Max-Planck-Institut für Mathematik in den Naturwissenschaften Leipzig

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Preprint no.: 112 2013

Physica A 392 (2013) 6596–6602

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Opinion evolution on a BA scaling network

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HIGHLIGHTS

- A model of opinion evolution with parameter q is introduced.
- There occurs a phase transition as the value of q is smoothly varied.
- The critical parameter value q_c is a monotone increasing function of m.
- The mean-field approximation can explain the transition nature.
- In the thermodynamic limit and for m very large, q_c can reach its upper limit, 0.5.

ARTICLE INFO

Article history:
Received 24 December 2012
Received in revised form 2 August 2013
Available online 5 September 2013

Keywords:
Opinion evolution
Mean-field theory
Average magnetization
Standard deviation
Critical point
Phase transition

ABSTRACT

In this paper, the dynamics of opinion formation is investigated based on a BA (Barabási–Albert) scale-free network, using a majority–minority rule governed by parameter q. As the value of q is smoothly varied, a phase transition occurs between an ordered phase and a disordered one. By performing extensive Monte Carlo simulations, we show that the phase transition is dependent on the system size, as well as on m, the number of edges added at each time step during the growth of the BA scaling network. Additionally, some theoretical analysis is given based on mean-field theory, by neglecting fluctuations and correlations. It is observed that the theoretical results coincide with results from simulations, especially for very large m.

1. Introduction

Opinion evolution dynamics has received much attention in recent decades [1–9]. Defining elementary processes that enable a group to reach a consensus is the main purpose of these models. Typically, these models consist of a number, say N, of agents, and the opinion of each of them is influenced by those of its neighbors. Highlighted and well-investigated examples of binary-choice opinion evolution models include the voter model [5], the majority-rule model [10], social impact theory [11], and the Sznajd model [12]. Extensive studies along this direction can be found in the literature [7,13–19].

Clifford and Sudbury [20] firstly considered the voter dynamics as a tool to study the competition among species, which was then named the "voter model" by Holley and Liggett [21]. The voter model is used very widely because it manifests a nonequilibrium stochastic process which can be exactly solved in systems with various dimensions [5]. The majority-rule model was proposed by Krapivisky and Redner [22] and was then applied to describing public debates [23]. The key point of this model is that all agents will absorb the majority opinion among the group of interest. Models based on social impact theory are used to describe how individuals feel the presence of people around them and how they in turn influence others. The Sznajd model was proposed based on the principle that it is easier to convince two or more persons than a single individual.

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0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.08.047

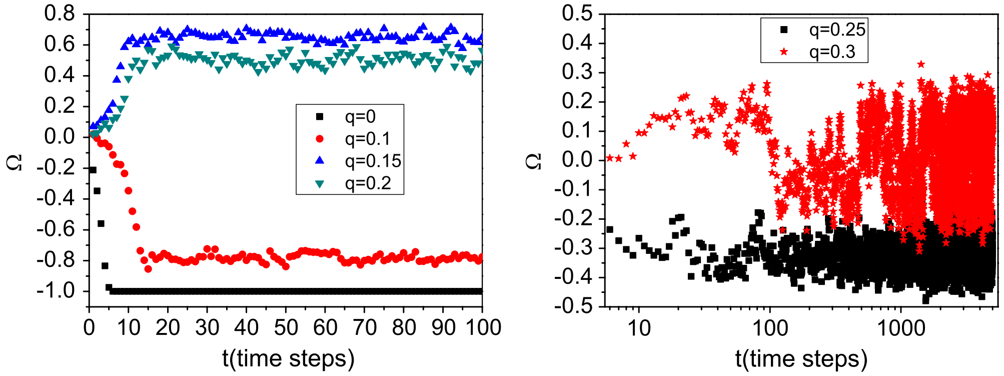

Fig. 1. The collective behavior (characterized by the average magnetization) of the system. Ω as a function of the number of time steps t for a BA scaling network ($N = 1000$ and $m = 4$), in which only one realization is given for each q.

Bing-Hong Wang et al. [24] investigated how social diversity affects the formation of a global consensus in opinion dynamics. By assigning each agent a weight, proportional to the power of its degree, they found that there exists an optimal power exponent that yields the shortest time interval for the consensus to be reached. This result enables one to identify the role of heterogeneous degree distributions of agents in the dynamics of opinion formation. Sampaio-Filho and Moreira [25] studied the dependence of the collective behavior of an entire system on the persuasive cluster spin (PCS) size, using the block voter model with noise based on two-dimensional square lattices. The majority rule and the minority rule were combined with the use of a single parameter q. The authors concluded that, at the critical threshold, there is a phase transition from an ordered state to a disordered one.

The goal of this work is to study the dynamics of opinion formation on a BA scaling network, which is then compared to a theoretical analysis. The rest of the paper is organized as follows. In Section 2, we introduce the model used for Monte Carlo simulations. In Section 3, we present our results and discussion, while in Section 4 we provide a theoretical analysis as a comparison to the simulations. Section 5 presents our conclusions.

2. Model

We modify a simple model proposed in Ref. [25] for the dynamics of opinion formation on a BA scaling network, whose generating process can be see in [26], with only nearest-neighbor interactions. Consider a set of N agents involved in a discussion about a certain topic. Let us indicate by s_i the opinion of agent i, who may take one of two opposite spin orientations, say "+1" or "−1", which stand for agree ($s_i = 1$), and disagree ($s_i = -1$), respectively. The change of s_i will depend on the opinions of its neighbors, namely who is linked to i. Define the number of neighbors of agent i as n_i. The sum of opinions of all agents among n_i at time step t is simply $W_i(t) = \sum_{j\epsilon n_i} s_j(t)$. Then the opinion of agent i, s_i, at $t + 1$ will be updated according to the following equation.

$$s_i(t+1) = \begin{cases} \mathrm{sgn}(W_i(t)) & \text{with probability } (1-q); \\ -\mathrm{sgn}(W_i(t)) & \text{with probability } q, \end{cases} \tag{1}$$

where $W_i(t) \neq 0$. When $W_i(t) = 0$, agent i will randomly select one value among ± 1 as its next-time opinion, each with probability $1/2$.

3. Results and discussion

Here, we introduce the average magnetization of system Ω to measure the collective behavior that opinion evolves over time.

$$\Omega(t) = \frac{1}{N}\sum_i s_i(t). \tag{2}$$

Additionally, Ω_s, the absolute saturated value of Ω at the stationary state, is adopted as an order parameter.

$$\Omega_s = \left|\frac{1}{N}\sum_i s_i\right|. \tag{3}$$

Extensive Monte Carlo simulations have been performed on a BA scaling network for a given value of m. Initially, the two opposing points of view are evenly distributed among all agents. Then agents update their opinions synchronously according

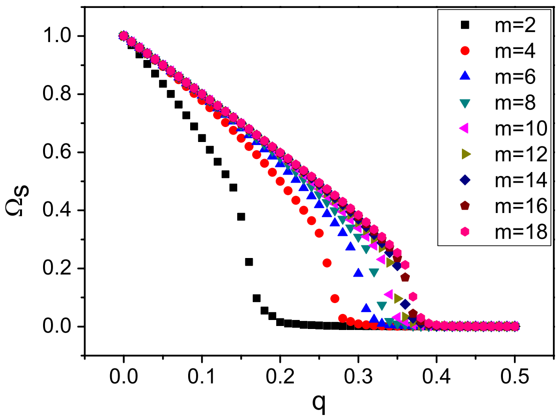

Fig. 2. The order parameter Ω_s is plotted as a function of q at different values of m for $N = 1000$. Each value of Ω_s is averaged over 10 000 time steps starting from 20 000th step. Each curve is an average over 50 independent simulations.

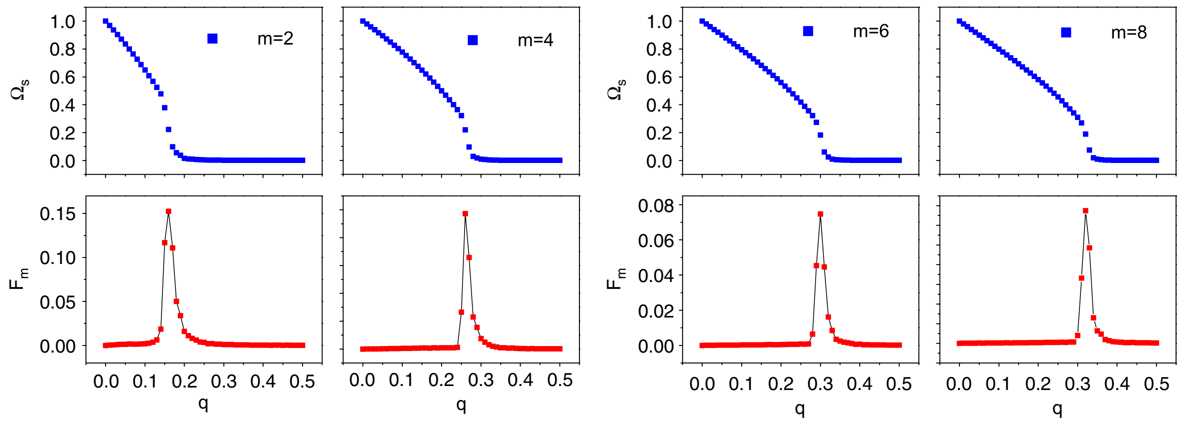

Fig. 3. F_m, the standard deviation of Ω_s, as a function of parameter q for $N = 1000$ and $m = 2, 4, 6, 8$.

to the rule of the model. This procedure is repeated indefinitely until the stationary state is reached. Ω is then calculated based on 10^4 time steps in the regime of stationary states.

We present simulation results of $\Omega(t)$ at various values of q in Fig. 1. It is observed that Ω rapidly reaches a steady state when q is not too large, except that, when $q = 0$, there are fluctuations in the evolution of Ω at the steady state. This reveals that the system is still alive after reaching the steady state: the opinions of agents keep changing but the overall sum of them is nearly constant. Fig. 1 displays an interesting phenomenon. That is, the difficulty for the system to reach an ordered state will increase as q does, for fixed N and m. If q takes its critical value, the system will never reach an ordered state.

Fig. 2 indicates the effect of parameter q on the order parameter Ω_s. In this figure, each curve for Ω_s reveals that there is a critical point q_c corresponding to a transition from an ordered state to a disordered one. Below q_c, the system possesses a nonzero magnetization showing two opposing opinions surviving together with different fractions. However, the average magnetization is equal to zero for $q \geq q_c$, which illustrates that the two different points of view are equally shared by agents. The reason behind this kind of behavior is that both the majority rule and the minority rule will eventually render all agents of the system to hold an identical opinion. Nonetheless, agents observe the opinion held by most agents and do not change their opinions in the case of majority rule. As for the minority rule, agents have the same point of view after a sufficiently long time; however, this point of view does not remain unchanged, but switches between $+1$ and -1. Consequently, two antithetic opinions coexist in the system when we introduce these two rules by a parameter q. The weights of these two disparate rules are not the same for $q < q_c$, which may give rise to nonzero average magnetization.

For the sake of obtaining a precise value of the critical parameter q_c, we analyze the standard deviation F_m of Ω_s for any given m.

$$F_m = \left[\frac{1}{G} \sum_{g=1}^{G} (\Omega_s(g) - \langle \Omega \rangle_c)^2 \right]^{1/2}, \tag{4}$$

where $G = 50$ is the number of independent simulations, and $\langle \Omega \rangle_c$ means the ensemble average.

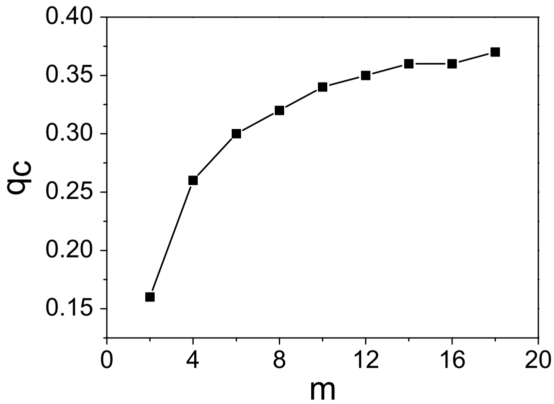

Fig. 4. The critical parameter q_c as a function of m for a BA scaling network ($N = 1000$). Each value of q_c corresponds to the peak of F_m at the same m (see Fig. 3). The plots suggest that q_c is a monotone increasing function of m.

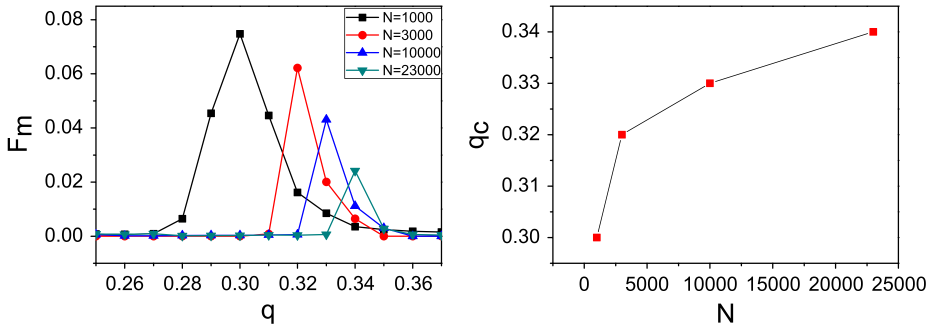

Fig. 5. The standard deviation of the order parameter as a function of q for a BA scaling network (N varies between 1000 and 23 000) (left panel), and the critical parameter q_c as a function of N (right panel), where $m = 6$. The plots suggest that F_m decreases as N increases, and that q_c is a monotone increasing function of N.

Fig. 3 reveals how the standard deviation F_m changes with parameter q, for $N = 1000$ and various values of m, where we only show the cases of $m = 2, 4, 6, 8$ for illustration. For a certain system, there exists a peak in F_m, corresponding to the critical point, namely q_c. Based on that, we plot q_c as a function of m in Fig. 4. As is seen, q_c is a monotone increasing function of m. m characterizes the strength of interactions between agents on a BA scaling network. The larger m, the stronger the interactions between agents. We have mentioned that an agent updates its opinion according to the minority rule with probability q and according to the majority rule with probability $(1 - q)$, respectively. Therefore, parameter q specifies the proportions of these two rules during the opinion evolution. From Fig. 4, we can see that, for a more dense network, the value of q_c is larger, which means that the difference between the weights of two rules is smaller. If m is sufficiently large or the network is sufficiently dense, q_c will reach its upper bound, 0.5. And in such a case there will be no phase transition.

We also perform some analysis about how the size of a system influences the value of the critical parameter q_c by checking F_m for different q when $m = 6$ and N varies between 1000 and 23 000. We plot the dependence of q_c on the size of system; see the right panel of Fig. 5. It can be inferred that q_c increases as N does. But F_m decreases as q_c increases; see the left panel of Fig. 5.

From Figs. 4 and 5, we can predict that the value of q_c will reach its upper limit 0.5 in the thermodynamic limit and for very large m. And the standard deviation F_m of the order parameter will be close to zero in that case.

4. Theoretical analysis

Here, we provide a mean-field theory study of the modified model. The probability that an agent observes opinion $+1$ and -1 at time step t is denoted by $p_+(t)$ and $p_-(t)$, respectively. On a BA scaling network, the probability that an agent i will stick to opinion $+1$ or -1, in the next time step, can be given by

$$p_+(t+1) = \sum_{k,a} P(k,a)\left[\Theta(k-2a)q + \Theta(2a-k)(1-q)\right],$$
$$p_-(t+1) = \sum_{k,a} P(k,a)\left[\Theta(k-2a)(1-q) + \Theta(2a-k)q\right], \quad (5)$$

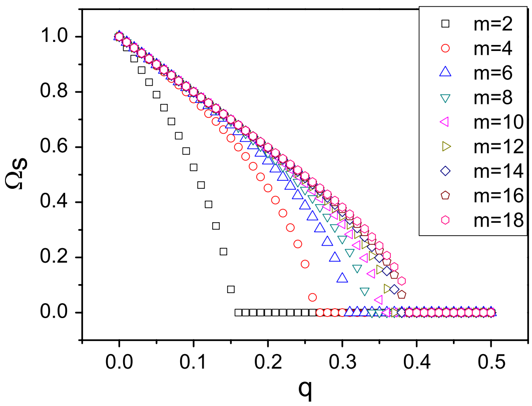

Fig. 6. Order parameter Ω_s as a function of parameter q. The results are attained by numerical solution of equations based on mean-field theory. The initial value of f is 0.45 for each m.

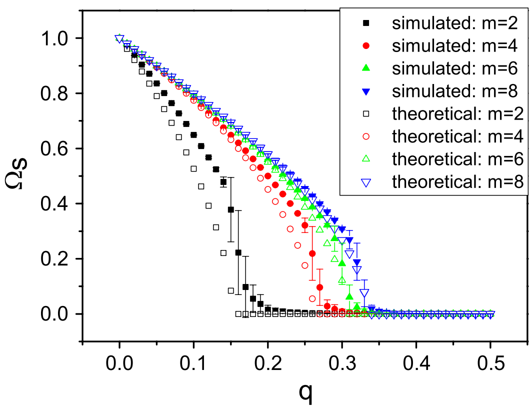

Fig. 7. Comparison between simulations and theoretical analysis. Ω_s as a function of parameter q for simulations (solid symbols) and theoretical analysis (hollow symbols), where $N = 1000$.

where

$$P(k, a) = P(k) \binom{k}{a} p_+(t)^a p_-(t)^{(k-a)}, \tag{6}$$

and

$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 1/2 & x = 0 \\ 0 & x < 0, \end{cases} \tag{7}$$

in which $k \in [1, k_{max}]$, $a \in [0, k]$, and $P(k)$ is the degree distribution of the BA scaling network.

Hence, at steady states, the order parameter of system Ω_s can be defined by

$$\Omega_s = |p_+(s) - p_-(s)|\,, \tag{8}$$

where $p_+(s)$ and $p_-(s)$ are the average probabilities of an agent with opinion $+1$ and -1, at stationary states.

By iterations, we obtain the relation between the order parameter Ω_s and parameter q; see Fig. 6. The figure shows that there exists a critical point, close to the one found in Monte Carlo simulations. The comparison between simulations and theoretical outcomes is given in Fig. 7, which shows certain agreement of the two. We plot $q_c(m)$ versus N for both simulations and theoretical analysis in Fig. 8. It is observed that the critical parameter q_c is independent of the size of the system in mean-field theory. However, it increases with N in Monte Carlo simulations. So, Fig. 8 tells us that mean-field theory can be used to explain opinion formation on a BA scaling network when the network is not too large. For large networks, we should take into account the heterogeneity of the BA scaling network.

Furthermore, we analyze the effects of the initial conditions on Ω_s by means of Monte Carlo simulations and theoretical analysis; see Fig. 9. It turns out there is little dependence on that.

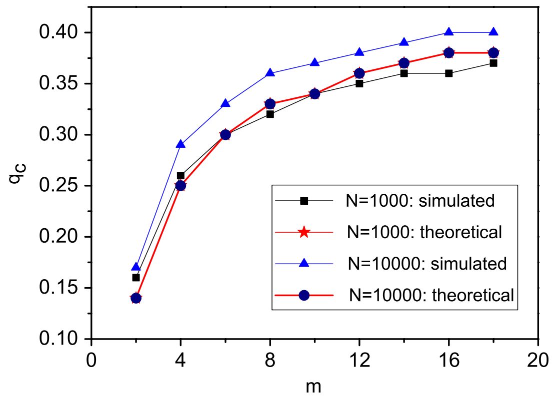

Fig. 8. The critical parameter q_c as a function of m, for Monte Carlo simulations and theoretical analysis with $N = 1000$ and $N = 10\,000$, respectively.

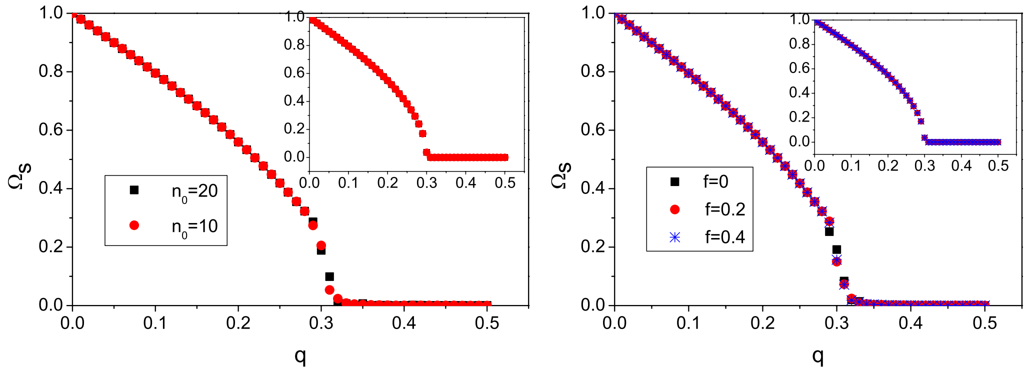

Fig. 9. Order parameter Ω_s as a function of parameter q, averaged over ten different simulations. In all cases, $N = 1000$ and $m = 6$. Left panel: Effect of different initial number of nodes, n_0. Right panel: Different initial distributions of opinion f (the fraction of opinion -1) are shown. Both of the insets are corresponding results of theoretical analysis.

5. Conclusions

In summary, we have studied the opinion diffusion dynamics on a BA scaling network by performing Monte Carlo calculations and mean-field approximations. Our main results suggest that there occurs a phase transition from an ordered state to a disordered one as one smoothly changes the value of parameter q. Furthermore, the mean-field approximation can explain the transition nature when the network is very dense. Simulation results of the model also show that in the thermodynamic limit and for m very large, the critical parameter q_c can reach its upper limit 0.5.

Acknowledgments

This work was supported by National Natural Science Foundation of China (grant nos. 10975057, 10635020, 10647125, and 10975062), and the Programme of Introducing Talents of Discipline to Universities under grant no. B08033.

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