Optimal redistribution with unobservable disability: welfarist versus non-welfarist social objectives

Pierre PESTIEAU[1] and María RACIONERO[2]

[1] CREPP, Université de Liège, CORE, PSE and CEPR.
[2] School of Economics, Australian National University, Australia.

Abstract

This paper examines the optimal non linear income and commodity tax when the same labor disutility can receive two alternative interpretations, taste for leisure and disability, but the disability is not readily observable. We compare the optimal policy under alternative social objectives, welfarist and non-welfarist, and conclude that the non-welfarist objective, in which the planner gives a higher weight to the disutility of labour of the disabled individuals, is the only reasonable specification. It has some foundation in the theory of responsability; further, unlike the other specifications it yields an optimal solution that may involve a lower labour supply requirement from disabled individuals.

1 Introduction

In a couple of papers [Marchand *et al.* (2003), Cremer *et al.* (2006)] the point was made that there are two ways of interpreting the disutility of labor in the standard optimal income taxation model. Either it represents some taste for leisure or it reflects the physical or psychological pain from work. In other words, in a two-good framework (disposable income and labor) the same utility function can represent the preferences of two different individuals: a leisure-prone one and a disabled one. If this is the case, a social planner who wants to distinguish them, more precisely discriminate in favor of the disabled, might be unable to do so in a setting of asymmetric information. In Marchand *et al.* (2003) the way out was to introduce a vector of consumption goods and posit that the two types of individuals behave differently towards some particular goods such as leisure goods or health-related goods. In Cremer *et al.* (2006) an audit technology was introduced allowing to sort out the two kinds of aversion to effort.

The fundamental question one faces with this issue of indistinguishability is what welfare criterion to use. More specifically, should one use a welfarist approach giving more weight to the disabled individuals or should one use a "paternalistic" non-welfarist approach? In this paper we argue that the correct approach is the second one and that it can be justified on the basis of the distinction made by Roemer (1998) between responsibility and luck.

Accordingly, individuals' outcomes may differ due partly to choice and partly to circumstances beyond their control, or luck. Roemer's view is that we should correct for differences across individuals that are a matter of luck, while preserving differences for which individuals are responsible. Two difficulties generally arise at this point. First, what is luck and what is responsibility? Second, how practically distinguish them? We will follow Roemer's approach and consider poor health as the result of bad luck and taste for leisure as the result of deliberate choice. Further, in order to take into account

differences in health and ignore differences in taste for leisure in the social criterion, we will use an average parameter for the latter.

The rest of the paper is organized as follows. In the next section we examine the non-welfarist approach, whereas in section 3 we present the welfarist approach, including the Rawlsian criterion as an extreme case, and we show why it does not provide desirable results. A final section concludes.

2   Non-welfarist approach

Let us consider a particular separable type of Mirrlees' utility function:

$$u\left(x,\ell\right) = u\left(x\right) - \alpha h\left(\ell\right) \tag{1}$$

where $u\left(\cdot\right)$ and $h\left(\cdot\right)$ are continuous, differentiable, strictly increasing and, respectively, concave and convex. x is disposable income, ℓ is labor supply and α is a parameter measuring the intensity of labor disutility. We can interpret $\alpha h\left(\ell\right)$ in two different ways. It can be viewed as measuring the physical or psychological hardship from working ℓ hours, or as the utility the individual gets from enjoying $\left(\overline{\ell} - \ell\right)$ hours of leisure, $\overline{\ell}$ being the total available time. One could encompass these two aspects by rewriting the utility function as:

$$u\left(x,\ell\right) = u\left(x\right) + \varphi_L\left(\overline{\ell} - \ell\right) - \varphi_H\left(\ell\right)$$

where $\varphi_L\left(\cdot\right)$ is the utility of leisure and $\varphi_H\left(\cdot\right)$ the disutility of effort. A particular form of this function could be:

$$u\left(x,\ell\right) = u\left(x\right) - \left(\alpha_L + \alpha_H\right) h\left(\ell\right)$$

where α_L is the leisure parameter and α_H the health parameter. From this expression, it is clear that two individuals can have the same $\alpha = \alpha_L + \alpha_H$ but different combinations of α_L and α_H.

Following Roemer (1998) we treat α_L as a choice variable and α_H as a variable stemming from luck. Consider first two types of individuals who have the same α, but different combinations of α_L and α_H. Type 1 is said to be disabled with a high α_H^1, relative to α_L^1, and type 2 is said to be leisure-prone with a high α_L^2, relative to α_H^2.

In their individual choice of ℓ both will use the same utility function (1). However the social planner may decide to take into account the difference in α_H but to ignore the difference in α_L, choosing to employ the same value of α_L for everyone, e.g. α_L^2. As a consequence, the objective function of the social planner would be:

$$n_1 \left[u\left(c_1\right) - \left(\alpha_H^1 + \alpha_L^2\right) h\left(\ell_1\right) \right] + n_2 \left[u\left(c_2\right) - \left(\alpha_H^2 + \alpha_L^2\right) h\left(\ell_2\right) \right] \tag{2}$$

where n_1 and n_2 are the relative number of types 1 and 2. Without loss of generality we can normalize these parameters such that $\alpha_H^1 + \alpha_L^2 = \beta_1 > 1$ and $\alpha_H^2 + \alpha_L^2 = \beta_2 = 1$.

The table below explains our normalization.

Type	Taste for leisure	Disutility of effort	Aggregate value for the individuals	Aggregate value for the social planner
1	$2-\beta$	$\beta - 1$	1	$\beta > 1$
2	1	0	1	1

So doing we clearly adopt a non-Paretian approach, which is at odds with the view of Fleurbaey and Maniquet (2005a,b) but not of Bossert and Van de Gaer (1999) or Schokkaert *et al.* (2004). These authors deal with the distinction between responsibility and luck by using a paternalistic view for the valuations of leisure.

This approach is also close to that used in behavioral economics when the social planner does not use, in its objective function, individual preferences but its own preferences for e.g. sin goods.[1]

Following Marchand *et al.* (2003) we consider also a third type of individual with higher productivity than types 1 and 2 but the same combination of α_L and α_H as type 2. The society consists then of three types of individuals: disabled and able low-productivity individuals, and high-productivity able individuals. We denote them by 1, 2 and 3, respectively. The crucial point is that the three types have the same formal labor disutility α but type 1 has a relatively higher α_H and, according to (2), the social planner attaches a higher weight to her labor disutility.

In addition, we consider the case where disabled and able individuals may use their disposable income, which was denoted above by x, in different ways. In order to capture this idea, we use the following utility function:

$$U_i = c_i + v\left(d_i - \overline{d}_i\right) - h\left(\ell_i\right) \qquad i = 1, 2, 3, \tag{3}$$

[1] O'Donoghue and Rabin (2003) and Kanbur *et al.* (2006).

where c_i and d_i represents two types of consumption goods. d_i is a good for which disabled individuals have relatively higher needs (like health care). Accordingly, we posit $\overline{d}_1 = \overline{d} > \overline{d}_2 = \overline{d}_3 = 0$. The utility function is quasi-linear (linear in consumption c_i), $v(\cdot)$ and $h(\cdot)$ are continuous, differentiable and strictly increasing functions. $v(\cdot)$ is strictly concave and $h(\cdot)$ is strictly convex.[2]

In a market economy, each individual maximizes:

$$U_i = (w_i\,\ell_i - d_i) + v\left(d_i - \overline{d}_i\right) - h\left(\ell_i\right),$$

where constant per-unit costs of production are assumed with one unit of effective labor being necessary to produce one unit of either good. w_i represents the productivity of individual i, where $w_1 = w_2 = w_\ell$ and $w_3 = w_h$, with $w_h > w_\ell$ The disposable income $y_i = w_i\,\ell_i$ is devoted to consumption of goods c_i and d_i. Clearly, $\ell_1 = \ell_2 < \ell_3$, $d_1 - \overline{d} = d_2 = d_3$ (hence, $d_1 > d_2 = d_3$) and $c_1 < c_2 < c_3$, where $c_1 = c_2 - \overline{d}$. Note that, in terms of income, $y_1 = y_2 < y_3$. That is, both disabled and able low-productivity individuals work and earn the same. The only difference is that they use their disposable income differently. Since disabled individuals have higher needs of commodity d, they enjoy lower consumption of commodity c than able ones. This may seem unfair and may provide some role for redistribution from able to disabled individuals.

As a benchmark let us look at the first-best solution. Given the quasi-linearity of individual utilities, we use a strictly concave social utility transformation $G(\cdot)$ that reflects aversion towards inequality. As just discussed the social planner acknowledges that the labor disutility of disabled individuals (i.e., type 1) does not have the same social cost as the labor disutility of the two other types and, accordingly, the labor disutility of disabled individuals is weighted more heavily in the social welfare function than that of able ones.

The problem of the planner is expressed now by the following Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{3} n_i \left[G\left(c_i + v\left(d_i - \overline{d}_i\right) - \beta_i h\left(\ell_i\right)\right) + \lambda\left(w_i\,\ell_i - c_i - d_i\right)\right],$$

where n_i is the relative number of individuals of type i, and β_i is the weight attached to the labor disutility of individuals of type i, with $\beta_1 = \beta > 1 = \beta_2 = \beta_3$. In what

---

[2] We choose a quasi-linear (linear in c) specification for simplicity. The qualitative results presented in the paper concerning the desirability of the non-welfarist approach over the welfarist approach in a SB setting with asymmetric information extend to a more general separable utility function.

follows, we will denote the argument of G by $\widetilde{U}_i$, which differs from U_i when the weight β_i differs from 1.

The first order conditions (hereafter FOCs) yield:

$$
\begin{aligned}
G'\left(\widetilde{U}_i\right) &= \lambda \quad \text{and thus} \quad \widetilde{U}_1 = U_2 = U_3; \\
v'\left(d_1 - \overline{d}\right) &= v'\left(d_2\right) = v'\left(d_3\right) = 1 \quad \text{and thus} \quad d_1 - \overline{d} = d_2 = d_3; \\
\beta \frac{h'\left(\ell_1\right)}{w_\ell} &= \frac{h'\left(\ell_2\right)}{w_\ell} = \frac{h'\left(\ell_3\right)}{w_h} \quad \text{and thus} \quad \ell_1 < \ell_2 < \ell_3.
\end{aligned}
$$

In contrast to the *laissez-faire*, disabled individuals work less than able ones. This follows from the fact that the planner attaches more weight to their disutility of labor.

To achieve the first-best solution perfect observability of individuals' characteristics is required. We now consider a second-best setting where the planner is able to observe consumption levels of c_i and d_i (although not needs $\bar{d}_i$), together with labor earnings y_i (that is, $y_i = w_i \ell_i$, but not w_i and ℓ_i separately). As usual, we express the utility function in terms of the variables the planer is able to observe (i.e., c_i, d_i and y_i):[3]

$$
U_i\left(c_i, d_i, y_i\right) = c_i + v\left(d_i - \bar{d}_i\right) - h\left(\frac{y_i}{w_i}\right).
$$

The first-best solution is no longer feasible because high-productivity individuals have incentives to mimic both low-productivity ones and, among low-productivity individuals, able individuals have incentives to mimic disabled ones.

In the second-best setting the social planner needs to take into account self-selection constraints (hereafter SSCs) in order to prevent individuals of a given type from applying for the tax-treatment designed for individuals of other types. Appendix A identifies the SSCs that need to be incorporated in the present framework. The second-best problem is then:

$$
\max_{\{c_i, d_i, y_i\}} \quad \sum_{i=1}^{3} n_i G\left[c_i + v\left(d_i - \bar{d}_i\right) - \beta_i h\left(\frac{y_i}{w_i}\right)\right]
$$

---

[3] This is standard use. In the second-best framework it is convenient to express the problem in terms of the variables the social planner can observe (see Stiglitz (1982)).

s.t.

$$(\mu) \quad : \quad \sum_{i=1}^{3} n_i \left(y_i - c_i - d_i \right) \geq 0$$

$$(\lambda_i) \quad : \quad c_i + v\left(d_i\right) - h\left(\frac{y_i}{w_i}\right) \geq c_{i-1} + v\left(d_{i-1}\right) - h\left(\frac{y_{i-1}}{w_i}\right), \quad i = 2, 3$$

$$(\lambda_{31}) \quad : \quad c_3 + v\left(d_3\right) - h\left(\frac{y_3}{w_h}\right) \geq c_1 + v\left(d_1\right) - h\left(\frac{y_1}{w_h}\right)$$

where μ is the Lagrange multiplier associated with the budget constraint, λ_i (for $i = 3, 2$) stand for the Lagrange multipliers associated with the adjacent downwards SSCs, namely 3 mimicking 2 and 2 mimicking 1, and λ_{31} is the Lagrange multiplier associated with the non-adjacent downwards SSC, namely 3 mimicking 1.

Rearranging the FOCs, given by equations (B.1.1) to (B.1.9) in appendix B, yields:

$$v'\left(d_3\right) = h'\left(\frac{y_3}{w_h}\right) \frac{1}{w_h} = 1, \tag{4}$$

$$v'\left(d_2\right) = \frac{\lambda n_2}{n_2 G_2' + \lambda_2 - \lambda_3} = 1, \tag{5}$$

$$h'\left(\frac{y_2}{w_\ell}\right) = w_\ell \left[1 - \frac{\lambda_3}{n_2 G_2' + \lambda_2} \left(1 - h'\left(\frac{y_2}{w_h}\right) \frac{1}{w_h} \right) \right] \leq w_\ell, \tag{6}$$

$$v'\left(d_1 - \bar{d}_1\right) - 1 = \frac{\lambda_2 + \lambda_{31}}{n_1 G_1'} \left(v'\left(d_1\right) - 1 \right), \tag{7}$$

$$h'\left(\frac{y_1}{w_\ell}\right) = w_\ell \left[1 - \frac{n_1 G_1' \left(\beta - 1\right) + \lambda_{31} \left(1 - h'\left(\frac{y_1}{w_h}\right) \frac{1}{w_h} \right)}{\beta n_1 G_1' - \lambda_2} \right] < w_\ell, \tag{8}$$

$$\mu = \frac{\sum_{i=1}^{3} n_i G_i'\left(\widetilde{U}_i\right)}{\sum_{i=1}^{3} n_i}. \tag{9}$$

The second-best levels of y_3, d_3 and d_2 coincide with the first-best ones. There is no efficiency gain in distorting the choices of the high-productivity individuals, nor the choice of commodity d by the low-productivity able individuals. There is an efficiency gain in distorting the labor supply decision of type 2 individuals if by doing so we prevent high-productivity individual from mimicking them. It can indeed be shown that the SSC that prevents type 3 from mimicking type 2 individuals binds, and $y_2^{SB} < y_2^{FB}$. In addition, d_1 is higher and ℓ_1 is smaller than in the first-best. It is optimal to distort the choice of commodity d of disabled individuals, so that they consume more than in

the first-best, and there are now two reasons why disabled individuals may be induced to work less: on the one hand, there is the concern of the planner for the labor disutility of this particular type (i.e., $\beta > 1$); on the other, it might be necessary to prevent high-productivity individuals from mimicking them.

The relationship between y_1 and y_2 (and, thus, between ℓ_1 and ℓ_2) is in principle ambiguous. A further analysis of the pattern of binding SSCs - see appendix B.1 for more detail - yields the following results. The three SSCs can only be simultaneously binding if $y_1^{SB} = y_2^{SB} < y_2^{FB} = y_1^{FB}$. If the SSCs that prevent high-productivity individuals from mimicking both low-productivity ones are binding, then it is possible that the downwards SSC that relates both low-productivity individuals holds with strict inequality if $y_1^{FB} > y_1^{SB} > y_2^{SB}$. If the adjacent downwards SSCs are binding, then it is possible that the non-adjacent downwards SSC holds with strict inequality if $y_2^{SB} > y_1^{SB}$. This last result seems particularly fair because the social planner wants the disabled individuals to work less than the able ones.[4] It can be shown that if $\bar{d}_1$ is sufficiently high and β sufficiently different from 1, the vector (c_1, d_1, y_1) that is not to be mimicked by type 2 individuals will include $y_1 < y_2$. This is pretty intuitive.

As it is standard, the optimal allocation resulting from solving the above problems can be decentralized by means of tax/subsidy schedules. We have so far presented the problem as if the planner confronted the individual with a choice of three bundles. Each bundle is composed by three terms: consumption of commodities c and d, and labor earnings y. The tax function must pass through the points $\{c_i, d_i, y_i\}$ for $i = 1, 2, 3$; and elsewhere must lie below the indifference curves through $\{c_i, d_i, y_i\}$. Given such a tax schedule, individual i will clearly choose the point $\{c_i, d_i, y_i\}$. As in Marchand *et al.* (2003), the tax system would consist of a combination of taxes/subsidies on labor earnings and on commodity d. The tax schedule with non-linear income and non-linear commodity taxes is $T_i = T(y_i) + t(d_i)$, where $T(.)$ and $t(.)$ are assumed to be differentiable. Labor earnings are devoted to the consumption of commodities c and d, and to the payment of income and commodity taxes. The following expressions

---

[4] $n_2 \geq n_3$ is a sufficient, although not necessary, condition for this case to hold (see condition in appendix B.1).

provide the marginal tax rates on income and commodity d, respectively:

$$
\begin{aligned}
T' &= 1 - \frac{1}{w_i} h'\left(\frac{y_i}{w_i}\right), \\
t' &= v'\left(d_i - \bar{d}_i\right) - 1.
\end{aligned}
$$

3   Welfarist approach

We have just seen that for the problem at hand the non-welfarist approach we use can be justified by the theory of responsibility. Furthermore it implies that in the second-best solution the disabled could be induced to work less than the leisure-prone individuals.

Yet, many economists have some problems with such an approach. They tend to prefer a welfarist approach wherein the disabled individuals would be given more weight than the two other types of individuals. We turn to this approach and show its implications in both the first- and second-best. The *laissez-faire* solution naturally does not change. Regarding the first-best, the problem of the planner is expressed by the following Lagrangian:

$$
L = \sum_{i=1}^{3} \alpha_i n_i G\left[c_i + v\left(d_i - \bar{d}_i\right) - h\left(\ell_i\right)\right] + \mu \sum_{i=1}^{3} n_i \left(w_i\,\ell_i - c_i - d_i\right),
$$

where α_i represents the weight given to individuals of type i. These weights are assumed to be non-increasing with i.

The FOCs yield:

$$
\begin{aligned}
\alpha_i G'_i &= \mu \text{ for all } i, \\
v'\left(d_1 - \overline{d}\right) &= v'\left(d_2\right) = v'\left(d_3\right) = 1 \text{ and thus } \quad d_1 - \overline{d} = d_2 = d_3, \\
\frac{h'\left(\ell_1\right)}{w_\ell} &= \frac{h'\left(\ell_2\right)}{w_\ell} = \frac{h'\left(\ell_3\right)}{w_h} \text{ and thus } \quad \ell_1 = \ell_2 < \ell_3.
\end{aligned}
$$

The equations for d and ℓ coincide with those obtained for the *laissez-faire*. In particular, disabled individuals are required to work the same amount as able ones. In order to determine c, note that:

$$
G'_i = \frac{\lambda}{\alpha_i} \text{ for all } i.
$$

The higher the weight α_i, the lower G'_i and, since $G\left(.\right)$ is concave, higher utility $c_i + v\left(d_i - \bar{d}_i\right) - h\left(\ell_i\right)$. When the social planner weighs more heavily the utility of the

disabled individuals (i.e., $\alpha_1 > \alpha_2$),

$$c_1 + v\left(d_1 - \bar{d}_1\right) - h\left(\ell_1\right) > c_2 + v\left(d_2\right) - h\left(\ell_2\right).$$

Since $d_1 - \bar{d}_1 = d_2$ and $\ell_1 = \ell_2$, we obtain $c_1 > c_2$.

This stems from the quasi-linear utility specification. The weight given to individual 1 is not reflected by a lower labor supply but by a higher level of consumption of good c. With a more general separable utility function $u\left(c\right) + v\left(d - \bar{d}\right) - h\left(\ell\right)$, ℓ_1 would be lower than ℓ_2.

It is worth noting that the Rawlsian criterion would amount to equalize the utilities of the three types, with $\ell_1 = \ell_2$.

Turning to the second-best, the problem of the social planner can be expressed as follows:

$$\max_{\{c_i, d_i, y_i\}} \quad \sum_{i=1}^{3} \alpha_i n_i G\left[c_i + v\left(d_i - \bar{d}_i\right) - h\left(\frac{y_i}{w_i}\right)\right]$$

s.t.

$$(\mu) \quad : \quad \sum_{i=1}^{3} n_i \left(y_i - c_i - d_i\right) \geq 0$$

$$(\lambda_i) \quad : \quad c_i + v\left(d_i\right) - h\left(\frac{y_i}{w_i}\right) \geq c_{i-1} + v\left(d_{i-1}\right) - h\left(\frac{y_{i-1}}{w_i}\right), \quad i = 2, 3$$

$$(\lambda_{31}) \quad : \quad c_3 + v\left(d_3\right) - h\left(\frac{y_3}{w_h}\right) \geq c_1 + v\left(d_1\right) - h\left(\frac{y_1}{w_h}\right).$$

Rearranging the FOCs, given by equations (B.2.1) to (B.2.9) in appendix B, we obtain:

$$v'\left(d_3\right) = h'\left(\frac{y_3}{w_h}\right)\frac{1}{w_h} = 1, \tag{10}$$

$$v'\left(d_2\right) = \frac{\lambda n_2}{\alpha_2 n_2 G_2' + \lambda_2 - \lambda_3} = 1, \tag{11}$$

$$h'\left(\frac{y_2}{w_\ell}\right) = w_\ell\left[1 + \frac{\lambda_3}{\alpha_2 n_2 G_2' + \lambda_2}\left(h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h} - 1\right)\right] \leq w_\ell, \tag{12}$$

$$v'\left(d_1 - \bar{d}_1\right) - 1 = \frac{\lambda_2 + \lambda_{31}}{\alpha_1 n_1 G_1'}\left(v'\left(d_1\right) - 1\right), \tag{13}$$

$$h'\left(\frac{y_1}{w_\ell}\right) = w_\ell\left[1 + \frac{\lambda_{31}}{\alpha_1 n_1 G_1' - \lambda_2}\left(h'\left(\frac{y_1}{w_h}\right)\frac{1}{w_h} - 1\right)\right] \leq w_\ell, \tag{14}$$

$$\mu = \frac{\sum_{i=1}^{3} \alpha_i n_i G_i'}{\sum_{i=1}^{3} n_i}. \tag{15}$$

As in the non-welfarist approach, the second-best levels of y_3, d_3 and d_2 coincide with the first-best ones. There is no efficiency gain in distorting the choices of the high-productivity individuals, nor the choice of commodity d by the low-productivity able individuals. However, it can be optimal to distort the labor choice of these individuals if by doing so high-productivity individuals are precluded from mimicking them. As before, the SSC that prevents type 3 from mimicking type 2 individuals is binding and $y_2^{SB} < y_2^{FB}$. Regarding low-productivity disabled individuals, it is optimal to distort their consumption of commodity d (i.e., $d_1^{SB} > d_1^{FB} > d_2^{FB} = d_3^{FB}$). However, in contrast with the non-welfarist approach, it is not always optimal to distort their labor supply decision. It depends on whether there are incentives for high-productivity individuals to mimic them (i.e., whether the SSC that prevents type 3 from mimicking type 1 individuals is binding).

A further analysis of the pattern of SSCs - see appendix B.2 for more detail - yields the following possibilities: either the three SSCs are simultaneously binding and $y_1^{SB} = y_2^{SB} < y_2^{FB} = y_1^{FB}$, or both SSCs that prevent the high-productivity individuals from mimicking both types of low-productivity individuals are binding, with the adjacent SSC that relates both types of low-productivity individuals holding with strict inequality. This latter case is only possible if $y_1^{FB} > y_1^{SB} > y_2^{SB}$ (i.e., among the low-productivity individuals the disabled are required to work more than the able ones). This is clearly at odds with the idea of alleviating the hardship of work for type 1 individuals.

Which one of these two regimes prevails? It will depend crucially on the parameters of the model (i.e., the degree of concavity of the social utility transformation $G(\cdot)$, the proportion of different types in the population, the wage gap, the particular weights given, etc.). It seems worthwhile to analyze two particular cases in further detail: namely, the utilitarian and Rawlsian specifications. In the utilitarian case where all α_i are equal the two regimes mentioned above are indeed possible. However, for the Rawlsian objective, which in this case corresponds to maximizing the utility of the disabled individual, the SSC that prevents low-productivity able individuals from

mimicking the disabled ones always binds and the only relevant possibility involves the three SSCs binding, with $y_1^{SB} = y_2^{SB}$ and, hence, $\ell_1^{SB} = \ell_2^{SB}$.

One interesting general result that emerges is that the weighted welfarist objective is unable to provide solutions in which disabled individuals are required to work less than their able counterparts, and they may indeed be required to work more. Disabled will instead be compensated with larger amounts of commodity d, because this is the only feature of their preferences a welfarist planner can use to distinguish them.

4 Conclusion

In this paper we have dealt with the design of an optimal social policy in a setting where different people have the same disutility for labor and thus in a *laissez-faire* economy they would work the same amount of time if they have the same productivity. Yet, the social planner would like to treat them differently because the labor disutility of some reflects a stronger preference for leisure whereas that of others stems from some physical or psychological pain from work.

The only way they can be distinguished is through their consumption basket that indicates the nature of their labor disutility: disability here implies some particular needs in consuming one of the goods. We consider two possible social criteria. One consists of giving more weight to the utility of the disabled and it thus generates a Paretian outcome. The other consists of giving more weight to the labor disutility of the disabled and this yields a non-Paretian outcome.

We show that the non-welfarist approach can be justified by using the dichotomy between characteristics of responsibility and characteristics of luck. Only the latter are used for the sake of redistribution. We also show that the non-welfarist approach can imply a lower labor supply requirement from disabled individuals whereas the welfarist approach is unable to generate such an outcome, regardless of the higher weight given to the utility of the disabled or the degree of concavity used to reflect redistributive concerns.

References

[1] Bossert, and D. Van de Gaer (1999) Responsibility, talent and compensation: a second-best analysis, *Review of Economic Design*, 4, 35-55.

[2] Cremer, H., J-M. Lozachmeur and P. Pestieau (2006), Disability testing and retirement, CORE DP 2006/16.

[3] Fleurbaey, M. and F. Maniquet (2005a), Fair income tax, *Review of Economic Studies*, forthcoming.

[4] Fleurbaey, M. and F. Maniquet (2005b), Help the low-skilled or let the hardworking thrive? A study of fairness in optimal income taxation, mimeo.

[5] Kanbur, R., J. Puttila and M. Tuomala (2006), Non welfarist optimal taxation and behavioral public economics, *Journal of Economic Surveys*, 20, 849-868.

[6] Marchand, M., Pestieau, P. and M. Racionero (2003), Optimal Redistribution when Different Workers are Indistinguishable, *Canadian Journal of Economics* 36, 911-922.

[7] Mirrlees, J. (1971), An exploration in the theory of optimal taxation, *Review of Economic Studies*, 38, 175-208.

[8] O'Donoghue, T. and M. Rabin (2003), Studying optimal paternalism, illustrated by a model of sin taxes, *American Economic Review*, 93, 186-191.

[9] Roemer, J. (1996), *Theories of Distributive Justice*, Harvard University Press.

[10] Stiglitz, J. (1982). Self-selection and Pareto efficient taxation, *Journal of Public Economics* 17, 213-240.

[11] Schokkaert, E., D. Van de Gaer, F. Vandenbroucke and R. Luttens (2006), Responsibility sensitive egalitarianism and optimal linear taxation, *Mathematical Social Sciences*, 48, 151-182.

A Relevant self-selection constraints

When relevant information about some individual characteristics is private, the social planner needs to take account of the so-called self-selection constraints. These constraints are incorporated in order to prevent individuals of a given type from applying for the tax-treatment designed for individuals of other types. The purpose of this section is to identify which SSCs need to be incorporated in the second-best problem. If the social objective involves redistribution towards the bottom we need only focus on downwards SSCs:

$$c_3 + v(d_3) - h\left(\frac{y_3}{w_h}\right) \geq c_2 + v(d_2) - h\left(\frac{y_2}{w_h}\right), \tag{A.1}$$

$$c_2 + v(d_2) - h\left(\frac{y_2}{w_\ell}\right) \geq c_1 + v(d_1) - h\left(\frac{y_1}{w_\ell}\right), \tag{A.2}$$

$$c_3 + v(d_3) - h\left(\frac{y_3}{w_h}\right) \geq c_1 + v(d_1) - h\left(\frac{y_1}{w_h}\right). \tag{A.3}$$

The first two constraints are the adjacent downwards SSCs (for $i = 3, 2$), whereas the last SSC is the non-adjacent one that relates individuals of types 3 and 1.

In most of the analysis of optimal non-linear income taxation, with social objectives that redistribute towards the bottom, the adjacent downwards SSCs are sufficient. However, it is necessary to check whether this is the case in the present framework, given the complexity added by the existence of differential needs for commodity d. From (A.1) and (A.2),

$$c_3 + v(d_3) - c_1 - v(d_1) \geq h\left(\frac{y_3}{w_h}\right) - h\left(\frac{y_2}{w_h}\right) + h\left(\frac{y_2}{w_\ell}\right) - h\left(\frac{y_1}{w_\ell}\right).$$

If

$$h\left(\frac{y_2}{w_\ell}\right) - h\left(\frac{y_1}{w_\ell}\right) \geq h\left(\frac{y_2}{w_h}\right) - h\left(\frac{y_1}{w_h}\right),$$

then (A.3) holds (Note this is a sufficient condition). Given the properties of the disutility of labor function $h(.)$ (in particular, $h' > 0, h'' \geq 0$) and $w_h > w_\ell$,

$$h\left(\frac{y_2}{w_\ell}\right) - h\left(\frac{y_1}{w_\ell}\right) \geq h\left(\frac{y_2}{w_h}\right) - h\left(\frac{y_1}{w_h}\right)$$

iff $y_2 \geq y_1$, and (A.3), the downwards SSC that relates non-adjacent individuals of types 3 and 1, holds. However, it is worthwhile emphasizing that it is possible that

$y_1 > y_2$ if the combination of (c, d) for a type 1 individual is such that the social planner can ask her to work harder than a type 2, in order to prevent type 2 individuals from mimicking type 1 individuals, and compensate her with larger amount of d, which a type 2 individual values relatively less. If $y_1 > y_2$,

$$h\left(\frac{y_3}{w_h}\right) - h\left(\frac{y_2}{w_h}\right) + h\left(\frac{y_2}{w_\ell}\right) - h\left(\frac{y_1}{w_\ell}\right) < h\left(\frac{y_3}{w_h}\right) - h\left(\frac{y_1}{w_h}\right).$$

It might still be possible that (A.3) is satisfied, except when (A.1) and (A.2) bind. In this case,

$$u\left(c_3\right) + v\left(d_3\right) - u\left(c_1\right) - v\left(d_1\right) = h\left(\frac{y_3}{w_h}\right) - h\left(\frac{y_2}{w_h}\right) + h\left(\frac{y_2}{w_\ell}\right) - h\left(\frac{y_1}{w_\ell}\right) < h\left(\frac{y_3}{w_h}\right) - h\left(\frac{y_1}{w_h}\right)$$

and (A.3) is violated. In view of this we need to incorporate the three SSCs in the formal second-best problem to make the mimicking behavior unattractive.[5]

B Analysis of the pattern of self-selection constraints

B.1 Non-welfarist objective

The FOCs of the social planner problem with the non-welfarist objective used in this paper are:

$$n_1 G_1' - \lambda_2^d - \lambda_{31}^d = \lambda n_1 \tag{B.1.1}$$

$$n_2 G_2' + \lambda_2^d - \lambda_3^d = \lambda n_2 \tag{B.1.2}$$

$$n_3 G_3' + \lambda_3^d + \lambda_{31}^d = \lambda n_3 \tag{B.1.3}$$

$$v'\left(d_1 - \bar{d}_1\right)\left(n_1 G_1'\right) - \left(\lambda_2^d + \lambda_{31}^d\right) v'\left(d_1\right) = \lambda n_1 \tag{B.1.4}$$

$$v'\left(d_2\right)\left(n_2 G_2' + \lambda_2^d - \lambda_3^d\right) = \lambda n_2 \tag{B.1.5}$$

$$v'\left(d_3\right)\left(n_3 G_3' + \lambda_3^d + \lambda_{31}^d\right) = \lambda n_3 \tag{B.1.6}$$

$$h'\left(\frac{y_1}{w_\ell}\right)\frac{1}{w_\ell}\left(\beta n_1 G_1' - \lambda_2^d\right) - \lambda_{31}^d h'\left(\frac{y_1}{w_h}\right)\frac{1}{w_h} = \lambda n_1 \tag{B.1.7}$$

$$h'\left(\frac{y_2}{w_\ell}\right)\frac{1}{w_\ell}\left(n_2 G_2' + \lambda_2^d\right) - \lambda_3^d h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h} = \lambda n_2 \tag{B.1.8}$$

---

[5] Marchand et al. (2003) focus on solutions in which the handicapped individuals are induced to work less than the lazy ones. In those circumstances, $y_2 > y_1$ and only the adjacent downwards SSCs are binding. For consistency, we incorporate the three downwards SSCs, including the non-adjacent that relates individuals of types 3 and 1, for all the objectives considered and highlight the circumstances under which the non-adjacent constraint is indeed redundant.

$$h'\left(\frac{y_3}{w_h}\right)\frac{1}{w_h}\left(n_3 G_3' + \lambda_3^d + \lambda_{31}^d\right) = \lambda n_3 \qquad \text{(B.1.9)}$$

Rearranging these conditions the second-best (hereafter SB) is characterized by equations (4) to (9) in the text. From (B.1.1) to (B.1.3), it follows that:

$$\lambda_2 + \lambda_{31} \geq 0 \Leftrightarrow G_1' \geq \lambda \Leftrightarrow G_1' \geq \frac{n_2 G_2' + n_3 G_3'}{n_2 + n_3},$$

$$\lambda_3 + \lambda_{31} \geq 0 \Leftrightarrow \lambda \geq G_3' \Leftrightarrow G_3' \leq \frac{n_1 G_1' + n_2 G_2'}{n_1 + n_2},$$

$$G_2' > \frac{n_1 G_1' + n_3 G_3'}{n_1 + n_3} \Leftrightarrow G_2' > \lambda \Leftrightarrow \lambda_3 - \lambda_2 > 0 \Rightarrow \lambda_3 > 0 \text{ and } \lambda_2 \geq 0,$$

$$G_2' < \frac{n_1 G_1' + n_3 G_3'}{n_1 + n_3} \Leftrightarrow G_2' < \lambda \Leftrightarrow \lambda_3 - \lambda_2 < 0 \Rightarrow \lambda_2 > 0 \text{ and } \lambda_3 \geq 0.$$

Moreover, the SSCs - equations (A.1) to (A.3) in appendix A - imply that:

$$u_3 > u_2 > u_1 > \tilde{u}_1 \rightarrow G_1' > G_2' > G_3'.$$

Hence,

$$G_1' > \frac{n_2 G_2' + n_3 G_3'}{n_2 + n_3} \rightarrow G_1' > \lambda \rightarrow \lambda_2 + \lambda_{31} > 0$$

$$G_3' < \frac{n_1 G_1' + n_2 G_2'}{n_1 + n_2} \rightarrow \lambda > G_3' \rightarrow \lambda_3 + \lambda_{31} > 0$$

which means that either SSC 21[6] and SSC 31 are binding, or SSC 21 and SSC 32 are binding, or SSC 31 and SSC 32 are binding, or the three constraint are binding. We hereafter analyze these possibilities in turn.

1. If SSC 32 and SSC 21 are binding, then $\lambda_3 > 0$ and $\lambda_2 > 0$. If $y_2 > y_1$, the SSC 31 holds with strict inequality. If $y_1 > y_2$, the SSC 31 is violated. In contrast to the weighted welfarist objective below, the SB equations (6) and (8) do not imply a clear-cut relationship between y_1 and y_2: either $y_1 > y_2$ (then the SSC 31 is violated) or $y_2 > y_1$. Using (6) and (8)

$$y_2 \geq y_1 \Leftrightarrow \frac{\lambda_3}{n_2 G_2' + \lambda_2}\left(1 - h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h}\right) \leq \frac{n_1 G_1'(\beta - 1)}{\beta n_1 G_1' - \lambda_2}.$$

Note that $y_2 > y_1$ is now possible because $\beta > 1$. Using (B.1.1) to (B.1.3), the equation can be rewritten as:

$$y_2 \geq y_1 \Leftrightarrow \frac{\frac{G'_1(\beta-1)}{G'_1(\beta-1)+\lambda}}{\frac{(\lambda-G'_3)}{(\lambda-G'_3)+\lambda\frac{n_2}{n_3}}} \geq 1 - h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h}.$$

Note that if

$$\frac{\frac{G'_1(\beta-1)}{G'_1(\beta-1)+\lambda}}{\frac{(\lambda-G'_3)}{(\lambda-G'_3)+\lambda\frac{n_2}{n_3}}} \geq 1,$$

then the condition holds and $y_2 > y_1$. Rearranging,

$$\frac{G'_1(\beta-1)}{\lambda-G'_3} \geq \frac{G'_1(\beta-1)+\lambda}{(\lambda-G'_3)+\lambda\frac{n_2}{n_3}}.$$

If $n_2 \geq n_3$, the sufficient condition holds. If $n_2 < n_3$, then the sufficient condition does not hold but it might still be possible that the overall condition for $y_2 > y_1$ holds.

2. If SSC 21 and SSC 31 are binding, then $\lambda_2 > 0$ and $\lambda_{31} > 0$. If $y_1 > y_2$, the SSC 32 holds with strict inequality. However, if $y_2 > y_1$, the SSC 32 is violated. The SB equations (6) and (8) imply that $y_2 > y_1$ for $\lambda_{31} > 0$ and $\lambda_3 = 0$. Hence, this pattern of SSCs is not possible because the SSC 32 is violated.

3. If SSC 32 and SSC 31 are binding, then $\lambda_3 > 0$ and $\lambda_{31} > 0$. If $y_1 > y_2$, the SSC 21 holds with strict inequality. However, if $y_2 > y_1$, the SSC 21 is violated. From SB equations (6) and (8) it is possible that $y_1 > y_2$. Working with (6) and (8), and using (B.1.1) and (B.1.2), we obtain:

$$y_1 > y_2 \Leftrightarrow \frac{(G'_2-\lambda)}{G'_2}\left(1 - h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h}\right) > \frac{G'_1(\beta-1)}{\beta G'_1} + \frac{(G'_1-\lambda)}{\beta G'_1}\left(1 - h'\left(\frac{y_1}{w_h}\right)\frac{1}{w_h}\right).$$

4. If the three constraints are binding, then $\lambda_3 > 0, \lambda_2 > 0$ and $\lambda_{31} > 0$. The three constraints can only be binding simultaneously if $y_1 = y_2 = y$. I.e.,

$$\frac{\lambda_3}{n_2 G'_2 + \lambda_2}\left(1 - h'\left(\frac{y}{w_h}\right)\frac{1}{w_h}\right) = \frac{n_1 G'_1(\beta-1) + \lambda_{31}\left(1 - h'\left(\frac{y}{w_h}\right)\frac{1}{w_h}\right)}{\beta n_1 G'_1 - \lambda_2}.$$

Using (B.1.1) and (B.1.2) and rearranging,

$$\left[\frac{n_1 G'_1(\beta-1) + \lambda n_1}{\beta n_1 G'_1 - \lambda_2} - \frac{\lambda n_2}{n_2 G'_2 + \lambda_2}\right]\left(1 - h'\left(\frac{y}{w_h}\right)\frac{1}{w_h}\right) = \frac{n_1 G'_1(\beta-1)}{\beta n_1 G'_1 - \lambda_2}.$$

16

Summing up, there are three possibilities: (i) either the three SSCs are binding and $y_1^{SB} = y_2^{SB} < y_2^{FB} = y_1^{FB}$, or (ii) the SSCs that prevent high-productivity individuals from mimicking both low-productivity ones are binding, with the downwards SSC that relates both low-productivity individuals holding with strict inequality, and $y_1^{FB} > y_1^{SB} > y_2^{SB}$, or (iii) the adjacent downwards SSCs are binding, with the non-adjacent downwards SSC holding with strict inequality, and $y_2^{SB} > y_1^{SB}$.

B.2 Weighted welfarist objective

The FOCs of the social planner problem with a weighted welfarist objective, assuming non-increasing weights, are:

$$\alpha_1 n_1 G_1' - \lambda_2 - \lambda_{31} = \lambda n_1, \tag{B.2.1}$$

$$\alpha_2 n_2 G_2' + \lambda_2 - \lambda_3 = \lambda n_2, \tag{B.2.2}$$

$$\alpha_3 n_3 G_3' + \lambda_3 + \lambda_{31} = \lambda n_3, \tag{B.2.3}$$

$$v'\left(d_1 - \bar{d}_1\right)\left(\alpha_1 n_1 G_1'\right) - \left(\lambda_2 + \lambda_{31}\right) v'\left(d_1\right) = \lambda n_1, \tag{B.2.4}$$

$$v'\left(d_2\right)\left(\alpha_2 n_2 G_2' + \lambda_2 - \lambda_3\right) = \lambda n_2, \tag{B.2.5}$$

$$v'\left(d_3\right)\left(\alpha_3 n_3 G_3' + \lambda_3 + \lambda_{31}\right) = \lambda n_3, \tag{B.2.6}$$

$$h'\left(\frac{y_1}{w_\ell}\right)\frac{1}{w_\ell}\left(\alpha_1 n_1 G_1' - \lambda_2\right) - \lambda_{31} h'\left(\frac{y_1}{w_h}\right)\frac{1}{w_h} = \lambda n_1, \tag{B.2.7}$$

$$h'\left(\frac{y_2}{w_\ell}\right)\frac{1}{w_\ell}\left(\alpha_2 n_2 G_2' + \lambda_2\right) - \lambda_3 h'\left(\frac{y_2}{w_h}\right)\frac{1}{w_h} = \lambda n_2, \tag{B.2.8}$$

$$h'\left(\frac{y_3}{w_h}\right)\frac{1}{w_h}\left(\alpha_3 n_3 G_3' + \lambda_3 + \lambda_{31}\right) = \lambda n_3. \tag{B.2.9}$$

Rearranging these conditions the SB is characterized by equations (10) to (15) in the text. From (B.2.1) to (B.2.3), it follows that:

$$\lambda_2 + \lambda_{31} \geq 0 \Leftrightarrow \alpha_1 G_1' \geq \lambda \Leftrightarrow \alpha_1 G_1' \geq \frac{n_2 \alpha_2 G_2' + n_3 \alpha_3 G_3'}{n_2 + n_3},$$

$$\lambda_3 + \lambda_{31} \geq 0 \Leftrightarrow \lambda \geq \alpha_3 G_3' \Leftrightarrow \alpha_3 G_3' \leq \frac{n_1 \alpha_1 G_1' + n_2 \alpha_2 G_2'}{n_1 + n_2},$$

$$\alpha_2 G_2' > \frac{n_1 \alpha_1 G_1' + n_3 \alpha_3 G_3'}{n_1 + n_3} \Leftrightarrow \alpha_2 G_2' > \lambda \Leftrightarrow \lambda_3 - \lambda_2 > 0 \Rightarrow \lambda_3 > 0 \text{ and } \lambda_2 \geq 0,$$

$$\alpha_2 G_2' < \frac{n_1 \alpha_1 G_1' + n_3 \alpha_3 G_3'}{n_1 + n_3} \Leftrightarrow \alpha_2 G_2' < \lambda \Leftrightarrow \lambda_3 - \lambda_2 < 0 \Rightarrow \lambda_2 > 0 \text{ and } \lambda_3 \geq 0.$$

In this case, the SSCs - equations (A.1) to (A.3) in appendix A - imply that

$$u_3 > u_2 > u_1 \rightarrow G_1' > G_2' > G_3'.$$

Hence, when the weights are non-increasing,

$$\alpha_1 G_1' > \frac{n_2 \alpha_2 G_2' + n_3 \alpha_3 G_3'}{n_2 + n_3} \rightarrow \alpha_1 G_1' > \lambda \rightarrow \lambda_2 + \lambda_{31} > 0,$$

$$\alpha_3 G_3' < \frac{n_1 G_1' + n_2 G_2'}{n_1 + n_2} \rightarrow \lambda > \alpha_3 G_3' \rightarrow \lambda_3 + \lambda_{31} > 0,$$

which means, as in the previous case, that either SSC 21 and SSC 31 are binding, or SSC 21 and SSC 32 are binding, or SSC 31 and SSC 32 are binding, or the three constraint are binding. We again analyze these possibilities in turn.

1. If SSC 32 and SSC 21 are binding, then $\lambda_3 > 0$ and $\lambda_2 > 0$. If $y_2 > y_1$, the SSC 31 holds with strict inequality. However, if $y_1 > y_2$, the SSC 31 is violated. The SB equations (12) and (14) imply that $y_1 > y_2$ for $\lambda_3 > 0$ and $\lambda_{31} = 0$. Hence, this pattern of SSCs is not possible because the SSC 31 is violated.

2. If SSC 21 and SSC 31 are binding, then $\lambda_2 > 0$ and $\lambda_{31} > 0$. If $y_1 > y_2$, the SSC 32 holds with strict inequality. However, if $y_2 > y_1$, the SSC 32 is violated. The SB equations (12) and (14) imply $y_2 > y_1$ for $\lambda_{31} > 0$ and $\lambda_3 = 0$. Hence, this pattern of SSCs is not possible because the SSC 32 is violated.

3. If SSC 32 and SSC 31 are binding, then $\lambda_3 > 0$ and $\lambda_{31} > 0$. If $y_1 > y_2$, the SSC 21 holds with strict inequality. However, if $y_2 > y_1$, the SSC 21 is violated. From SB equations (12) and (14) it is possible that $y_1 > y_2$. Working with (12) and (14), and using (B.2.1) and (B.2.2), we obtain:

$$y_1 > y_2 \Leftrightarrow \frac{(\alpha_2 G_2' - \lambda)}{\alpha_2 G_2'} \left(1 - h'\left(\frac{y_2}{w_h}\right) \frac{1}{w_h} \right) > \frac{(\alpha_1 G_1' - \lambda)}{\alpha_1 G_1'} \left(1 - h'\left(\frac{y_1}{w_h}\right) \frac{1}{w_h} \right).$$

4. If the three constraints are binding, then $\lambda_3 > 0, \lambda_2 > 0$ and $\lambda_{31} > 0$. The three constraints can only be binding simultaneously if $y_1 = y_2 = y$. I.e.,

$$\frac{\lambda_3}{\alpha_2 n_2 G_2' + \lambda_2} = \frac{\lambda_{31}}{\alpha_1 n_1 G_1' - \lambda_2}.$$

18

Summing up, there are only two possibilities now: (i) either the three SSCs are simultaneously binding and $y_1^{SB} = y_2^{SB} < y_2^{FB} = y_1^{FB}$, or (ii) both SSCs that prevent the high-productivity individuals from mimicking both types of low-productivity individuals are binding, with the adjacent SSC that relates both types of low-productivity individuals holding with strict inequality. This last case is only possible if $y_1^{FB} > y_1^{SB} > y_2^{SB}$.