms in both markets. For instance, the paper by Salinger assumes (without being fully explicit about this assumption) that downstream firms have a constant returns technology. Similarly, Gaudet and Van Long suppose an even stronger technical relationship between the input and output: they simply assume that one unit of input is transformed homogeneously in one unit of output !

In order to better understand how the effects of mergers and entry in successive markets depend on technology, we first propose a model which makes explicit how the downstream and upstream markets are linked to each other *via* the *technology* used by the downstream firms to transform the input into the output. For this purpose, we define two markets as *technology- linked* whenever the good produced and exchanged in one of them is transformed, *via* some technology, into another good, and then exchanged by the firms operating in the other market. Then, we consider two examples of technology-linked markets. The first corresponds to a decreasing returns technology while the other uses a constant returns technology, as in the case of Salinger and Gaudet and Van Long. In the framework of these examples, we highlight some features accompanying the entry of new firms. *These features differ from one technology to the other, underlying the crucial role played by the technology which links the upstream and downstream markets.* In particular, we show that, under decreasing returns, free entry in both markets does not entail the usual tendency for the input price to adjust to its marginal cost while it does under constant returns. Also we show that some merger may result profitable under one technology but brings losses with the other. We stress also the differences obtained in our framework with those appearing in the papers referred to above.

Another outcome of our approach consists in studying the role played by the existence of an upstream market on collusive agreements among firms in the downstream one. The analysis of horizontal mergers has often been performed assuming that firms face an exogenous cost function for producing the output, as in Salant, Schwitzer and Reynolds (1983). Here we make this cost function endogenous, taking into account the technology used by the downstream firms when producing the output. It turns out that the consequences of collusive agreements can be very different.

Finally, our framework can also be used to analyze other economic issues like product differentiation as in Belleflamme and Toulemonde (2003), or mergers' stability, as in d'Aspremont *et al.* (1983). Furthermore, it can be used as well to analyse mergers in the spirit of Neumann, Fell and Reichel (2005).

The rest of the paper is organized as follows. In the next section we present the model, assuming a given number of firms in the upstream and downstream markets. In Section 3, we consider two examples of technology-linked markets corresponding to two different technologies, and highlight the differentiated effects resulting from entry. Section 4 provides the analysis of collusion; furthermore, we revisit at the light of our analysis a paper by Salant, Switzer and Reynolds (1983) devoted to horizontal integration. Section five concludes.

2 The model

We consider two technology-linked markets, the downstream and upstream markets, with n *downstream* firms $i, i = 1, ...n$, in the first producing the output, and m *upstream* firms j, $j = 1, ...m$, in the second, producing and selling the input. The n downstream firms face a demand function $\pi(Q)$ in the downstream market, with Q denoting aggregate output. Firm i owns technology $f_i(z)$ to produce the output, namely

$$q_i = f_i(z),$$

with z denoting the quantity of the sole input used in the production process and bought by firm i in the upstream market. The m *upstream* firms each produce the input z at a total cost $C_j(z), j = 1, .., m$.We assume that this situation gives rise to two games. The players in the first game, *the downstream game*, are the n downstream firms with output strategies q_i, while the players in the second, *the upstream game*, are the upstream firms with input strategies s_j. The two markets are linked to each other as follows. In the downstream game, firms select strategically their output levels q_i which determines their individual demand $z_i(p)$ of input *via* both the production function f_i, namely $z_i = f_i^{-1}(q_i)$, and the input price p. Consequently, the downstream firms while behaving strategically in the output market, are assumed to be price takers in the input market. Faced with the input demand schedule $\Sigma_{i=1}^n z_i(p)$ resulting from aggregating individual demands, firms in the upstream game select non cooperatively the quantities of input s_j they offer for sale; the input market price at which upstream firms evaluate their profits is assumed to clear the input market, namely, it satisfies

$$\Sigma_{i=1}^{n} z_i(p) = \Sigma_{j=1}^{m} s_j.$$

Given an input price p, the payoff in the downstream game for the i_{th} firm at the vector of strategies (q_i, q_{-i}) obtains as

$$\Pi_i(q_i, q_{-i}; p) = \pi(q_i + \Sigma_{k \neq i} q_k) q_i - p f_i^{-1}(q_i).$$

Expressed in terms of input, this payoff rewrites as

$$\Pi_i(z_i, z_{-i}; p) = \pi\left(f_i(z_i) + \Sigma_{k \neq i} f_k(z_k)\right) f_i(z_i) - p z_i.$$

Given these payoffs and a price p in the input market, the best reply, $z_i(z_{-i}; p)$ of firm i in the downstream game, obtains as a solution (whenever it exists) to the problem

$$\underset{z_i}{Max}\ \Pi_i(z_i, z_{-i}; p).$$

A Nash equilibrium in the downstream game (whenever it exists) writes as an output vector $(q_1^*, .., q_n^*) = (f_1(z_1^*(p)), ..., f_n(z_n^*(p)))$, where $z_i^*(p)$ solves

$$\underset{z_i}{Max}\ \Pi_i(z_i, z_{-i}^*(p)); p)$$

for all $i, i = 1, .., n$.

In the upstream game, firms select their selling strategies $s_j, j = 1, .., m$. Assuming a Nash equilibrium in the downstream game, they face a total demand $\Sigma_{i=1}^{n} z_i^*(p)$ of input. Given a n-tuple $(s_1, ..., s_j, ..s_m)$, the input price $p\left(\Sigma_{k=1}^{m} s_k\right)$ thus satisfies

$$\Sigma_{i=1}^{n} z_i^*(p) = \Sigma_{k=1}^{m} s_k.$$

Accordingly, the payoff function $\Gamma_j(s_j, s_{-j})$ of firm j in the upstream game writes as

$$\Gamma_j(s_j, s_{-j}) = p(\Sigma_{k=1}^{m} s_k) s_j - C_j(s_j)$$

whenever it is defined for all admissible values of p. Denote by $(s_1^*, ..., s_m^*)$ a Nash equilibrium in the upstream game (whenever it exists). We define an *industry equilibrium* as a $(m+n)$-tuple vector $(q_1^*, ..., q_n^*; s_1^*, ..., s_m^*)$ and an input price p^* such that (i) $(q_1^*, ..., q_n^*)$ is a Nash equilibrium in the downstream game (ii) $(s_1^*, ..., s_m^*)$ is a Nash equilibrium in the upstream one, and (iii) $p^* = p(\Sigma_{k=1}^{m} s_k^*)$. An industry equilibrium is a situation in which both the downstream and upstream markets exhibit Cournot equilibria, and where the quantity of input demanded at equilibrium in the first market exactly balances the quantity supplied in the second.

We have defined an industry equilibrium for the case of two technology-linked markets with the link reducing to a technology which uses a single input. Nonetheless, there is no reason to restrict this concept by putting constraints on the number of markets linked and the number of inputs through which these markets are linked. Concerning the number of markets linked by technology, we could easily set up a model with a chain of markets $1, 2, .., r, r+1, ...M$, where commodity $r+1$ is produced and exchanged in market $r+1$, and serves as an input for the firms operating in market r : then, markets are two-by-two technology-linked. Concerning the number of inputs, it would not be difficult to extend our analysis to a technology embodying two factors, one of them being viewed as a fixed input. Also more elaborate technology-linked markets' networks could be investigated, which do not reduce to two-by-two linked markets, but also link one, or several, market(s) to several others, corresponding to the various inputs used in the production of the good exchanged in the former while bought in the latter.

3 Exploring industry equilibria: entry

It is difficult to analyze industry equilibria at the full level of generality underlying the above model. This is why we try to get some insight into the relationship between technology and competition observed at an industry equilibrium by looking at two significant examples. The first corresponds to a situation in which downstream firms are endowed with a decreasing returns technology while the second is characterized by constant returns. Furthermore, we assume in both examples a linear demand function in the downstream market, as in Salinger (1988) and Gaudet and Van Long (1996). We also assume that firms operating in the upstream (resp. downstream) market are all identical. Entry and competition are analyzed through the asymptotic properties of the industry equilibria when the number of firms in the technologically-linked markets is increased by expanding the economy, as in Debreu and Scarf (1963). The two examples are now considered successively and their conclusions are compared.

3.1 Decreasing Returns

The n *downstream* firms are assumed to face a linear demand $\pi(Q) = 1 - Q$ in the downstream market. They share the same technology $f(z)$ to produce the output, namely

$$q = f(z) = z^{\frac{1}{2}}.$$

The m upstream firms each produce the input z at the same linear total cost $C_j(s_j) = \beta s_j$, $j = 1, .., m$. As in the general formulation above, we assume that this situation gives rise to two games. The players in the first game are the n

downstream firms with output strategies q_i, while the players in the second are the m upstream firms with input strategies s_j.

The profits of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{k \neq i} q_k) q_i - p q_i^2.$$

As a result of the strategic choice q_i, each firm i sends the input quantity signal $z_i(p) = q_i^2$ to the upstream market. When aggregating these signals, we get the demand function of input over which the upstream firms select their selling strategies s_j. The j_{th} upstream firm's profit Γ_j at the vector of strategies (s_j, s_{-j}) writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j}) s_j - \beta s_j, \tag{1}$$

with $p(s_j, s_{-j})$ such that $\Sigma_{k=1}^m s_k = \Sigma_{k=1}^n z_i(p)$.

Given a price p in the input market, the best reply of downstream firm i in the downstream game obtains as

$$q_i = \frac{1 - \Sigma_{k \neq i} q_k.}{2p + 2} \tag{2}$$

Clearly, these best replies depend on the upstream market price p and we may compute the symmetric Nash equilibrium of the above game, contingent on the price p. Defining $q_i = q$ for $i = 1...n$, re-expressing equation (2) and solving it in q, we get at the symmetric solution

$$q_i^* = \frac{1}{(n + 2p + 1)} \tag{3}$$

so that we obtain

$$z_i^*(p) = z^*(p) = \frac{1}{(n + 2p + 1)^2};\ i = 1...n. \tag{4}$$

The upstream firms then face a total demand $\sum_{i=1}^{n} z_i^*(p)$ of input equal to $nz^*(p)$. At a given an n-tuple $(s_1, ..., s_j, ..s_m)$ of input strategies chosen by the upstream firms in the upstream game, the input price clearing the upstream market must satisfy

$$\frac{n}{(n + 2p + 1)^2} = \Sigma_{k=1}^m s_k$$

so that we get

$$p(\Sigma_{k=1}^{m} s_k) = \sqrt{\frac{n}{4\Sigma_{k=1}^{m} s_k}} - \frac{n+1}{2}. \tag{5}$$

Substituting (5) into (1), the payoff function $\Gamma_j(s_j, s_{-j})$ of the upstream firm j in the upstream game rewrites as

$$\Gamma_j(s_j, s_{-j}) = (\sqrt{\frac{n}{4\Sigma_{k=1}^{m} s_k}} - \frac{n+1}{2})s_j - \beta s_j,$$

Notice that the profit function $\Gamma_j(s_j, s_{-j})$ is concave in $s_j, j = 1, ...m$, so that we can use the first order necessary and sufficient conditions to characterize an equilibrium. Accordingly, at the symmetric Nash equilibrium of the upstream game, we obtain

$$s^*(m, n) = \frac{n\left(2m-1\right)^2}{4m^3\left(2\beta+1+n\right)^2}.$$

Hence the profit $\Gamma_j(m, n)$ of an upstream firm at the symmetric equilibrium of the upstream game obtains as

$$\Gamma_j(m, n) = \frac{n(2m-1)}{8\left(n+1\right)m^3}.$$

Finally, the equilibrium price $p^*(m, n)$ in the input market obtains as

$$p^*(m, n) = \frac{n+1+4m\beta}{2\left(2m-1\right)}.$$

Consequently, substituting this equilibrium price into the equilibrium quantities $z^*$of input bought by each downstream firm, as given by (4), we get

$$z^*(m, n) = \frac{\left(2m-1\right)^2}{4m^2\left(2\beta+n+1\right)^2}$$

so that, from (3), we obtain

$$q_i^*(m, n) = q^*(m, n) = \frac{2m-1}{2m\left(2\beta+n+1\right)}.$$

Therefore, the resulting output price $\pi^*(m, n)$ in the downstream market obtains as

$$\pi^*(m, n) = 1 - \frac{n\left(2m-1\right)}{2m\left(2\beta+n+1\right)}.$$

The profit $\Pi_i(m,n)$ of a downstream firm at equilibrium in the corresponding game is thus equal to

$$\Pi_i(m,n) = \frac{1}{8}\left(4m\beta + 4m + n - 1\right)\frac{2m-1}{m^2\left(2\beta + n + 1\right)^2}.$$

Notice that $\Pi_i > 0$, - a requirement needed to guarantee the survival of firms in the downstream market.

3.1.1 Comparative statics

Taking the derivatives of input and output prices, we get

$$\frac{\partial \pi^*}{\partial m} < 0 :$$

when the number of upstream firms increases, the output price decreases. This reflects the fact that an increase in the number of upstream firms leads to a decrease in the equilibrium input price which, in turn, induces downstream firms to produce more at equilibrium.

Furthermore, it can be checked:

Proposition 1 *The profit $\Pi_i(m,n)$ of a downstream firm when $n \leq 3$ always increases with the number of upstream firms. On the contrary, when $n > 3$, the profit of a downstream firm decreases as the number of upstream firms increases, if, and only if, the condition*[1]

$$m > \frac{n-1}{n-3} \tag{6}$$

is satisfied[2].

An increase in the number of upstream firms influences both the revenue and the cost sides of the profit of a downstream one. On the one hand, it decreases the output selling price; on the other, it decreases the unit cost of production which is equal to $p^{\frac{1}{2}}$. But the unit cost $p^{\frac{1}{2}}$ is a function of m as well as of n; hence, whether the first or the second effect of an increase in m on the downstream firms' profit prevails, also depends on n, as clarified by Proposition 1. Intuitively, one would expect that the downstream firms should always benefit from an increase in competition in the upstream market since it is expected to lower the price of the input they use. Nonetheless, this intuition only holds when the downstream market is strongly concentrated ($n \leq 2$), while

[1]The derivative of the profit of a downstream firm is $\frac{3m+n-mn-1+2m\beta}{4(n+1)^2m^3}$. Hence, the sign depends only on the sign of the numerator. The derivative is positive iff $\beta > \frac{mn+1-3m-n}{2m}$, which is satisfied when $n < 3$ or $3 < n$ and $m \geq \frac{n-1}{n-3}$.

[2]Notice that condition (??) becomes redundant when the number of downstream firms n exceeds 5 !

it does not whenever the degree of competition gets higher in the downstream market! Of course, this paradoxical outcome is related to our specific decreasing returns technology. But even so, it should attract the interest of scholars on the type of interaction existing between technology and the degree of competition in the market.

In order to evaluate how the downstream firms' profits vary when both m and n are increased in the same proportion, we calculate the total derivative of $\Pi_i(m,n)$, that is

$$\frac{\partial \Pi_i(m,n)}{\partial m} + \frac{\partial \Pi_i(m,n)}{\partial n}$$

It is easily checked that $\frac{\partial \Pi_i(m,n)}{\partial n} < 0$. Therefore, whenever $n > 5$ according to proposition 1, a proportional increase in n and m causes a decrease in the profit of a downstream firm. In the remaining cases when $\frac{\partial \Pi_i(m,n)}{\partial m} > 0$, the sign of the total derivative can be positive or negative.

Similarly, we get that $\frac{\partial \Gamma_j(m,n)}{\partial n} > 0$, reflecting the fact that an increase in n generates an increase in input demand and, accordingly, an increase in the input price which, in turn, increases the profit of the upstream firm. As for the total derivative of $\Gamma_j(m,n)$, $\frac{\partial \Gamma_j(m,n)}{\partial n} + \frac{\partial \Gamma_j(m,n)}{\partial m}$, we observe again that the sign of the total derivative depends on m and n. Whenever $m > 1$, the total derivative can have both signs depending on n.

3.1.2 Asymptotic properties of input and output prices

It is interesting to examine the effects of entry on equilibria in the successive markets. We choose to model entry by replicating $k-$times the basic economy, as in Debreu and Scarf (1963). In the $k-$th replica, there are kn downstream and km upstream firms. We consider successively the following situations.

1.Perfect competition

We compute

$$\lim_{k\to\infty} \pi^*(km, kn) = 0$$

and

$$\lim_{k\to\infty} p^*(km, kn) = \frac{1}{4}\frac{n}{m} + \beta$$

Furthermore we get

$$\lim_{k\to\infty} q^*(km, kn) = 0.$$

Proposition 2 *Under decreasing returns, when both the number of upstream and downstream firms tend simultaneously to infinity, the equilibrium input price does not converge to upstream firms' marginal cost, but exceeds it by an amount which decreases with the ratio of the number of firms* $\frac{n}{m}$. *However, the equilibrium output price converges to the competitive output price.*

The usual practice when increasing the number of firms in the market consists in comparing the resulting price with a *fixed* marginal cost. The novelty here is that the marginal cost of the downstream firms does not remain fixed when increasing the number of firms in the downstream and upstream markets simultaneously. Importantly, notice that, whatever k, the marginal cost of producing the input, which is equal to β, is lower than the input price by an amount of $\frac{1}{4}\frac{n}{m}$. This looks as a surprise since this context, for large values of k, corresponds exactly to perfect competition. It is as if the downstream firms would be charged a constant tax per unit of input over the marginal cost of producing the input, β. In fact, when k is close to ∞, $q^*(m,n)$ is close to zero, implying an infinitesimal individual demand of input from each downstream firm and, accordingly, a marginal product of the input which tends to infinity with k. In particular, if the price of input were set at the marginal cost β, the quantity of input demanded by the downstream firms would exceed the quantity which would be offered by the upstream firms at the same price, preventing thereby the equality of supply and demand, as required by the definition of a competitive equilibrium [3]. Notice however that, even though upstream firms get the amount of the tax, it *does not* prevent the quantity of input exchanged in the input market to correspond exactly to the quantity required to produce an aggregate output corresponding to the competitive equilibrium output. More than that: the burden of this tax is even required in order to induce downstream firms to reduce their input demand in order to produce exactly the competitive equilibrium output level! Notice also that the presence of this subsidy does not bring any extra profits to the upstream firms themselves: their profit tends to zero when k tends to infinity. Consequently, this limit value of the input price, including the existence of the subsidy, does not preclude the limit economy to be in a Pareto optimal state simultaneously in both markets. The existence of this transfer, through the input price from the downstream to the upstream firms, reveals the interlinkage between the competitive and technological effects resulting from the *simultaneous* increase in the number of firms in both markets. Furthermore, notice that, if the economy would be replicated at a different speed

[3]The total quantity demanded by the downstream firms at the downstream Cournot equilibrium if $p = \beta$ obtains from the solution of the problem

$$\underset{q_i}{Max}(1 - q_i - \Sigma_{k\neq i}q_k)q_i - \beta q_i^2$$

from which we easily obtain:

$$nz^* = \frac{n^2}{(n + 2\beta + 2)^2}.$$

Thus $\underset{k\to\infty}{lim}\ \{knz^*\} = 1$. On the other hand, the amount of input offered by the upstream firms at price β is $\frac{n}{2(\frac{1}{2}+\beta+\frac{1}{2})^2}$. This amount tends to zero when km and kn tend to inifnity, and not to 1 in the downstream and upstream markets, this discrepancy between marginal cost and input price may disappear. In fact, when the upstream market is replicated infinitely faster than the downstream one, this discrepancy disappears at the limit. For instance, when the downstream market is replicated at speed k, while the upstream market is replicated at speed k^2, the limit input price is equal to the marginal cost β. In other words, the power of upstream firms should be diluted much faster than the downstream firms' one in order to force the competitive outcome !

2. Upstream competition and downstream oligopoly

We compute

$$\lim_{k\to\infty} \{\pi^*(km, n)\} = \frac{1+2\beta}{n+1+2\beta}$$

and

$$\lim_{k\to\infty} \{p^*(km, n)\} = \beta.$$

Proposition 3 *When the number of upstream firms tends to infinity while the number of downstream remains fixed,*

(i) the equilibrium input price converges to upstream firms' marginal cost;

(ii) the equilibrium output price converges to the output price $\frac{1+2\beta}{1+n+2\beta}$ *corresponding to the Cournot equilibrium with* n *downstream firms producing the good at a unit cost* β.

Thus, differently from proposition 2, proposition 3 fits with the standard asymptotic results obtained in the usual Cournot framework of a single market. In fact, the technological effects, present when n and m tend simultaneously to infinity, disappear when n is fixed: the production level of each downstream firm does not tend to zero, so that, whatever m, the marginal product of the input remains bounded away from infinity. Then no tax is needed to dampen the incentive to overproduce the output. On the contrary, the competitive effects are still operating since the input price now tends to the marginal cost.

3. Downstream competition and upstream oligopoly

We easily compute

$$\lim_{k\to\infty} \{\pi^*(m, kn)\} = \frac{1}{2m}$$

and

$$\lim_{k\to\infty} \{p^*(m, kn)\} = \infty.$$

Proposition 4 *When the number of downstream firms* n *tends to infinity while the number* m *of upstream ones remains fixed, the output price converges to the marginal cost of producing the output when* m *firms are operating in the input market. In this case, the input price gets arbitrary large.*

This immediately follows from the fact that the marginal cost of producing the output at the equilibrium in the downstream market when m firms operate in the upstream market is equal to $2pq$, with $p = p^*(m,n)$ and $q = q^*(m,n)$. The output price exactly reflects the market power existing in the upstream market, which is transferred in the downstream market through its dependence on the number of upstream firms, m. This sheds some further light on the interaction between two technology-linked markets under Cournot competition. Even if the competitive conditions are met in the downstream market, since $MC \cong \pi^*(m,n)$, the output price encompasses the non competitiveness in the input market. The usual analysis of Cournot competition in a market does not allow this type of consideration because the relationship of costs to market power in the input market cannot be taken into account when the cost function is exogenous.

In fact, as in the case of pure competition considered above, when n is close to ∞, $q^*(m,n)$ is again close to zero, implying an infinitesimal individual demand of input from each downstream firm and, accordingly, a marginal product of the input which tends to infinity with m and n. This leads downstream firms' demand to increase beyond any limit, forcing in turn the input price to increase itself beyond any limit when the number of upstream firms remains fixed.

3.2 Constant returns

We consider exactly the same case as above, with the exception that the technology $f(z)$ shared by the downstream firms is now given by

$$f(z) = \alpha z, \ \alpha > 0$$

as in Salinger and Gaudet and Van Long (with α equal to 1 in the latter case).We assume that $\alpha \geq \beta$: this assumption guarantees that the marginal cost of producing the input does not exceed its marginal product in the production of output. The profits $\Pi_i(q_i, q_{-i})$ of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) now obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{k \neq i} q_k)q_i - pz_i.$$

As a result of the strategic choice q_i, each firm i sends an input quantity signal $z_i(p) = \frac{q_i}{\alpha}$ to the upstream market. Given the price p in the input market, the best reply of downstream firm i in the upstream game obtains as

$$z_i(z_{-i}; p) = \frac{\alpha - p - \alpha^2 \Sigma_{k \neq i} z_k}{2\alpha^2}, i = 1, ..., n. \tag{7}$$

We may compute the symmetric Nash equilibrium of the above game contingent on the price p. Defining $z_i = z$ for $i = 1...n$, re-expressing equation (**??**) and solving it in z, we get at the symmetric solution

$$z^*(p) = \frac{\alpha - p}{(n+1)\alpha^2}; \tag{8}$$

so that

$$q^* = \frac{\alpha - p}{(n+1)\alpha}. \tag{9}$$

Given a n-tuple $(s_1,..., s_j, ..s_m)$ of input strategies chosen by the upstream firms in the second stage game, the input price clearing the upstream market must satisfy

$$\frac{n(\alpha - p)}{(n+1)\alpha^2} = \Sigma_{k=1}^m s_k$$

so that, for this example, we get

$$p(\Sigma_{k=1}^m s_k) = \alpha - \alpha^2 \frac{n+1}{n} \Sigma_{k=1}^m s_k. \tag{10}$$

Substituting (**??**) into the payoff function $\Gamma_j(s_j, s_{-j})$ we have

$$\Gamma_j(s_j, s_{-j}) = \left(\alpha - \alpha^2 \frac{n+1}{n} \Sigma_{k=1}^m s_k\right) s_j - \beta s_j,$$

leading to the best response function

$$s_j(s_{-j}) = \frac{n(\alpha - \beta)}{2\alpha^2(n+1)} - \frac{(1+n)\alpha^2 \Sigma_{k \neq j} s_k}{2\alpha^2(n+1)},\ j = 1, ..., m.$$

Accordingly, at the symmetric equilibrium of the second stage game, we obtain

$$s^*(m,n) = \frac{n\,(\alpha - \beta)}{\alpha^2(n+1)(m+1)}.$$

Finally, the equilibrium price in the input market obtains as

$$p^*(m,n) = \frac{\alpha + m\beta}{m+1}. \tag{11}$$

Consequently, substituting this equilibrium price into the equilibrium quantities z_i^* of input bought by each downstream firm, as given by (**??**), we get

$$z^*(m,n) = \frac{m(\alpha - \beta)}{\alpha^2\,(n+1)\,(m+1)},$$

so that

$$q_i^*(m,n) = \frac{m(\alpha - \beta)}{\alpha\,(n+1)\,(m+1)}.$$

Accordingly, the resulting output price $\pi^*(m,n)$ in the downstream market obtains as[4]

$$\pi^*(m,n) = \frac{\alpha(1+m+n)+mn\beta}{\alpha\,(n+1)\,(m+1)}$$

3.2.1 Comparative statics

Taking the first derivative of the input and output prices and taking into account that $\alpha > \beta$, we get

$$\frac{\partial p^*}{\partial m} < 0 \text{ and } \frac{\partial \pi^*}{\partial m} < 0 :$$

under constant returns, an increase in the number of upstream firms leads to a decrease in the equilibrium input price which, in turn, induces downstream firms to produce more at equilibrium only if the marginal cost of producing the input is lower than the marginal productivity of the input at the downstream level.

The profit $\Pi_i(m,n)$ of a downstream firm at equilibrium in the downstream game writes as

$$\Pi_i(m,n) = \frac{m^2\,(\beta-\alpha)^2}{\alpha^2\,(n+1)^2\,(m+1)^2}.$$

It is easily seen that $\frac{\partial \Pi_i(m,n)}{\partial m} > 0$; consequently, in spite of the decrease in the output price stemmed from an increase in m, the profit of a downstream firm increases with m. In order to evaluate how the downstream firms' profits vary when both m and n are increased in the same proportion, we calculate the total derivative of $\Pi_i(m,n)$, that is

$$\frac{\partial \Pi_i(m,n)}{\partial m} + \frac{\partial \Pi_i(m,n)}{\partial n} = \frac{2\left(n-m-m^2+1\right)m\,(\beta-\alpha)^2}{\alpha^2\,(n+1)^3\,(m+1)^3}.$$

We see that the sign of the total derivative depends on the number of downstream and upstream firms. Whenever $\frac{1+n}{1+m} > m$ is satisfied, then a proportional increase in n and m causes an increase in the profit of a downstream firm.

Similarly, we get that

$$\Gamma_j(m,n) = \frac{(\beta-\alpha)^2\,n}{(n+1)\,(m+1)^2\,\alpha^2}$$

[4] Notice that, in order to have $\pi^*(m,n) \geq p^*(m,n)$, - the requirement needed to guarantee the survival of firms in the downstream market -, no condition on α is required.

where $\frac{\partial \Gamma_j(m,n)}{\partial n} > 0$, reflecting the fact that an increase in n generates an increase in input demand. As for the total derivative of $\Gamma_j(m,n)$, we get

$$\frac{\partial \Gamma_j(m,n)}{\partial m} + \frac{\partial \Gamma_j(m,n)}{\partial n} = \frac{m+1-2n^2-2n}{\alpha^2 (n+1)^2 (m+1)^3}.$$

We observe again that the sign of the total derivative depends on m and n. Whenever $\frac{1+n}{1+m} < \frac{1}{2n}$ is satisfied, then a proportional increase in n and m causes an increase in the profit of an upstream firm. Since the conditions $\frac{1+n}{1+m} < \frac{1}{2n}$ and $\frac{1+n}{1+m} > m$ are not compatible, the profit of an upstream firm should always decrease when the profit of a downstream firm increases, and *vice versa*, as m and n increase in the same proportion.

3.2.2 Asymptotic properties of input and output prices

Again consider successively the following assumptions:

1.Perfect competition

We compute

$$\lim_{k\to\infty,} \pi^*(km, kn) = \frac{\beta}{\alpha}$$

and

$$\lim_{k\to\infty,} p^*(km, kn) = \beta$$

Proposition 5 *Under constant returns, when both n and m tend to infinity, the equilibrium output price converges to its marginal cost, and similarly for the input price. Furthermore both prices converge to their competitive counterpart.*

2. Upstream competition and downstream oligopoly

We compute

$$\lim_{k\to\infty} \{p^*(km, n)\} = \beta$$

and

$$\lim_{k\to\infty} \{\pi^*(km, n)\} = \frac{\alpha + n\beta}{\alpha(1+n)}.$$

Therefore,

Proposition 6 *Under upstream competition and downstream oligopoly,*

(i) the equilibrium input price converges to the competitive price;

(ii) the equilibrium output price converges to the output price corresponding to the Cournot equilibrium with n downstream firms producing the good at cost β.

3. Downstream competition and upstream oligopoly
We compute

$$\lim_{n\to\infty}\{\pi^*(m,n)\} = \frac{\alpha+m\beta}{\alpha(1+m)}$$

and

$$\lim_{n\to\infty}\{p^*(m,n)\} = \frac{\alpha+m\beta}{1+m}$$

Therefore,

Proposition 7 *Under downstream competition and upstream oligopoly, the output price converges to the marginal cost of production when m firms are operating in the input market.*

Proof. The increase in cost following the production of a further unit of output is equal to $\frac{p}{\alpha}$, with p denoting the input price. We have seen in (7) that the input price when m upstream firms operate in the input market is equal to $\frac{\alpha+m\beta}{1+m}$. ■

To summarize, while the output price converges to $\frac{\alpha+n\beta}{\alpha(1+n)}$ under upstream pure competition, it tends to $\frac{\alpha+m\beta}{\alpha(1+m)}$ when downstream pure competition is the case ! Consequently, the degree of market power existing in the upstream (resp. downstream) market determines the discrepancy between the output competitive price and the actual output price.

Let us provide a brief summary of our findings concerning entry. This summary highlights the crucial role played by the degree of competition in the input market both on the demand and supply sides of the downstream market, as well as the importance of the technology used by downstream firms. *(i)* Entry in the upstream game generates lower prices for consumers in the downstream market, regardless of the type of technology used at the downstream level; *(ii)* entry in the upstream game always increases downstream firms' profits under constant returns; on the contrary, under decreasing returns, profits are higher under monopoly and duopoly while they are lower for oligopolies with a larger number of downstream firms; *(iii)* simultaneous entry in both markets, -i.e. higher m and n -, entails different effects on profits of upstream and downstream firms depending on the type of technology. Under constant returns, the sign of profits' variation depends on the ratio $\frac{n}{m}$; under decreasing returns, the sign of profits' variations only depends on the number of downstream firms, n; *(iv)* whatever the type of technology, the output price opposed to consumers is not equal to the marginal cost of producing the output, even when the number of downstream firms is infinitely large, *unless* the upstream market is itself competitive; *(v)* the input price at equilibrium always exceeds the marginal cost of producing the input and tends to infinity when the number of downstream firms becomes arbitrarily large.

All these results cannot be directly derived in a framework which does not take explicitly into account *(i)* the interlinkage between the downstream and upstream markets through the technology used by the output producers, and *(ii)* the degree of competition in each of these markets.

4 Exploring industry equilibria: collusion

4.1 Modelling collusion

For the sake of analyzing collusion in technology-linked markets, we restrict ourselves to constant returns technology. Assume that k downstream firms i, $i = 1, ..., k$, say, and h upstream firms $j, j = 1, ..., h$, say, integrate vertically and maximize joint profits. We assume that $k < n$ and $h < m$[5]. After this merger, we move from an initial situation comprising globally $n+m$ firms to a new one, with $n-k+1$ firms in the downstream market and $m-h$ in the upstream one[6]. Indeed, the integrated entity now internalizes output production by using the input provided by the h upstream firms belonging to the new entity. This general formulation covers as particular cases mergers including either only downstream firms, or only upstream ones, which correspond to the usual case of horizontal merging of firms.

Let us first consider the game played among the $n-k+1$ firms operating in the downstream market after collusion takes place[7]. The payoff of the integrated firm I is given by

$$\Pi_I(q_I, q_{-I}) = (1 - q_I - \sum_{k \neq I} q_k)q_I - \beta\frac{q_I}{\alpha}$$

As for the downstream firms $i, i \neq I$, not belonging to the integrated entity, they have as payoffs

$$\Pi_i(q_i, q_I, q_{-i}) = (1 - q_i - \sum_{k \neq i}^{n} q_k)q_i - p(\frac{q_i}{\alpha})^8. \qquad (12)$$

It is clear from the above payoffs that the main difference between the collusive and non collusive members in the downstream market comes from the fact that

[5]This assumption guarantees that there always exists at least one unintegrated firm on each side of the upstream market so that the integrated entity cannot exclude the uintegrated downstream firms to have access to the input. A similar asssumption in another approach to collusion has been used by Gabszewicz and Hansen (1971).

[6]Differently from Gaudet and Van Long (1996) who consider only pairwise mergers consisting of a single downstream and upstream firms, we allow for collusive agreements embodying an arbitrary number of them.

[7]Notice that, as in Salinger (1988), we assume complete foreclosure: the entity does not sell input to the unintegrated downstream firms.

the former pay their input at marginal cost β while the latter buy it at the input price p. Since Π_I is concave, we may use the first order condition to get the best response function of the integrated entity in the downstream market game as

$$q_I(q_{k\neq I}) = \frac{1-\frac{\beta}{\alpha}-\sum_{k\neq I} q_k}{2}.$$

As for the downstream firms i, $i \neq I$, their best reply in the downstream market is conditional on the input price p realized in the upstream market, namely

$$q_i(q_I, q_{-i}, p) = \frac{1-\frac{p}{\alpha}-\left(q_I+\sum_{k\neq i,k\neq I} q_k\right)}{2}.$$

Assuming a symmetric equilibrium *between the unintegrated firms*, we get the resulting Cournot equilibrium in the downstream market, namely

$$q_I^*(k,h) = \frac{\alpha-\beta+(n-k)(p-\beta)}{\alpha(n-k+2)}$$

and

$$q^* = q_i^*(p;k,h) = \frac{\alpha-2p+\beta}{\alpha\,(n-k+2)}. \tag{13}$$

Consequently, as expected, the downstream equilibrium is conditional on the input price obtained in the upstream market as a result of supply and demand in this market. There are $n-k$ firms with total demand equal to $\sum_{h\neq I} z_i(p) = (n-k)(\frac{\alpha-2p+\beta}{\alpha(n-k+2)})$. As for the supply, it comes from the strategies s_j, $j\neq I$, selected by the unintegrated upstream firms in this market. Consider the j_{th} upstream firm which does not belong to the entity. Its profit Γ_j at the vector of strategies (s_j, s_{-j}) writes as

$$\Gamma_j(s_j,s_{-j}) = p(s_j,s_{-j})s_j - \beta s_j,$$

with $p(s_j, s_{-j})$ such that $\sum_{k\neq I} s_k = \sum_{h\neq I} z_h(p)$, namely

$$p(s_j,s_{-j}) = \frac{(\alpha+\beta)(n-k)-\alpha(n-k+2)\sum_{k\neq I} s_k}{2\,(n-k)} \tag{14}$$

Accordingly, the payoff of the j-th upstream firm writes as

$$\Gamma_j(s_j,s_{-j}) = \left(\frac{(\alpha+\beta)(n-k)-\alpha(n-k+2)\sum_{k\neq I} s_k}{2\,(n-k)}\right)s_j - \beta s_j.$$

Therefore , at the symmetric equilibrium in the upstream market, each unintegrated firm supplies a quantity $s_j^*$of input which obtain as

$$s_j^*(k,h) = \frac{(\alpha-\beta)\,(n-k)}{\alpha(n-k+2)(m-h+1)}$$

Substituting the expression of s_j^* in (**??**) we get the equilibrium input price

$$p^*(k,h) = \frac{\alpha+\beta+2\beta(m-h)}{2\,(m-h+1)} \tag{15}$$

Substituting (**??**) in (**??**) we get the output supply of each unintegrated downstream firm, namely

$$q_i^*(k,h) = \frac{(m-h)\,(\alpha-\beta)}{\alpha(n-k+2)(m-h+1)}.$$

Similarly, we get

$$q_I^*(k,h) = \frac{(\alpha-\beta)(n-k+2(m-h+1))}{2\alpha(n-k+2)(m-h+1)}.$$

Hence, the resulting output price $\pi^*$is given by

$$\pi^*(k,h) = \frac{(2(m-h)+n-k+2)\,(\alpha+\beta)+2\beta(n-k)(m-h)}{2\alpha\,(n-k+2)\,(m-h+1)}.$$

For later use, we also compute the profit $\Pi_I(k,h)$ of the integrated firm

$$\Pi_I(k,h) = \frac{(\alpha-\beta)^2\,(2(m-h)+n-k+2)^2}{4\alpha^2\,(n-k+2)^2\,(m-h+1)^2}, \tag{16}$$

and the profit $\Pi_i(k,h)$ of an unintegrated downstream firm, namely

$$\Pi_i(k,h) = \frac{(\alpha-\beta)^2\,(m-h)^2}{\alpha^2(n-k+2)^2(m-h+1)^2}. \tag{17}$$

It is interesting to compare the input and output prices with and without collusive agreements. It is easy to show that the input price in the first case is lower than in the latter if, and only if, the number h of collusive upstream firms is smaller than half of the total number of upstream firms. Hence the price of the input is lower when the merger takes place, even though the number of firms supplying the input in the upstream market is smaller. Therefore, a collusive agreement involving a number of upstream firms smaller than half the total number does not bring a rise in the downstream rivals' costs *à la* Salop and Scheffman (1983, 1987). Nevertheless, it is easy to check that the *output* price is always smaller when no mergers take place, whatever the number of upstream firms inside the entity.

4.2 Horizontal mergers: Salant, Schwitzer and Reynolds revisited

In their 1983 paper, Salant, Schwitzer and Reynold (SSR) point out a bizarre result of horizontal mergers when firms play Cournot and produce a homogeneous good: "*some exogenous mergers may reduce the endogenous joint profits of the firms that are assumed to collude. In the Cournot case this is surprising since the merged firm always has the option of producing exactly as its components did in the pre-merger equilibrium*".

The model of SSR considers only one market where the good is traded and produced at a constant exogenous marginal cost. In this market, the consequences of mergers are determined by the strategic interaction of collusive firms and outsiders. Namely, the collusive firms internalize inframarginal losses between them; so, as a merged entity, they decrease their final output while the non colluding firms expand theirs. There exist then the possibility that the increase of production of outsiders may decrease the profit of collusive firms making it even smaller than the profit each (collusive) firm can get producing independently. SSR find also that *(i)* when the profit per firm in the entity is lower than the profit obtained without merger, higher the number of firms in the collusive entity, higher the loss from the merger; *(ii)* the merger that completely monopolizes the market is always profitable per firm, compared with the sum of per firm profits with no merger at all.

Do the same conclusions remain valid when we consider explicitely technology-linked markets? Indeed, profits or losses from horizontal mergers in the downstream market may well behave quite differently when making explicit the interaction with the upstream firms, *even when none upstream firm is participating to the merger.* We denote by $g(k,h)$ the increase in profit of a downstream firm that results if k and h firm collude, respectively in the downstream and upstream markets. Then, using equations (**??**) and (**??**) that refer to the constant returns technology (the one also used by SSR), we obtain

$$g(k,h)=\frac{(2(m-h)+n-k+2)^2-4\,(m-h)^2\,(h+k)}{4\,(n-k+2)^2\,(m-h+1)^2\,(h+k)}.$$

Isolating horizontal mergers by assuming $h=0$, we find that

$$\begin{aligned}\frac{\partial g(k,0)}{\partial k} &> 0;\frac{\partial g(0,h)}{\partial h}<0, \quad if\ m\geq n;\\ \frac{\partial g(k,0)}{\partial k} &\gtreqless 0;\frac{\partial g(0,h)}{\partial h}0 \gtreqless if\ m<n.\end{aligned}$$

Thus, contrary to SSR, the presence of the upstream linkage may generate situations in which losses do not appear at all. Take for instance, an industry composed of 6 downstream firms and 6 upstream ones. It can easily be checked that, if $h=0$, $g(k,h)>0$ for every k. In turns out that now *the existence of losses due to mergers depends on the number of upstream firms m in the economy compared with the number of downstream firms n.* Furthermore, when the profit per firm in the entity is lower than the profit obtained without merger, the property that the higher the number of firms in the collusive entity, the higher the loss from the merger *only holds* when $m \geq n$. In the opposite case, a higher number in the collusive entity may well entail a smaller loss from the merger.

On the other hand, the property that a merger, which would completely monopolize the market is always profitable per firm, compared with the sum of per firm profits with no merger at all, no longer holds when the upstream linkage is taken into account.Consider for example a industry with $m = 1$ and $n = 6$. It can be checked that any horizontal merger up to 5 downstream firms gives profits. The only merger that gives losses to collusive firms is the total merger of the downstream firms. Thus, in this example, the *only* merger producing losses is the merger that SSR claim to give always profits- the monopoly merger.

Finally, differently from SSR, $g(k, 0)$ is not convex. Hence, if a merger of k firms causes losses, a merger of $k+1$ firm may generates profits. For example, in a industry of 12 firms producing the output and 12 firms producing the input, a merger of 6 downstream firms causes gains but not a merger of 7 downstream firms.

To conclude, introducing the upstream linkage considerably modifies the effects of horizontal merging as analyzed by Salant, Schwitzer and Reynolds.

5 Conclusion

In this paper, we have tried to clarify how entry and collusion affect successive markets when the technology linking these markets is made explicit. We have differentiated the effects of entry in these markets according to the nature of the technology : constant and decreasing returns, making explicit several properties which differ in both cases. Moreover, we have highlighted the role of upstream linkage on the profitability of horizontal mergers *à la* Salant, Switzer and Reynolds.

Our exploration of industry equilibria deserves to be continued. First, as in the existing literature, we have kept the assumption of price taking agents in the demand side of the markets. This assumption is not very satisfactory because it is difficult to justify the fact that an economic agent behaves strategically in one market but not in another. A full treatment would require downstream firms behaving strategically simultaneously in the downstream and upstream markets. This constitutes our next point on our research agenda. In particular, it would be interesting to examine whether the effects of entry which are specific to each type of technology would still be observed with more sophisticated downstream firms, behaving strategically in both markets. Also, even if welfare implications of entry and collusion can be derived from our framework, they were not our main concern in this paper. This does not mean that they should not deserve more attention in future work. Another avenue for potential research would consist in analysing the stability of collusive agreements, as in d'Aspremont *et al.(1983),* using the framework identified in the present paper. Finally, as claimed above at the end of the presentation of the model, the analysis could be extended to chains of technology-linked markets and to technological contexts involving more than one factor. All this looks like a promising research territory for a better understanding of industry equilibria in technology linked markets.

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