Fear of ruin and longevity enhancing investment

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Abstract

Rectangularization of the survival probability seems to be an ongoing process. It results from a higher concentration of the ages at death; but it can be reversed by a continuous increase in the limit of life time. In this paper, we assume that these two factors are endogenous and we show that risk averse decision makers exhibit a bias towards rectangularization. More specifically, the importance of the bias depends upon the intensity of the "fear of ruin" which is another measure of the degree of absolute risk aversion.

Keywords: longevity, fear of ruin.

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1   Introduction

There is not much debate as to the idea that increases in life expectancy will continue in the near future. There is however much debate on two related issues. Are we going to observe an increasing rectangularization of the survival curve or rather a parallel shift of this curve to the right? What are in private actions those which favor one kind of displacement of the survival curve or the other?

To formalize these displacements it is convenient to focus on two parameters: one representing the limit of human life span and one representing the probability of reaching that age limit after a certain age. If this probability increases we get an increasingly rectangular survival curve. If, instead, the age limit increases, we have a parallel shift of the survival curve. In empirical studies the question of rectangularization is often cast in terms of the relative decrease of the mortality rate at 60 and at 90. A decrease at 60 implies rectangularization and a decrease at 90 a parallel shift to the right.

The distinction between those two ways of increasing longevity raises interesting normative problems. To take an example, we assume that a representative individual lives for sure till 50 and at 50 he has a probability 1/2 of dying immediately and a probability 1/2 to live up to the age limit of 90. This implies a longevity of 70. Now he has the possibility of increasing this longevity by 10 years either by aiming at an age limit of 110 with the same survival probability (1/2) or at a survival probability of 0.75 with the same age limit. What is he going to choose? The purpose of this letter is to show that under reasonable assumptions about the utility function and the technology, risk averse decision makers will tend to favor the increase in the survival probability (and hence rectangularization). In this framework, risk aversion is measured through the concept of "fear of ruin" which was first proposed by Aumann and Kurz (1977) and was recently analyzed in depth by Foncel and Treich (2005).

Our letter is organized as follows. We first describe the technology available to the decision maker as well as his utility function. Then we analyze his optimal choices and present the main result of the letter. In a third section we discuss the intuition behind the result through a comparison with the related concepts of self protection and self insurance as defined by Ehrlich-Becker (1972) in a unidimensional context.

2   The technology and the preferences

Each individual is assumed to live for sure during the first period, which has a length time normalized to 1. He faces at the begining of the second period a probability of death denoted π. If he survives at the begining of the second period, he will live for a time length denoted h with $h \leqslant 1$.

Both π and h can be influenced by choices made at the beginning of the first period. We denote by x efforts that are undertaken to increase π and, as is usual, we assume that $\pi'(x)$ is positive while $\pi''(x)$ is negative. Similarly, y are efforts chosen ex ante to increase h and we have $h'(y)$ positive and $h''(y) \leqslant 0$.

To illustrate the meaning of x and y, x corresponds to membership in one (or more) fitness club(s) or to the choice of an expensive diet. Both activities reduce the probability of death at a later age. The investment in y can be thought as corresponding to a frequency of screening procedures (mammographies, etc.) that make possible an early diagnosis and more efficient treatment of a disease so that life time can be extended.

Besides the investment choices x and y made at the beginning of the first period, the individual has also to decide upon his saving level s. Given his earnings available at the beginning of the first period and denoted w, his current consumption of the first period c is expressed as:

$$c \equiv w - s - x - y \tag{1}$$

In the second period his consumption level d is determined by the rate of return of the savings that had been invested in an annuity market which is more or less fair.

The rate of return of this annuity is given by $R = \dfrac{1+r}{(\pi h)^{\alpha}}$ where r is the rate of interest and $\alpha \in (0,1)$ reflects the fairness of the annuity. With $\alpha = 0$, there is no annuity and with $\alpha = 1$, the annuity is actuarially fair. Another issue is whether the individual when choosing x and y sees the effect of his choice on the annuity returns. (See Becker and Phillipson (1998)). In any case, in this note, we take R as given.

The utility function of the decision maker in each period (u) has two arguments: consumption and length of life. His intertemporal expected utility U is then given by:

$$U = u(w - s - x - y, 1) + \pi(x) u(sR, h(y)). \tag{2}$$

If the individual dies at the beginning of the second period, his current utility is equal to zero (because there is no bequest motive). Then by

symmetry, when $h(y)$ tends to zero, $u(sR, h(y))$ also tends to zero so that necessarily

$$u(d, 0) = 0. \tag{3}$$

Such an assumption that will turn out to be crucial in the next section is very reasonable. Indeed, when h tends to zero, it is equivalent to death at the beginning of the second period so that U tends to zero.

We assume that each first derivative of u is strictly positive while each direct second derivative is non positive. As a benchmark case to be developed in section 3, we will consider the special case where

$$u(d, h) = u(d) \cdot h. \tag{4}$$

In this special case the second derivative of u with respect to h (denoted u_{22}) is equal to zero and the decision maker is risk neutral with respect to lotteries on survival time.

3   The main result

The first order conditions (FOC) with respect to s, x and y associated to the objective function defined in (2) are given by:

$$-u_1(c, 1) + \pi(x) R u_1(d, h(y)) = 0 \tag{5}$$

$$-u_1(c, 1) + \pi'(x) u(d, h(y)) = 0 \tag{6}$$

$$-u_1(c, 1) + \pi(x) h'(y) u_2(d, h(y)) = 0. \tag{7}$$

Since we are going to focus exclusively on the link between x and y, we take s as given.

With the benchmark case

$$u(d, h(y)) = u(d) \cdot h(y), \tag{8}$$

equations (3) and (4) become:

$$-u_1(c, 1) + \pi'(x) u(d) h(y) = 0 \tag{9}$$

$$-u_1(c, 1) + \pi(x) u(d) h'(y) = 0 \tag{10}$$

and at the optimum:

$$\frac{\pi'(x)}{\pi(x)} = \frac{h'(x)}{h(y)}. \tag{11}$$

3

This optimal rule corresponds to the behavior of a decision maker who maximizes his expected life time[1] for a given budget $(w - s)$ to be spent on x and y. Indeed

$$Max \quad \pi(x) h(y)$$
$$s.t. \quad x + y \leqslant w - s$$

yields as an optimum precisely $eq(11)$. Hence, if the utility function is multiplicative and linear in h as in (8), the individual cares only about his expected survival.

Let us now return to the more general case. Comparison of equations (6) and (7) yields:
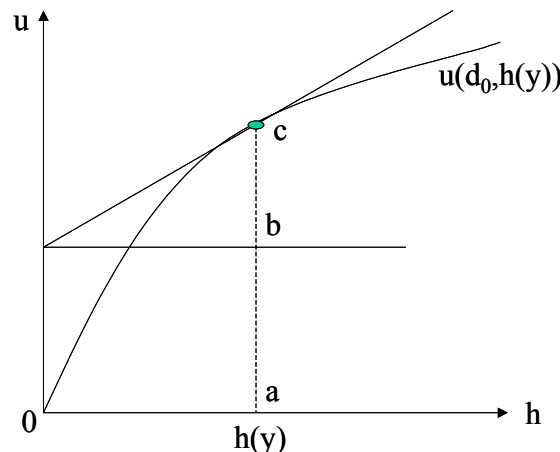
$$\frac{\pi'(x)}{\pi(x)} = \frac{h'(y)}{h(y)} \cdot \frac{u_2(d, h(y)) \cdot h(y)}{u(d, h(y))}. \tag{12}$$

Hence we obtain

$$\frac{\pi'(x)}{\pi(x)} \lessgtr \frac{h'(y)}{h(y)} \quad \text{whenever} \quad \frac{u_2 \cdot h}{u} \lessgtr 1 \tag{13}$$

where $\frac{u_2}{u}$ is the inverse of the "fear of ruin" (denoted FR) with respect to $h(y)$, the length of life.[2] As shown by Foncel and Treich, FR is an index of risk aversion.

When utility is concave in $h(y)$ and when $u(d, 0) = 0$ (for reasons indicated before) it is easily shown that $\frac{u_2 \cdot h}{u} < 1$ as can be seen from Figure 1.

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[1] Indeed expected life time $= \pi(x) h(y)$.

[2] For a utility function $u(w)$ such that $u(0) = 0$, FR at $w > 0$ is defined by $\frac{u(w)}{u'(w)}$. For more details see Foncel and Treich.

Figure 1

In Figure 1 and for a given d (denoted d_0) $u_2 \cdot h = bc$ while $u = ac$ so that, because of the concavity of u, $\dfrac{u_2 h}{u} < 1$.

As a result, when the decision maker becomes risk averse with respect to lotteries on survival time, one has:

$$\frac{\pi'(x^*)}{\pi(x^*)} < \frac{h'(y^*)}{h(y^*)} \tag{14}$$

and because of the assumption made about the technology this implies inside a budget constraint that x^* increases and y^* falls. (See appendix 1).

Notice that when risk aversion (i.e. here FR) increases, $\dfrac{u_2 h}{u}$ falls and x^* goes on increasing.

4   The intuition

To justify the result obtained above, a comparison with the concepts of self-protection and self-insurance as proposed by Ehrlich and Becker (1972) (henceforth E-B) is useful.

Clearly in our model increasing x induces exactly the same effect as self-protection activities described in E-B. Indeed, in both cases, more effort increases the probability of occurence of the best outcome (no financial loss in E-B, no death at the end of the first period here). As is well known also, the variance of the final outcome (total wealth in E-B, survival length here) is non-monotonic in π and behaves as indicated in Figure 2.
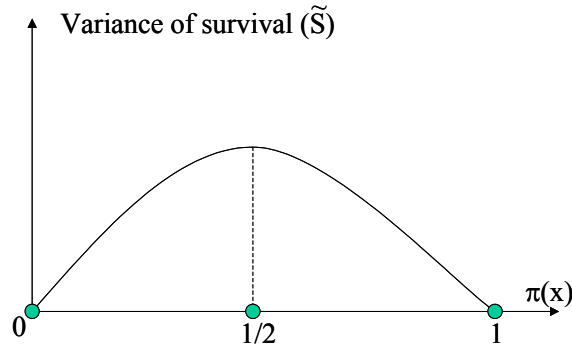

Figure 2

While x in our model plays the same role as self protection in E-B, matters are quite different for y. Self-insurance in E-B corresponds to efforts made

in order to reduce the financial loss if it occurs, i.e. in the bad state. In the longevity model however y is an effort that increases the benefit $(h(y))$ in the good state. As a result while self-insurance in E-B's world reduces the variance of the outcome (final wealth), investment y in the longevity model increases the variance of survival. Indeed in this paper, $\tilde{S}$ is characterized by
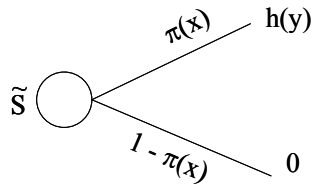
so that $E\left(\tilde{S}\right) = \pi(x)h(y)$ and

$$Var\left(\tilde{S}\right) = (h(y))^2 \cdot (\pi(x))(1 - \pi(x)). \tag{15}$$

Hence $Var\left(\tilde{S}\right)$ is an increasing and convex function in h. Consequently, increases in y affect $Var\left(\tilde{S}\right)$ much more significantly than do increases in x. As a consequence, risk averse decision makers in the longevity model will systematically prefer increases in x relative to increases in y, which explains their preference for rectangularization.

5   Conclusion

In this paper we use the concept of fear of ruin to explain why free-willing individuals would prefer to increase their longevity in the direction of rectangularization of their survival curve instead of an upward shift of the curve. Our paper is related to Bommier (2006) who rejects the traditional life-cycle models (here the benchmark case) because they imply risk neutrality with respect to length of life. He adopts a formulation (more general than ours) in which time discounting is directly related to preference over length of life.

References

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Appendix 1

Since $\pi(x)$ is increasing and concave in x

$$\frac{d}{dx}\left(\frac{\pi'(x)}{\pi(x)}\right) = \frac{\pi\pi" - (\pi')^2}{\pi^2} < 0 \qquad (A.1.1)$$

and a similar property holds for $h(y)$:

$$\frac{d}{dy}\left(\frac{h'(y)}{h(y)}\right) < 0 \qquad (A.1.2)$$

As we know, under risk aversion, $\dfrac{\pi'(x)}{\pi(x)}$ must be smaller than $\dfrac{h'(y)}{h(y)}$ (see eq(14)) while they were equal under risk neutrality (eq(12)). Besides, because of the budget constraint x and y must move in opposite directions. Hence, since risk aversion requires a fall in $\dfrac{\pi'(x)}{\pi(x)}$ relative to $\dfrac{h'(y)}{h(y)}$, this result can be obtained only with an increase in x and a reduction in y.

8