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Technological breakthroughs and asset replacement

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Technological breakthroughs and asset replacement

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Abstract

The authors analyze the optimal replacement of assets under continuous and discontinuous technological change. They investigate the variable lifetime of assets in an infinite-horizon replacement problem. Due to deterioration, the maintenance cost increases when the asset becomes older. Because of technological change, both maintenance and new capital costs decrease for a fixed asset age. The dynamics of the optimal lifetime is investigated analytically and numerically under technological change in the cases of one and several technological breakthroughs. It is shown that the breakthroughs cause irregularities (anticipation echoes) in the asset lifetime before the breakthrough time.

Keywords: asset replacement, technological change, optimal lifetime, anticipation echoes.

JEL Classification: C61, L23, O14, O33

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1 Introduction

The paper analyzes the impact of discontinuous *technological change* (TC) on the optimal replacement of assets (capital equipment, machines) in a framework traditional for Operations Research (OR). The TC is understood in OR as an exogenous decrease of the operating and maintenance costs of possible replacement assets (challengers). It is usually described as:

- *continuous TC* as a continuous improvement in the newer vintages of equipment (Grinyer 1973; Sethi and Chand 1979; Bean *et al* 1994; Regnier *et al* 2004; Hritonenko and Yatsenko, 1996b, 2005) or
- *discontinuous TC* in the form of instantaneous changes in the technological parameters of assets (Rogers and Hartman 2005; Hopp and Nair 1991; Rajagopalan *et al* 1998).

The optimal asset replacement *under the continuous TC* has been intensively analyzed in the OR literature, see (Grinyer 1973; Bean *et al* 1994; Hartman 2000; Regnier *et al* 2004; Hritonenko and Yatsenko 2007, 2008b, 2008c; Sethi and Chand 1979; Yatsenko and Hritonenko 2008) and the references therein. The case of the discontinuous TC is less explored. At the same time, there is economic evidence that both TC types are present in economic and engineering reality (Goolsbee 1998; Rogers and Hartman 2005).

Two fundamental categories of the OR equipment replacement models are:

- *serial replacement* of a single asset (machine),
- and *parallel replacement* of many dependent assets that operate in parallel.

The majority of research on the equipment replacement under TC (e.g., Bethuyne 1998, Regnier *et al* 2004, Rogers and Hartman 2005) considers the serial replacement (of a single machine). Also, the TC technological breakthroughs supposedly affect the levels of exogenous technological parameters (Rogers and Hartman 2005, Hartman and Rogers 2006).

This paper is inspired by Rogers and Hartman (2005) who numerically analyzed the joint impact of the continuous TC and periodic technological breakthroughs on the *constant* asset lifetime in a *serial replacement* model. They showed that the optimal lifetime of assets is *always* smaller under more intensive continuous TC and is *usually* smaller under more intensive discontinuous TC (with some exceptions). Following the above mainstream of OR replacement models, the present paper assumes:

- a single asset,
- jumps in the level of technological parameters (rather than their rates).

While mostly focusing of the study of a single breakthrough, we also consider the case of several successive breakthroughs to address the periodic breakthroughs of Rogers and Hartman (2005) and non-periodic breakthroughs in Hartman and Rogers (2006).

The new contribution of the paper to the OR replacement literature is the *variable asset* lifetime, a general continuous TC as decreasing capital and maintenance costs, and a general discontinuous TC as in the form of several non-periodic technological breakthroughs. As shown by Hritonenko and Yatsenko (1996a, 2005, 2007, 2008b) and Regnier *et al* (2004), the optimal lifetime of assets can be *variable* even under the continuous exponential TC. Obviously, it is true for the discontinuous TC.

In general, the issue of technological replacement under breakthroughs goes much beyond the OR literature. More theoretical papers on this issue consider technological breakthroughs as radical innovations caused by the substitution of one *general-purpose technology* for another. Starting with Bresnahan and Trajtenberg (1995), such breakthroughs explain economy-wide structural changes. The steam engine, gasoline engine, electric power, semiconductors were the examples of such general-purpose (enabling) technologies. As Aures (2005) argues, the radical innovations are necessary for continued long-term economic growth. Many economic papers interpret and analyze the more recent IT revolution as a major breakthrough as well (Greenwood and Yorukoglu 1997, Boucekkine and de la Croix 2003).

Returning to the scope of this paper, radical innovations obviously affect the rational decision on replacing or upgrading related capital. From this viewpoint, the single machine used in this paper can be considered as a metaphor for the general problem of replacing the old technology at the dawn of a new general purpose technology. However, such a macroeconomic interpretation raises new issues to take into consideration. In particular, the considered below breakthrough shocks in level do not cover all cases observed in reality. Shocks in the TC growth rates seem to be more important and are observed in the ICT literature (Greenwood and Yorukoglu, 1997, for example).

To fully address the macroeconomic link between technological breakthroughs and optimal capital replacement, we need to employ vintage capital models of economic growth theory (Boucekkine *et al* 1997, 1999, 2008; Boucekkine and de la Croix 2003; Hritonenko and Yatsenko 1996b, 2005, 2008a; Yorukoglu 1998). Such analysis is the subject of a forthcoming paper of the authors.

The present paper is organized in the following manner. The next section introduces a replacement model for a single asset with variable lifetime over the infinite horizon. Section 3 exposes preliminary analytic results such as an extremum condition and properties of the optimal lifetime under continuous TC. Section 4 investigates the model under discontinuous TC with one and several breakthroughs analytically and numerically on an industrial data sample about car replacement. It demonstrates how the variable optimal lifetime of this single asset is impacted by technological breakthroughs. The obtained results are new in the OR asset replacement theory. Section 5 concludes.

2 Model of Serial Asset Replacement under TC

Let us suppose that a production shop (plant, enterprise) keeps one asset (machine) of a certain type. The shop periodically sells the old asset and buys a new replacement asset. We consider this replacement process in continuous time t using the following notations:

- $t_0=0$: the starting point of planning horizon;
- $\tau_0 \le 0$: the given (initial) purchase time of the asset (0^{th} replacement time);
- τ_k, $k=1,2,\ldots$: the unknown time of the k-th replacement;
- L_k, $k=1,2,\ldots$: the unknown lifetime of the k-th replaced asset, $L_k = \tau_k - \tau_{k-1}$,
- $p(t)$, $t\in[0,\infty)$: the capital cost (purchase price & installation cost) of an asset bought at time t;
- $q(t,u)$, $t\in[\tau_0,\infty)$, $u\in[0,\infty)$: the operating and maintenance (O&M) cost at the time u for the asset bought at time t, $u\ge t$;
- r, $r>0$: the instantaneous discount rate.

Because of deterioration, the O&M cost $q(t,u)$ increases in u at a fixed t when the asset age $u-t$ increases (the asset becomes older). At this point, we make a general TC assumption that $p(t)$ and $q(t,u)$ decrease in t for any fixed asset age $u-t$ (newer assets are less expensive and require less maintenance). More specific TC assumptions will be considered in Sections 5 and 6.

The *replacement policy* is the set $\pi=\{\tau_i,\ i=1,2,\ldots\}$ of the sequential replacement times τ_i. The present value of the total cost of the replacement policy over the infinite horizon $[0,\infty)$ can be expressed as

$$J(\pi)=\sum_{i=1}^{\infty} e^{-r\tau_i}\, p(\tau_i)+\sum_{i=0}^{\infty}\int_{\tau_i}^{\tau_{i+1}} e^{-ru} q(\tau_i,u)du \qquad (1)$$

(see Hritonenko and Yatsenko 2008b, 2008c; Grinyer 1973; Regnier et al. 2004; Rogers and Hartman 2005). The first sum in (1) is the discounted total capital cost and the second sum is the discounted total O&M cost[3]. We formulate the replacement problem as finding the optimal policy $\pi^*=\{\tau_i^*, i=1,2,\ldots\}$ that minimizes the replacement cost (1):

$$J(\pi^*) = \min_{\tau_i, i=1,\ldots,\infty} J(\pi) \quad . \tag{2}$$

3 Preliminary Results

In the section, we summarize some theoretical results essential for the further analysis. No proofs are provided, since this optimal program has been investigated in (Yatsenko and Hritonenko 2008b, 2008c).

Theorem 1 (*necessary condition for an extremum*). *If an optimal policy* $\pi^*=\{\tau_i^*, i=1,2,\ldots\}$ *exists, then every component* τ_i^*, $0<\tau_i^*<\infty$, *satisfies the condition*

$$\partial J/\partial\tau_i = 0, \qquad i=1,2,\ldots, \tag{3}$$

where

$$\frac{\partial J}{\partial \tau_i} = e^{-r\tau_i}\left\{ \; p'(\tau_i) - rp(\tau_i) - q(\tau_i,\tau_i) + q(\tau_{i-1},\tau_i) + \int_{\tau_i}^{\tau_{i+1}} e^{-r(u-\tau_i)} \frac{\partial q(\tau_i,u)}{\partial \tau_i} du \; \right\} \tag{4}$$

is the partial derivative of function (1) in τ_i.

In the case $q(t,u)$=const, $p(t)$=const with no TC and deterioration, $\partial J/\partial\tau_1<0$ by (4) and the optimal policy is trivial: $\tau_1^* = \infty$ (no replacement).

A continuous equation for the optimal replacement times. By Theorem 1, the optimal set $\{\tau_i^*, i=1,2,\ldots\}$ for a *given* initial replacement time τ_0 (if it exists) should satisfy the equality

$$p'(\tau_i) - rp(\tau_i) - q(\tau_i,\tau_i) + q(\tau_{i-1},\tau_i) + \int_{\tau_i}^{\tau_{i+1}} e^{-r(u\,\tau_i)} \frac{\partial q(\tau_i,u)}{\partial \tau_i} du = 0 \tag{5}$$

[3] Expression (1) omits possible salvage values. A possible impact of salvage values in the model (1) with continuous TC is discussed in (Hritonenko and Yatsenko 2008b, 2008c).

for all i=1,2,… . To investigate the replacement process in continuous time *for any* generic τ_0, let us assume that the current time t is a replacement time τ_i^* and consider the *previous replacement time* $R(t)=\tau_{i-1}$ (of the asset *replaced* at time $t=\tau_i$) as a function of t. By (5), the unknown function $R(t)$ satisfies the nonlinear equation

$$p'(t) - rp(t) + q(R(t),t) - q(t,t) + \int_t^{R^{-1}(t)} e^{-r(u-t)} \frac{\partial q(t,u)}{\partial t} du = 0\,,\ \ t \in [0,\infty), \qquad (6)$$

where the inverse function $R^{-1}(t)$ of $R(t)$ corresponds to the replacement time τ_{i+1} of the asset *purchased* at t.

Theorem 2 (Yatsenko and Hritonenko 2008b)**.** *If an optimal policy* $\pi^*=\{\tau_i^*, i=1,2,\ldots\}$ *exists and the nonlinear integral equation (6) has a unique solution* $R(t)$, $t\in[0,\infty)$, *then*

$$\tau_{i-1}^* = R(\tau_i^*), \qquad i = 2,3,\ldots\ . \qquad (7)$$

The policy π uniquely determines the set $\{L_i, i=1,2,\ldots,\}$ of the lifetimes L_i of sequentially replaced assets. By Theorem 2, the optimal lifetime $L_i^{\ *}=\tau_i^*-\tau_{i-1}^*$ of the asset *replaced* at time $t=\tau_i$ is equal to $L(t) = t-R(t)$, hence by (7)

$$L_i^*=L(\tau_i^*) \qquad \text{at} \qquad \tau_i^* = \tau_{i-1}^* + L_i^*, \qquad i=1,2,\ldots. \qquad (8)$$

So, if we know a solution to the nonlinear equation (6), we can find the optimal asset lives L_i^*, i=1,2,…, for *any* initial replacement time $\tau_0<0$. Similar equations have been analytically and numerically studied for different smooth functions p and q by Hritonenko and Yatsenko (1996a, 2008b), Yatsenko and Hritonenko (2005). Here we assume the exponential TC and deterioration:

$$q(t,u)=\tilde{q}(t,u)= q_0 e^{c_d(u-t)} e^{-c_q t}, \quad p(t)=\tilde{p}(t)=p_0 e^{-c_p t}, \quad c_q+c_d>0,\ c_p+c_d\geq 0. \qquad (9)$$

The TC rate in the capital and O&M costs can be different in (9): $c_p\neq c_q$. These costs can increase ($c_p<0$ or $c_q<0$) but slower than the deterioration rate c_d. Usually, both costs decrease: $c_p>0$ and $c_q>0$. We will refer to the case *of equal rates* $c_q=c_p$ as *the proportional TC.*

The properties of equation (6) allow us to analyze the dynamics of the *variable* optimal asset lifetime L_i^* in the serial replacement model (1)-(2) under exponential TC and deterioration (9).

Theorem 3 (Yatsenko and Hritonenko 2008b). *Under (9), the optimization problem (1)-(2) possesses a unique optimal policy* $\{L_i^*, i=1,2,\ldots\}$ *such that:*

(a) If $c_q = c_p$, *then* $L_i^* = L^*$, $i=1,2,\ldots$, *where the constant* $L^*>0$ *is uniquely determined from the non-linear equation*

$$r\,e^{(c_q+c_d)L} + (c_q+c_d)\,e^{-rL} = (r+c_q+c_d)[1+rp_0/q_0] \tag{10}$$

and, $L^* \approx [2p_0/(q_0(c_q+c_d))]^{1/2}$ *at small* c_q+c_d *and* r .

(b) If $c_p > c_q$, *then* $L_{i+1}^* < L_i^*$, $i=1,2,\ldots$, *and* L_i^* *strives to 0 as* $i\to\infty$.

(c) If $-c_d < c_p < c_q$, *then* $L_{i+1}^* > L_i^*$, $i=1,2,\ldots$, *and* $L_{i+1}^* \cong L_i^*(c_q+c_d)/(c_p+c_d)$ *as* $k\to\infty$.

Theorem 3 leads to the following ***qualitative conclusions*** (Yatsenko and Hritonenko 2008b):

- In the case of *the proportional TC*, $c_q=c_p=c$, the optimal asset lifetime is constant: $L_i^* = L^*$, $i=1,2,\ldots$. The optimal lifetime L^* is shorter when the proportional TC is more intense (when c is larger).
- If the O&M cost $q(t,u)$ decreases in t at the fixed age u-t slower than the capital cost $p(t)$, then the optimal lifetime decreases, $L_i^*>L_{i+1}^*$, $i=1,2,\ldots$ (and converse).
- For the same O&M cost rate c_q, the optimal lifetime L_i^* is *shorter* when the capital cost rate c_p is larger (i.e., when the TC in capital cost is more intense).
- For the same capital cost rate c_p, the optimal lifetime L_i^* is *longer* when the O&M cost rate c_q is larger (i.e., when the TC in O&M cost is more intense).

4 Optimal Asset Lifetime under Discontinuous TC

The *discontinuous TC* is understood as an instantaneous change in technological parameters due to the introduction of a new breakthrough vintage (model) of machines (Hopp and Nair 1991; Bean et al. 1994; Rajagopalan et al. 1998; Rogers and Hartman 2005). Rogers and Hartman (2005) consider the case of periodic breakthroughs in the maintenance cost at the presence of continuous TC. Other authors assume their stochastic appearance (stochastic times or sizes) (Hopp and Nair 1991; Rajagopalan et al. 1998).

In model (1), the dynamics of the *variable* optimal asset lifetime under discontinuous TC requires solving the nonlinear equation (6) for *non-smooth* functions p and q. To do that, we will combine analytic investigation and numeric simulation.

To analyze model (1) on real replacement data, the authors have developed numeric algorithm and software for solving equation (6). The algorithm is based on the *rolling horizon idea* and assumes that the trend of the continuous TC remains the same in some future (Hritonenko and Yatsenko 2008d). It is implemented in MS Excel/VBA and is provided to all interested readers at request. The dataset for numeric simulation is taken from a discrete model (Regnier et al, 2004) used to simulate the optimal car replacement on automotive industry data for 1985-1998. We employed this data for the analysis of the continuous model (1) in (Yatsenko and Hritonenko 2008b). The basic dataset in model (1), (9) is

$$c_p=0, \quad c_q=0.05, \quad c_d=1.34, \qquad r=0.14. \tag{11}$$

At (11), the optimal lifetime is approximately constant and found from the nonlinear equation (10) as $L^*\approx 10.5$ years (Yatsenko and Hritonenko 2008b).

We assume that the TC breakthroughs can appear at given instants on the background of the continuous exponential TC and deterioration (9). The breakthroughs will normally impact both TC parameters $p(t)$ and $q(t,u)$. To understand the dynamics, we start with the simplest case of one breakthrough.

4.1 Case of one technological breakthrough in capital cost

Let us assume that q is exponential (9) and the discontinuous TC causes the discontinuity in the capital cost $p(t)$ at instant t_1:

$$p(t)=\begin{cases}\tilde{p}(t) & \text{if} \quad t<t_1, \\ B_p\tilde{p}(t) & \text{if} \quad t\geq t_1,\end{cases} \qquad\qquad B_p<1, \tag{12}$$

where $\tilde{p}(t)$ is given by (9). Then, equation (6) has the form

$$q_0e^{-(c_q+c_d)R(t)}-q_0e^{-c_qt}-(c_q+c_d)q_0\int_t^{R^{-1}(t)}e^{-(r-c_d)(u-t)}du-e^{c_dt}(rp(t)-p'(t))=0, \tag{13}$$

$t\in[0,\infty)$, or after evaluating the integral,

$$e^{-(c_q+c_d)(t-R(t))}=F(t,\tau^{-1}(t),p(t),p'(t)), \tag{14}$$

where

$$F(t,R^{-1}(t),p(t),p'(t))=e^{-c_qt}+(c_q+c_d)[1-e^{-(c_q+c_d)(R^{-1}(t)-t))}]/r+e^{c_dt}[rp(t)-p'(t)]/q_0,$$

Equation (13) represents a recurrent relation between the optimal $R(t)$ and its inverse $R^{-1}(t)$. In (Yatsenko and Hritonenko 2005), it is treated as a nonlinear delay equation with respect to the unknown function $R(t)$, $t\in[0,\infty)$. Namely, if we know $R(t)$ (or its approximation) starting some instant τ_f, on $[\tau_f, R^{-1}(\tau_f)]$ or a longer interval, the we can solve (14) sequentially from right to left and obtain $R(t)$ at $t\in[\tau_0, \tau_f)$. In the case (12) with a jump in $p(t)$ at $t=t_1$,

$$p'(t)=\begin{cases}-c_pp_0e^{-c_pt} & \text{if} \quad t<t_1, \\ p_0e^{-c_pt}[-c_p+(1-B_p)\delta(t-t_1)] & \text{if} \quad t=t_1, \\ -c_pB_pp_0e^{-c_pt} & \text{if} \quad t>t_1,\end{cases} \tag{15}$$

where $\delta(t)$ is the *Dirac delta-function*. Substituting (12) and (15) into (13), we obtain the exact recurrent formula

$$R(t) = (c_q+c_d)^{-1}[\ \ln q_0-\ln(F(t, R^{-1}(t), p\ (t), p'(t)))\ ] \tag{16}$$

for the (13) solution R. Formula (16) worked well in (Hritonenko and Yatsenko, 2008b) in the case (9) of continuous p and q. Now, (16) produces a continuous optimal $R(t)$ on the interval (t_1,∞) right to the jump. However, by (15), the function $F(t, R^{-1}(t), p(t), p'(t))$ includes the delta-function $\delta(t-t_1)$. Correspondingly, the unknown optimal $R(t)$ also includes $\delta(t-t_1)$ and $R(t_1)=-\infty$.

So, the solution $R(t)$ of our *unconstrained* optimization problem (1),(2) in case (15) includes a jump to $-\infty$ at time t_1, which is not feasible from both theoretical and practical viewpoints. In practice, the optimal $R(t)$ recommends the use of a very old machine at $t=t_1$ for a negligibly short period of time. The theoretical problem is that $R(t)$ is not monotonic at $t=t_1$, hence, the unique inverse $R^{-1}(t)$ does not exist at $t<R(t_1)$ and $R(t)$ cannot be constructed by (16) at $t<R(t_1)$. Hence, the replacement problem (1),(2) has no solution in the jump case (15). We would like to emphasize that this situation is specific only for technological breakthroughs.

To correct the situation and use equation (14) for simulating optimal replacement policies in the presence of TC jumps, we have to impose an additional constraint on the replacement problem (1),(2) in the jump case (15). A natural idea is to keep the regeneration time $R(t)$ monotonic (non-decreasing). Technically, it means solving equation (14) with restriction $R'(t)\geq 0$.

Let introduce the smoothened *monotonic replacement* trajectory $\hat{R}(t)$, $t\in[0,\infty)$. The function $\hat{R}(t)$ is obtained by removing the jump to $-\infty$ in a neighborhood of t_1 from the trajectory $R(t)$. Analytically, we replace the derivative $p'(t)$ in (15) with its smoothened monotonic version

$$\hat{p}(t)=\begin{cases} -c_p p_0 e^{-c_p t} & \text{if} \quad t<t_1, \\ -c_p B_p p_0 e^{-c_p t} & \text{if} \quad t\geq t_1, \end{cases} \tag{17}$$

Then, the corresponding $F(t, R^{-1}(t), p(t), \hat{p}(t))$ in (16) does not include the delta-function $\delta(t-t_1)$ and the corresponding optimal $\hat{R}(t)$, $t\in[0,\infty)$ is monotonic[4].

[4] Another smoothing technique is required for parallel asset replacement models (Hritonenko and Yatsenko, 2003, 2005).

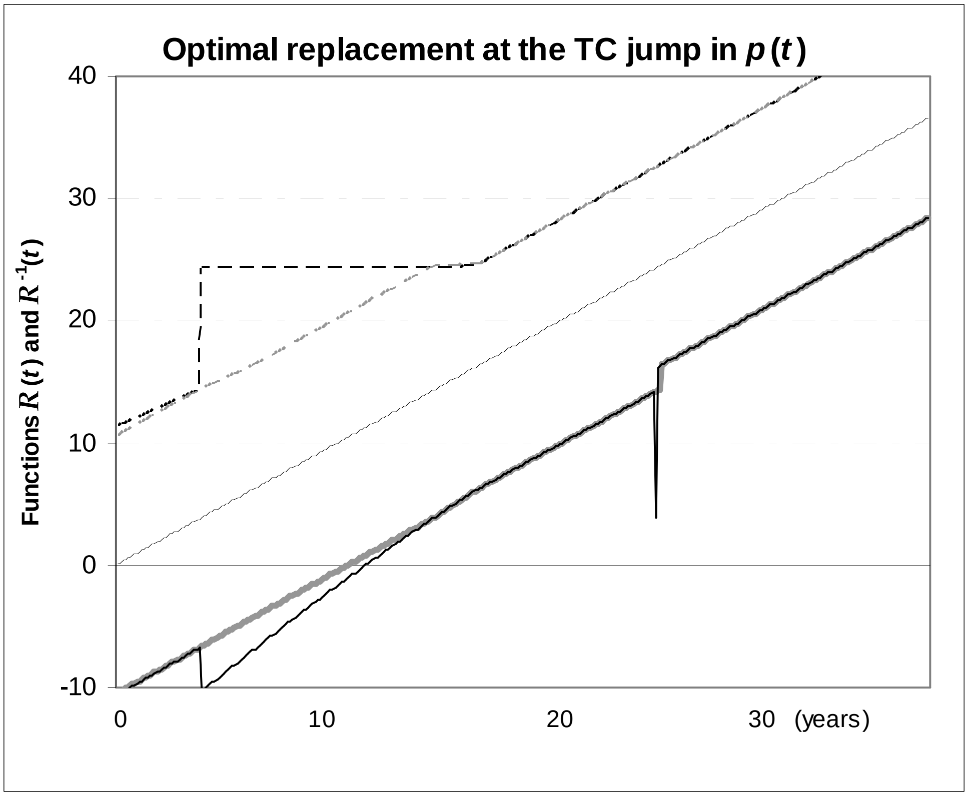

Figure 1. Optimal asset replacement in the case of the TC jump B_p=0.35 at t_1=25 (and the TC rates c_p=c_q=0.05). The solid line corresponds to the previous replacement time $R(t)$ and the dashed line is the inverse $R^{-1}(t)$. The gray lines describe the monotonic replacement time $\hat{R}(t)$ and its inverse $\hat{R}^{-1}(t)$.

To observe the actual dynamics of $R(t)$ and $\hat{R}(t)$, we provide a numeric simulation of equation (13). The parameters are B_p=0.35, t_1=25, the initial τ_1=0, the horizon length is T=60 years, the discretization step h=0.1, and the other parameters are as in (11). The solid line in Figure 1 demonstrates the simulated solution $R(t)$ of (13) and the dashed line shows the inverse $R^{-1}(t)$. As expected, the functions $R(t)$ and $R^{-1}(t)$ are symmetric with respect to the (dotted) straight line $y=t$ also shown in Figure 1. The behaviour of $R(t)$ is similar to the one predicted by formula (16). Since the simulation is done with the finite discretization step, the delta function in (16) at $t=t_1$ is replaced with the negative $R(t)$ jump of a finite size because of numeric differentiation. The size of the jump essentially depends on the value of h (it $\to -\infty$ at $h\to 0$). So, in simulation, the TC jump (12) at $t=t_1$ is compensated in (13) by the "approximate" delta-function in $R(t)$ at $t=t_1$.

The monotonic replacement trajectory $\hat{R}(t)$ is indicated in Figure 1 with the gray line. Then, the left–hand side of equation (13) has a positive jump at $t=t_1$ because it is not longer compensated by the delta-function in $R(t_1)$. Hence, the constructed $\hat{R}(t)$ is not optimal at the moment $t=t_1$. At $t=t_1$, instead of bringing an older asset, the policy is to wait until a cheaper challenger becomes available.

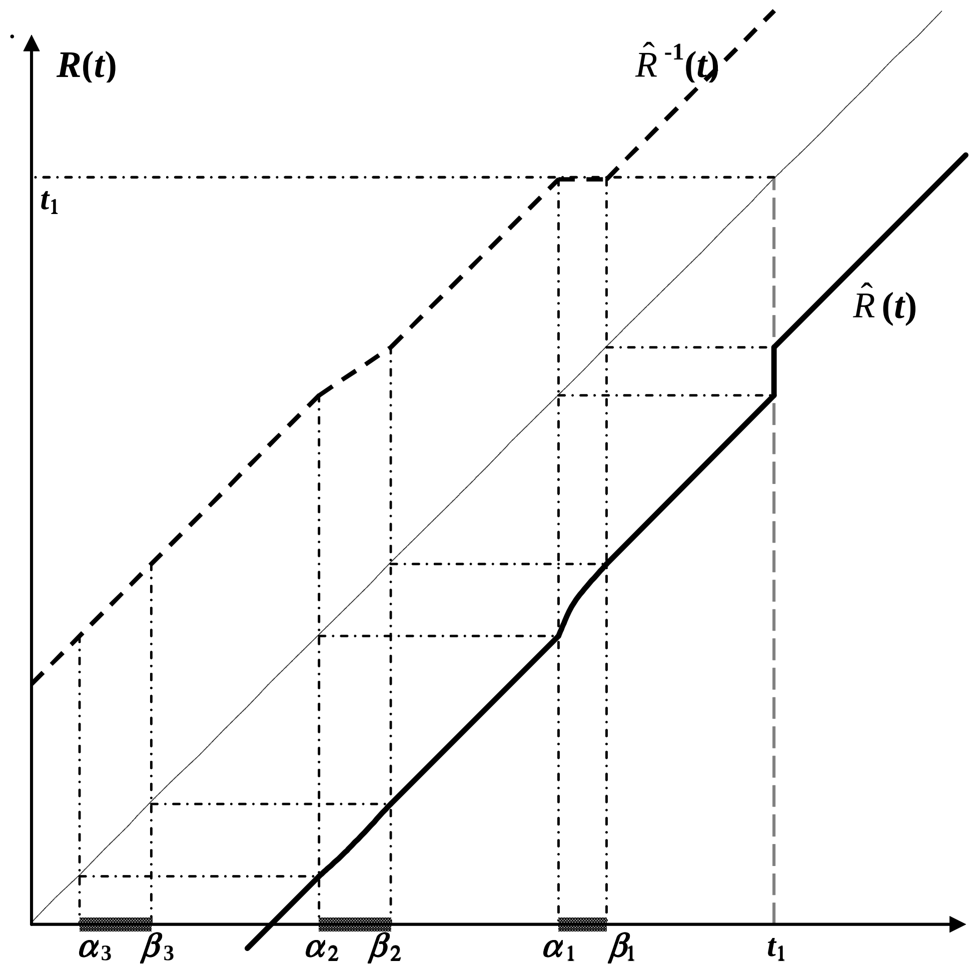

Figure 2. The optimal asset replacement in the case of the TC jump in $p(t)$ at t_1. The solid line is the monotonic replacement time $\hat{R}(t)$, the dashed line is the inverse $\hat{R}^{-1}(t)$, and the straight 45° line highlights the symmetry between them. The marked intervals are $\Delta_1=[\alpha_1, \beta_1]$, $\beta_1=\hat{R}(t_1)$, $\Delta_2=[\alpha_2, \beta_2]$, $\Delta_3=[\alpha_3, \beta_3]$, where the trajectory $\hat{R}(t)$ is not optimal (see Theorem 5 below).

Let us discuss the impact of our smoothing technique on the optimality of the solution to problem (1)-(2). The monotonic replacement trajectory $\hat{R}(t)$ is depicted in Figure 2 in more details. By construction, $R(t)$ at $t=t_1$ determines its inverse $R^{-1}(t)$ at $t=R(t_1)$. The non-optimality of $\hat{R}(t)$ at $t=t_1$ leads to its non-optimality in the neighborhoods of the times $R(t_1)$, $R(R(t_1))$, .., and so on, such that $\ldots<R(R(t_1))< R(t_1)< t_1$. Namely, by (14), the vertical segment $[\alpha_1, \beta_1]$ of $\hat{R}(t)$ at $t=t_1$ in Figure 2 produces a horizontal segment $\hat{R}^{-1}(t)\equiv t_1$ in $\hat{R}^{-1}(t)$ at $t\in\Delta_1=[\alpha_1, \beta_1]$ such that the next replacement time is t_1 for all $t\in\Delta_1$. By (14), this segment causes the irregular part

$$\hat{R}(t) = (c_q + c_d)^{-1}[\ \ln q_0 - \ln(F(t, t_1, p(t), \hat{p}(t))]$$

in $\hat{R}(t)$ at $t\in\Delta_1$. If a replacement time $\tau_k^*\in\Delta_1$, then the next replacement time is $\tau_{k+1}=t_1$, which is not optimal. In turn, the irregular $\hat{R}(t)$ on Δ_1 leads to a perturbation in $\hat{R}^{-1}(t)$ and in $\hat{R}(t)$ over the interval $\Delta_2=\hat{R}(\Delta_1)$. If the time t_1 is large enough, such echoed perturbations appear (from right to left) at $\hat{R}(t_1)$, $\hat{R}(\hat{R}(t_1))$, $\hat{R}(\hat{R}(\hat{R}(t_1)))$,…, until $\Delta_{M+1}<\tau_0$ for some number $M>0$.

We will refer to such perturbations as the *anticipation echoes* because they appear *before* the jump time t_1 in *the anticipation of the future TC breakthrough* at t_1. The anticipation echoes disseminate from right to left, starting at the jump time. Such irregularities represent a common pattern in serial optimal replacement models (Hritonenko and Yatsenko 1996b, 2005, 2008a; Yatsenko and Hritonenko 2005). First three echoes Δ_1, Δ_2, Δ_3 are shown in Figure 2. Thus, the constructed policy involves a finite set Δ of intervals

$$\Delta = \{\Delta_M, ,\ldots, \Delta_2, \Delta_1: \text{ if } \tau_k^*\in\Delta_l, \text{ then } \tau_{k+l}=t_1\}. \tag{18}$$

The original policy π is impacted and becomes non-optimal *if only if* one of the replacement time τ_k falls inside Δ.

We summarize the above outcome with the following analytic conclusion that relates the constructed monotonic replacement trajectory $\hat{R}$ to the optimal replacement policy π^*.

Theorem 4. In the case of discontinuous TC with jump (12), if a policy $\pi^*=\{\tau_k^*, k=1,2,\ldots\}$ satisfies (7) and all the times τ_k^*, k=1,2,…, do not belong to the set (18) and do not coincide with the jump time t_1, then π^* is an optimal policy in the problem (1)-(2).

Proof follows directly from (14)-(17). In this case, the above solution procedure produces a unique monotonic replacement trajectory $\hat{R}(t)$ over $[0,\infty)$, shown in Figure 2. The functions (15) and (17) differ only at $t=t_1$. Correspondingly, by (14) and (16), the smoothened recurrent trajectory $\hat{R}(t)$ does not satisfy equation (13) only at t_1 and on the intervals Δ_1 , Δ_2 ,…, Δ_M. If no times τ_k^*, k=1,2,…, coincide with the time t_1, then all the times τ_k^*, k=1,2,…, are optimal. The theorem is proved.

Thus, we give up the optimality during the anticipation echoes (18) preceding the jump time t_1 and shown in Figure 2 to be able to produce an optimal replacement strategy over the infinite horizon $[0,\infty)$. The anticipation echoes deteriorate fast. In Figure 1, the first anticipation echo Δ_1 is visible at $t\approx15$ years, the second echo Δ_2 is barely visible at $t\approx6$ years, and the third echo Δ_3 is out of the graph range and is too smooth to see.

To analyze the dependence of the replacement process on the TC intensity, we have solved equation (6) in case (9) for c_q=0.05 and five different values c_p=0.15, 0.1, 0.05, 0, –0.02. Figure 3 displays the asset lifetime $L(t)=t-\hat{R}(t)$ for different scenarios $c_p<c_q$, $c_p\neq c_q$, and $c_p>c_q$. Several important effects are visible in Figure 3. Namely, the optimal lifetime of assets:

- is variable in the general case,
- decreases or increases depending on the sign of c_p-c_q,
- monotonically increases in time immediately before the TC jump (*anticipation effect*);
- produces quickly weakening replacement echoes during the regeneration periods preceding the TC jump (*anticipation echoes*).
- *is shorter for a larger capital cost rate* at every point of the planning horizon.

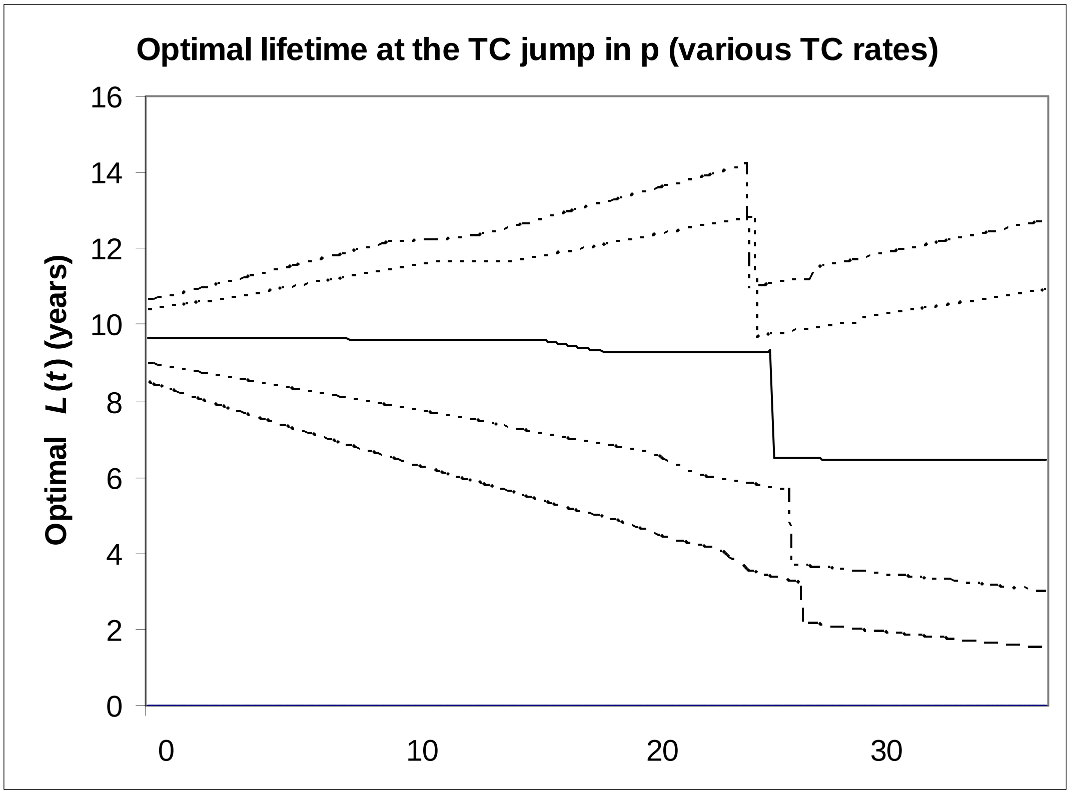

Figure 3. The smoothened optimal asset lifetime in the case of the TC jump B_p=0.35 at t_1=25, the O&M cost rate c_q=0.05, and the different capital cost rates c_p=0.15, 0.1, 0.05, 0, –0.02 (shown from top to bottom).

In the presence of technology jumps, more intensive TC in the capital cost requires more frequent replacements (the same holds for continuous TC by Theorem 3). When the optimal asset lifetime decreases, the echoes appear more frequently (see two lower curves in Figure 3).

Let us focus on the third (solid) line in Figure 3 that corresponds to the case c_p=c_q=c=0.95 of *the proportional TC*. It appears that, under the jump (23), the lifetime permanently changes from the *constant value* L_b≈ 9.7 years before the jump to the *constant* L_a≈ 8.2 years after the jump.

At the proportional TC with no technology jumps, the optimal lifetime is constant and known analytically by Theorem 3. We can prove an analytic result for the proportional TC with jumps.

Theorem 5. If c_p=c_q, the function $p(t)$ has the jump (12) at t=t_1, and all the replacement times τ_k^* do not belong to the set (18) and do not coincide with t_1, then the optimal lifetimes

$L_k^* = \tau_k^* - \tau_{k-1}^* \equiv L_b$ while $\tau_k^* > t_1$, and the optimal $L_k^* \to L_a$ when $\tau_k^* < t_1$ and k decreases to 1. The constants L_a and L_b are found from equation (10) at $p \equiv \tilde{p}$ and $p = B_p \tilde{p}$ correspondingly.

Proof. By Theorem 3, equation (6) has a unique solution $L(t) \equiv L_b$=const on the infinite interval (t_1, ∞), where $p(t)$ is the exact exponent $\tilde{p}(t)$ from (9). Then by Theorem 4, the optimal L_k^* coincides with L_b while $\tau_k^* > t_1$. Now, since $R(t) = t - L_b$ is uniquely known at $t \in (t_1, \infty)$, we can determine the smoothened solution $\hat{R}(t)$ of equation (13) over the interval $[0, t_1)$ using formulas (16) and (17). As shown in (Yatsenko and Hritonenko 2005), the constructed $R(t)$ by (16) converges and strives to $t - L_a$, when $t \to 0$ and t_1 is large. The constant L_a is the unique constant solution of (13) at $p \equiv B_p \hat{p}$. By Theorem 4, the optimal L_k^* coincides with $\tau_k^* - R(\tau_k^*)$. The theorem is proven.

4.2 A single breakthrough in Q&M cost

Now let us assume that the discontinuous TC causes the discontinuity in $q(t,u)$ at instant t_1:

$$q(t,u) = \begin{cases} \hat{q}(t,u) & \text{if} \quad t < t_1, \\ B_q \hat{q}(t,u) & \text{if} \quad t \ge t_1, \end{cases} \qquad B_q < 1, \tag{19}$$

whereas $\hat{q}$ and p are the exponents given by (9).

The numeric simulation of this case has been provided at B_q=0.35, t_1=20, and shown in Figures 4 and 5. The qualitative picture is similar to the one shown in Section 4.1 with some additional complications. Namely, as opposed to Figure 1, the first discontinuity in $R(t)$ happens at the instant $R^{-1}(t_1)$. Indeed, instead of (13), now the left–hand side of equation (6) is instantly changed by factor B_q when $a(t_c) = t_1$, i.e., at $t_c = a^{-1}(t_1)$. The corresponding small $R(t)$ jump is clearly visible in Figure 4 at $t_c \approx 30$. It is preceded by the numeric "delta-function" in the optimal $R(t)$ at $t = t_1 = 20$ similar to shown in Figure 1. The delta-function has similar causes as in the case (12) of discontinuous $p(t)$. It is removed in the smoothened *monotonic replacement time* $\hat{R}(t)$ using the solution technique of Section 4.1.

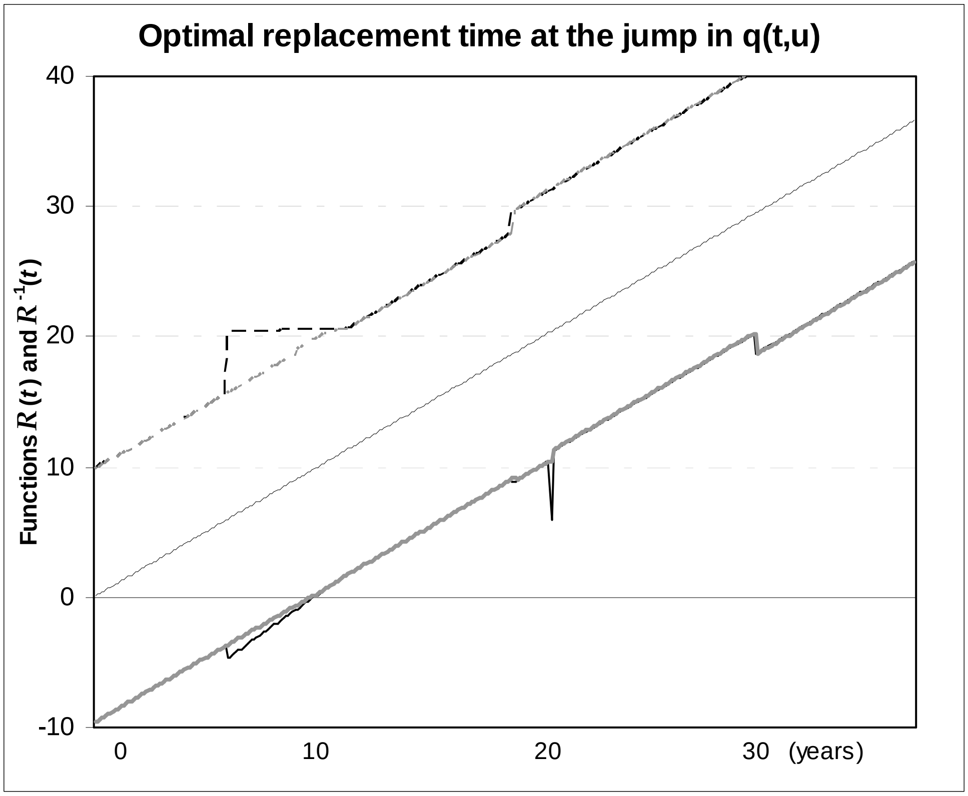

Figure 4. Case of the TC jump B_q=0.35 in the Q&M cost at t_1=20. The solid line is the previous replacement time $R(t)$ and the dashed line is the inverse $R^{-1}(t)$, and the solid gray line is the "smoothened" replacement $\hat{R}(t)$.

Figure 5 illustrates the smoothened lifetime $L(t)=t-\hat{R}(t)$ of assets in the case of the TC jump B_q=0.6 at t_1=15 for c_q=0.05 and five different values c_q=0, 0.02, 0.05, 0.08, 0.1. Figure 5 demonstrates that, at the fixed c_p, *the optimal dynamic lifetime is always longer for a larger O&M cost rate* c_q (for both continuous and discontinuous TC). Also, the optimal asset lifetime increases in t when the O&M cost rate c_q is larger than c_p.

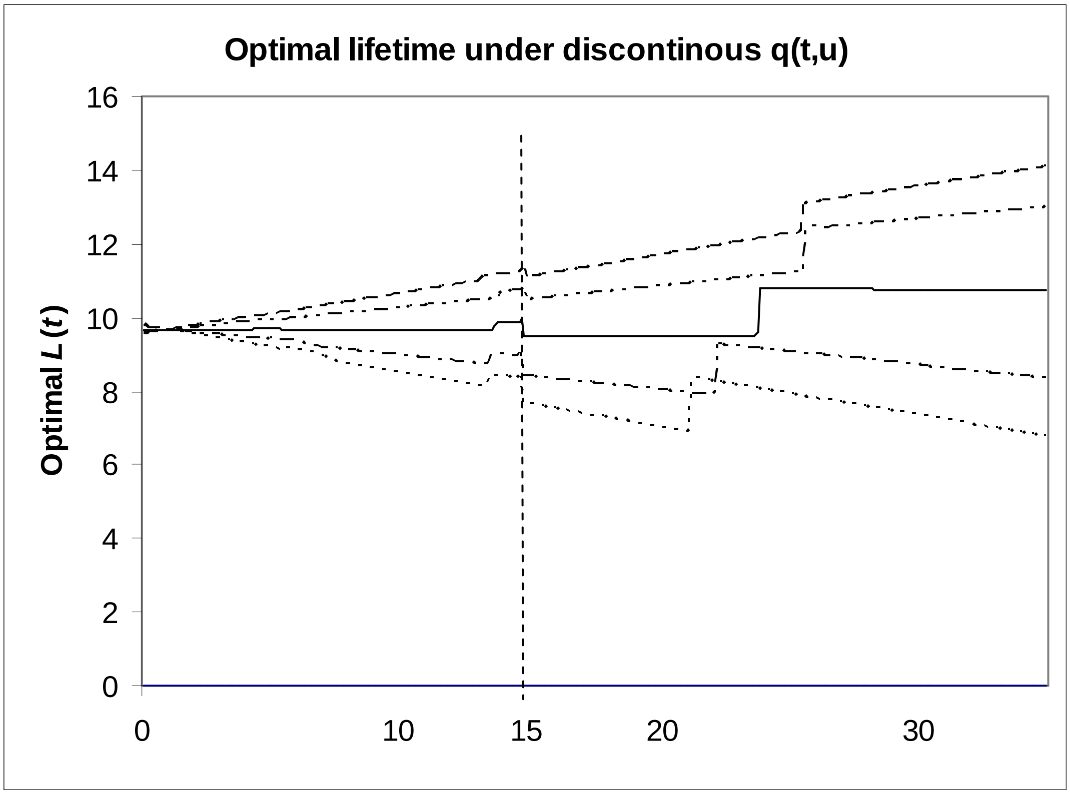

Figure 5. The smoothened optimal asset lifetime in the case of TC jump B_q=0.6 at t_1=15, the capital cost rate c_q=0.05, and the different O&M rates c_q=0, 0.02, 0.05, 0.08, 0.1 (shown from top to bottom).

As in Section 4.1, the optimal lifetime $L(t)$ possesses anticipation echoes before the jump time, that disseminate to the left. The first anticipation echo is visible just before $t\approx15$ years, the second echo is much smaller but also visible around t=3-7 years, and the third echo is out of the graph range and too small to see. The relative jumps are smaller than for similar values of B_p.

4.3. A single breakthrough in both Q&M and capital costs

Now let us assume that the discontinuous TC causes the discontinuities (12) and (19) in both $p(t)$ and $q(t,u)$ at the instant t_1. The qualitative picture remains essentially the same as above. Figure 6 illustrates the variable optimal lifetime for the TC jump B_p=B_q=0.65 in both p and q at

t_1=15, c_p=0.95, and various rates c_q=0.86, 0.9, 0.95, 1, 1.02. As in the previous case, the optimal asset lifetime *is larger* when the continuous TC rate in *O&M cost* is larger.

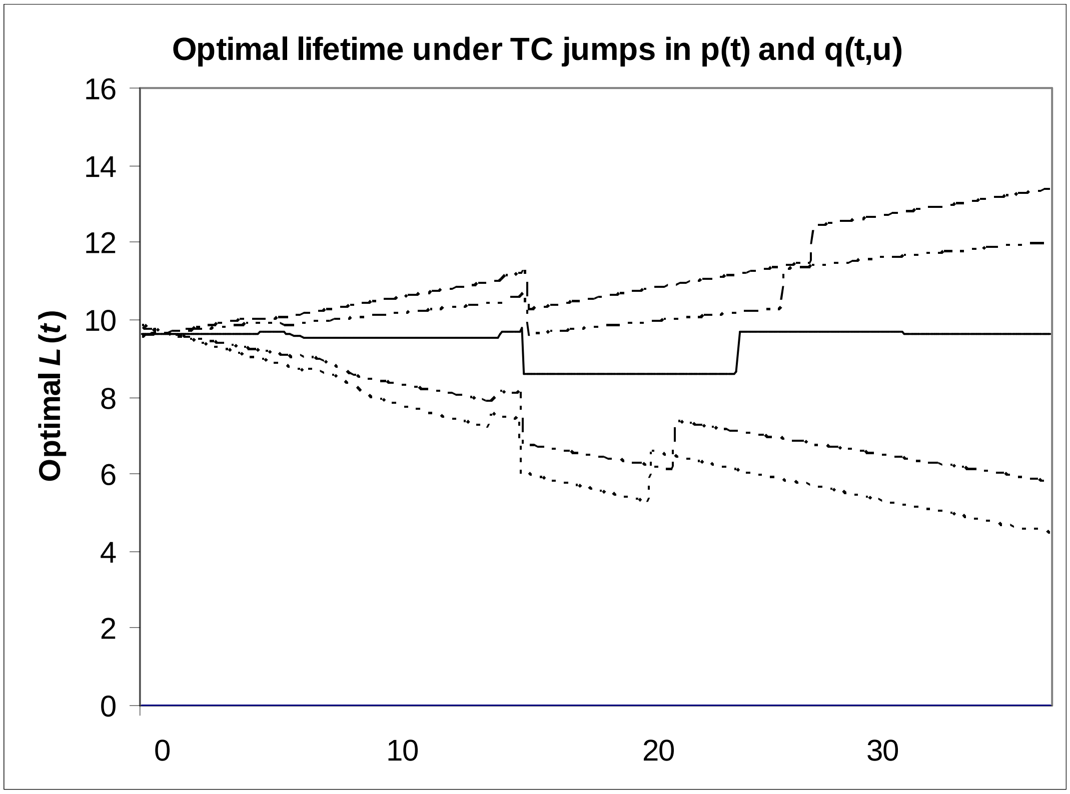

Figure 6. The smoothened optimal asset lifetime in the case of TC jump B_p=B_q=0.65 in both Q&M and capital costs at t_1=15, and the various rates of continuous TC c_p=c_q=0.15, 0.1, 0.05, 0, –0.02 (shown from top to bottom).

Figure 6 demonstrates that the "proportional" TC jump B_p=B_q does not impact the optimal lifetime $L(t)$ permanently and $L(t)$ returns to the previous trajectory after one regeneration period. It is especially clear in the case when *the continuous TC* is also *proportional*, c_p=c_q=0.95 (the third solid line in Figure 6). Then, by Theorem 2, the optimal lifetime is constant, $L_k^* = \tau_k^* - \tau_{k-1}^* \equiv L^*$, k=1,2,…, and is found from equation (10) as L^*≈9.6 years when there is no TC jumps. In the case of the TC jump, the optimal variable $L(t)$ is initially constant (9.6 years) and returns to this constant value when time $t > a^{-1}(t_1)$≈25 years. This fact can be proven analytically using equation (16). Namely, the following property holds.

Theorem 6. If $B_p=B_q$, $c_p=c_q$, $p(t)$ has the jump (12) at $t=t_1$, $q(t,u)$ has the jump (19) at $t=t_1$, and all the replacement times τ_k^*, k=1,2,…, do not belong to the set (18) and do not coincide with t_1, then the optimal $L_k^*\equiv L^*$ while $\tau_k^*>a^{-1}(t_1)$, and $L_k^*\rightarrow L^*$ when $\tau_k^*<t_1$ and k decreases to 1.

Proof is similar to the proof of Theorem 5. The difference is that, in this case, the optimal $L(t)= L^*$=const is determined from the nonlinear equation (10) in the absence of TC jump, if both p and q are exponential on $[0,\infty)$. Equation (6) has a unique solution $L(t)\equiv L^*$ on the infinite interval $(a^{-1}(t_1),\infty)$. So, by Theorem 5, the optimal L_k^* coincides with L^* while $\tau_k^*>a^{-1}(t_1)$. As shown in Section 4.3, the first discontinuity in $a(t)$ happens at the instant $a^{-1}(t_1)$.

Since $\tau(t)$ is known at $t\in(t_1,\infty)$, we determine the unique $\tau(t)=t-L(t)$ on the previous interval $(0, t_1)$ using the recurrent expression (16) from the right to the left. In this case, the iterations (16) converge and strive to $t-L^*$, when $t\rightarrow 0$ and t_1 is large. The optimal L_k^* coincides with $\tau_k^*-R(\tau_k^*)$ by Theorem 4.

The theorem is proven.

The sizes of the instantaneous jumps B_p are B_q in the car prices and Q&M costs have been intentionally chosen too large to better illustrate the nature of the response. In the considered real example, more reasonable numbers for these jumps in the range 0.9-0.95 are not visible in the above figures.

4.4 Case of several breakthroughs

Many technological breakthroughs can arise at different times. In model (1), the case of several TC jumps is handled similarly to the above Sections 4.1-4.3 with single TC jumps. It appears that in our framework the presence of several TC jumps does not add a new complexity to the analytic and numeric investigation.

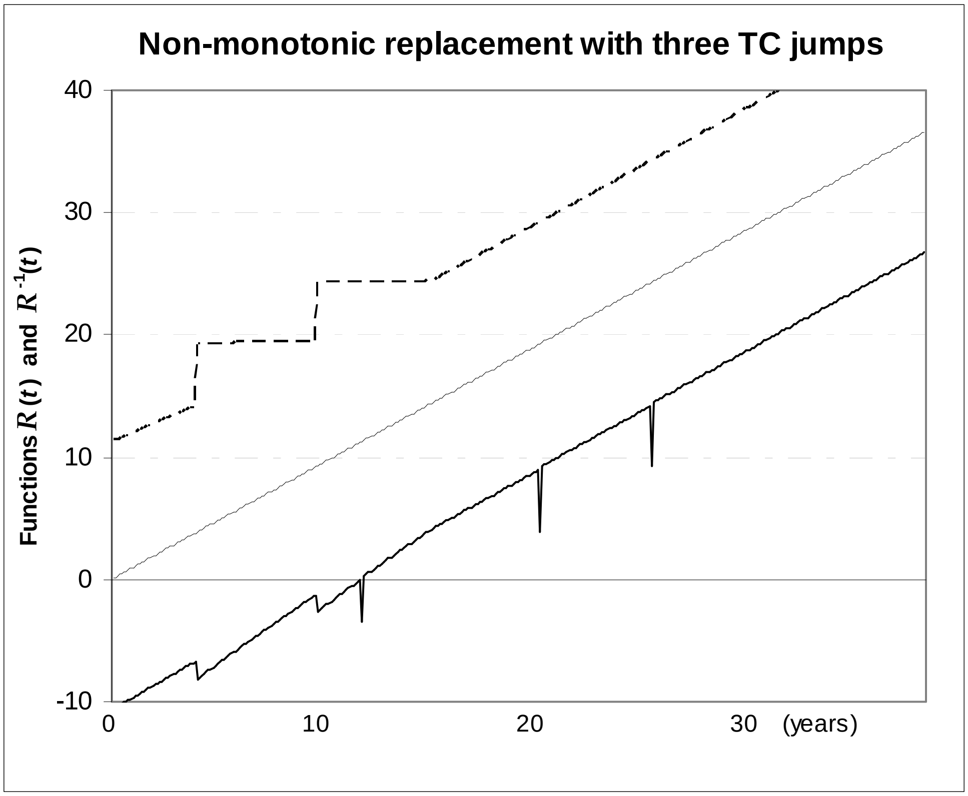

Figure 7. The non-monotonic optimal replacement time $R(t)$ in the case with three TC jumps: B_p=0.93 at t=12, B_p=0.1 at t=20, and B_p=0.12 at t=25 (and the continuous TC rates c_p= c_q=0).

We have provided a series of experiments with several breakthroughs. The dynamics of the optimal replacement has been analyzed for several scenarios. Figures 7 and 8 illustrate that, in both considered cases of the *non-monotonic* (original) and *monotonic* (smoothened) regeneration time, the irregularities caused by earlier jumps sequentially superimpose on the top of irregularities and echoes caused by the later jumps. Since the echoes caused by every jump weaken fast, the process strives to the continuous TC dynamics. The jump sizes and times have been chosen arbitrarily in these figures.

Summarizing the results of this section, we notice that *the TC jump causes repetitive irregularities (echoes)* in the optimal asset lifetime *before the jump time* t_1. The *echoes* in the optimal asset lifetime disseminate to the left of t_1 and are damped pretty fast (the only one preceding echo is visible in Fig. 2, 4, and 5). If the time before jump increases, the optimal lifetime returns quickly to a continuous trajectory for the continuous TC.

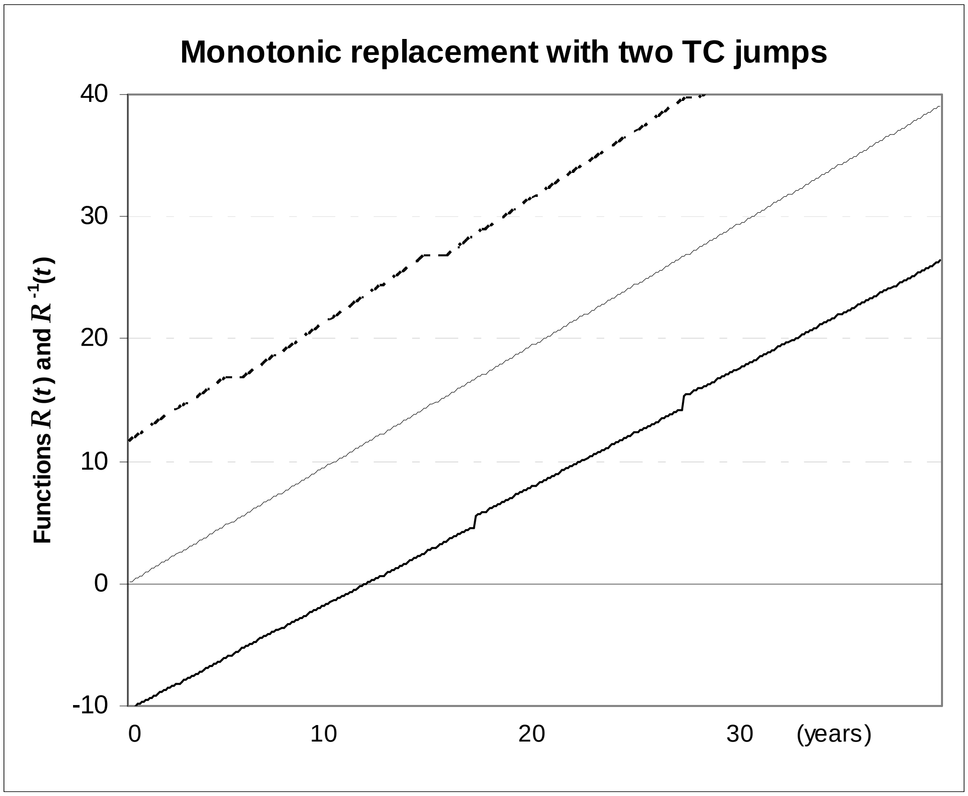

Figure 8. The smoothened optimal asset replacement time $\hat{R}(t)$ in the case of two TC jumps B_p=0.65 at t=21 and B_p=0.6 at t=27 (and the continuous TC rates c_p=0, c_q= 0.05).

Following (Rogers and Hartman, 2005), we have also simulated periodical technological breakthroughs arising after the same time period. The behavior is similar if the jump size and frequency are moderate. The echoes caused by every jump weaken fast (when the time *before* the jump increases). The replacement process strives to the continuous TC dynamics, except for the jump points and the corresponding anticipation echoes (where no policy is optimal).

5 Concluding Remarks

The paper considers a model of the optimal asset replacement on the infinite horizon under general assumptions of discontinuous TC. The model involves the variable lifetime of assets. The variable lifetime allows providing a more refined analysis of the replacement problem. The employed technique from (Hritonenko and Yatsenko 2005, 2008b, 2008c) does not directly solve the formulated optimization problem but analyzes a nonlinear equation derived from extremum conditions.

We have shown that there is no feasible optimal replacement decision exactly at the time of TC jump (a technological breakthrough). More exactly, the optimization problem recommends the use of an infinitely old machine at that time for a negligibly short period of time. Every TC jump also creates a set of *anticipation echoes* in the optimal asset lifetime during the regeneration periods preceding the jump. During these echoes, the replacement is impacted by the future TC jump and no optimal policy exists. When the time before the jump increases, the echoes decline quickly and the optimal asset lifetime strives to an optimal trajectory for the continuous TC. So, the optimal lifetime of assets appears to be stable under the TC jumps (except for the jump times).

We have provided theoretic and numeric analysis of the optimal asset replacement under various assumptions about continuous and discontinuous TC. The results indicate that, in the cases of both continuous and discontinuous TC, the *optimal asset lifetime*:

- is *always smaller for more intensive TC in the capital cost*;
- is *always smaller for more intensive proportional TC* (with equal rates of capital and Q&M costs);
- is *always larger for more intensive TC in the Q&M cost only*.

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