The Scaling of Several Public Transport Networks in China

by

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THE SCALING OF SEVERAL PUBLIC TRANSPORT NETWORKS IN CHINA

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Abstract
Public transport networks (PTNs) are often researched without reference to their geographical embedding. The question arises if there is any underlying structure or principle characterizing the observed behavior of geographically embedded transport routes. Here, we focus on the scaling properties of PTNs in Space L through fractal analysis and consider the effect of the real bus routes, which reflects the human movement between stations indirectly. We find that the PTN in Space L is the better basal one to mimic the human migration in city. Furthermore, we also research the scaling property of the correlation between stations and the distribution of node’s weight, which shows the heterogeneous property of human activity between different stations. Our present work provides some new perspective and tools to realize the human migration on spatial networks.

Keywords: Fractal Dimension; Spatial Property; Scaling; Public Transport Networks.

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1. INTRODUCTION

Many real social phenomena, such as the epidemics, rumor spreading and the opinion formation, have been studied on complex networks and got some interesting results. As is well known, the evolution of social phenomena is due to the human migration, which occurs in the two- or three-dimensional space. Recently, many empirical analyses have shown that human migration share the spatial-temporal scaling property and provided some basic explanations. The spatial-temporal scaling plays an important role in the social dynamics, such as the spreading dynamics of SI model under the temporal scaling of human migration patterns. Here, we focus on the network reconfiguration of the space of human migration, and show the heterogeneous properties of human migration in city.

It is shown that human migration depends on the transportation, such as the subway, the public bus and the railway. For example, the bus routes reflect the human migration from one place to another indirectly. From a graphical point of view, a station, which is mapped to a vertex in network, has its local spatial property, and an edge between two stations exists if they are stops (Space P) or consecutive stops (Space L) by the same bus route. Several previous empirical analyses of the transportation networks have been done in the space P and space L, respectively. They found that those networks share the heterogeneous properties of the degree distribution (the power-law distribution or the exponential decay distribution) and the small-world effect. For example, Sui et al. analyzed the space evolution of an urban public transport network using empirical evidence and a simulation model validated on the data. To uncover the role of human migration in the spatial heterogeneous property of society, Jiang et al. studied the aggregate flow assigned to individual streets on the road networks. And Lämmer et al. studied the scaling laws of the urban road networks in the spatial structure, defining the betweenness centrality according to the real vehicular traffic and the heterogeneous property of road networks. Could the PTNs in Space L be used to coarsely describe the spatial properties of city? If yes, how can the weights of stations and the strength of spatial correlation between stations according to the human migration be quantified? Our present work aims to solve the two questions mentioned above.

In order to solve the first question, we calculate the fractal dimension of PTNs using the fractal analysis. Some previous works have been done to show the fractal feature of PTNs, which are introduced in Ref. 17. The prevailing part of these former studies concern the density of stations or the total length of track as a function of the distance from the center of the network. Here, the fractal dimension is calculated using the method introduced by Shanler. The average density of stations follows a better power-law function of the distance r with the fractal dimension \( d_f \approx 2 \), which is an intrinsic property that is unrelated to the human migration. We also examine the spatial heterogeneous property of human activity and migration between stations.

2. EMPIRICAL RESULTS

Here, we analyze the statistical properties of four PTNs in China, and the data are downloadable at http://www.mapbar.com. Those four cities are Beijing (the capital of China), Wuhan (an inland city with the Yangtse River flowing across), Tianjin (a coastal city) and Taiyuan (one of the great industrial cities of China and lies on the Fen River, north of its fertile upper basin). The general parameters of those PTNs are shown in Table 1. Those PTNs are sparse ones with the small-world effect and the scaling property of the degree distribution, see Fig. 1.

In our daily life, most of the time we travel only over short distances, between home and work, and occasionally we take long trips. Our movements occur on the planar geo-spaces, and are commonly dependent of the public transportation, such as the bus, taxi and subway. Hence, to studying the emergence of social phenomena, the fundamental problem is mapping the geographic position of city using the graph theory. In order to do this, the station, which is mapped to node in network, represents the local spatial region coarsely, and an edge between two stations exists if they are consecutive stops by the same bus route, which is the space of stops and called Space L. On the other hand, in order to quantify the human activity at station and human migration between stations, we define the weight of station (edge) according to the real bus routes and assume that the bus route plays the same role in the stations where it stops. Hence, the weight of station (edge) is defined as the number of the bus routes,
Table 1 General Parameters of the Four Public Transport Networks.

<table>
<thead>
<tr>
<th>Cities</th>
<th>N</th>
<th>E</th>
<th>N_b</th>
<th>C</th>
<th>l</th>
<th>γ</th>
<th>d_f (w/unw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>9182</td>
<td>27601</td>
<td>1066</td>
<td>0.163</td>
<td>23.9</td>
<td>2.86(0.21)</td>
<td>2.44(0.04)/2.59(0.04)</td>
</tr>
<tr>
<td>Wuhan</td>
<td>1692</td>
<td>5036</td>
<td>283</td>
<td>0.167</td>
<td>16.2</td>
<td>2.42(0.08)</td>
<td>1.90(0.02)/1.88(0.05)</td>
</tr>
<tr>
<td>Tianjin</td>
<td>3219</td>
<td>9002</td>
<td>298</td>
<td>0.161</td>
<td>25.2</td>
<td>2.68(0.13)</td>
<td>2.12(0.02)/2.13(0.03)</td>
</tr>
<tr>
<td>Taiyuan</td>
<td>1136</td>
<td>3017</td>
<td>154</td>
<td>0.110</td>
<td>15.4</td>
<td>3.04(0.11)</td>
<td>2.19(0.02)/2.14(0.03)</td>
</tr>
</tbody>
</table>

Note: N is the number of stations, E is the number of edges, N_b is the number of bus lines, C is the clustering coefficient, l is the average shortest path length, γ is the exponent of the degree distribution and d_f is the fractal dimension of weighted (w) or unweighted (unw) PTNs.

Fig. 1 The degree distribution of PTNs in the log-log representation. The dotted line, shown as a visual guide represents a power-law decay with the exponent γ = 3.

Fig. 2 The fractal dimensions of those four weighted (open signs) and unweighted PTNs (solid signs). The dash line, shown as a visual guide represents a power-law decay with the exponent d_f = 2.

which stop (pass through) the corresponding station (edge).

In order to show the spatial property of PTNs in Space L, we analyze the fractal dimensions of those four PTNs using the method proposed by Shanker.\textsuperscript{18} We define the distance d_ij between nodes, says i and j, as the shortest path length from node i to node j. And in the weighted PTNs, the distances of edges between nodes are not the same and are instead assigned by their weights. We set all the nodes as the seeds in turn and a cluster of nodes centered at each seed within the box of the linear size r is calculated. Then, the average “density” ⟨N(r)⟩, defined as the ratio of the number of nodes in all the boxes to the number N of the boxes with the same size r, is calculated as a function of r:

\[ ⟨N(r)⟩ \sim r^{d_f}, \]

where d_f is defined as the fractal dimension, which reflects the scaling about some physical quantities and has many applications, such as to study the surface roughness of particles in porous media.\textsuperscript{19} Figure 2 shows the evolution of ⟨N(r)⟩ as a function of r about the four PTNs, and the d_f s are recorded in Table 1. We find that the d_f ≃ 2, which is unrelated to the dynamical weighted method (i.e. the human activity), is the intrinsic property of PTNs. Hence, the PTN in Space L is the better basal one to mimic human dynamics in the plane space of cites. Surprisingly, d_f of PTN in Beijing is much larger than 2, which is due to more shortcuts (i.e. the BRTs and many branches of the same bus lines) in PTN of Beijing, which can be explained in our previous works about the fractal dimension of complex network.\textsuperscript{20}

Furthermore, we also analyze the statistical properties of weight of nodes (edges), which reflect the distribution and intensity of migration between stations about the population. Figure 3 displays the cumulative distributions of the weights of nodes (left) and edges (right). We find that those weighted distributions follow the power-law functions with the exponents \( \tau = 0.88 \) and \( \alpha = 1.35 \), which are the same in the four PTNs and reflect the universal scaling of PTNs, for stations and edges respectively.
Fig. 3 The cumulative distribution of the weights of nodes (left) and edges (right). The solid lines, shown as a guide to eyes, represent the power-law decay with the exponents $\tau = 0.88$ and $\alpha = 1.35$ for nodes and edges respectively.

Fig. 4 The cumulative distribution of the correlation strength between stations. The dash line and the solid line, shown as a guide to eyes, represent the power-law decay with the exponents $\beta_1$ and $\beta_2$ for different part.

3. CONCLUSION

In summary, we analyze the statistical properties of the weighted PTNs in China in Space L. The definition of weight is based on the real bus routes, which reflect the human activity and migration in city indirectly. The topology of PTNs describes the spatial properties of cities coarsely in Space L, and the weighted method cannot affect the fractal dimension of PTNs. On the other hand, we find the universal scaling properties of the statistical weights of nodes (edges), which show the heterogeneous properties (i.e. the scaling distribution) of human activity and migration in city. Furthermore, the weights of stations reflect the spatial density of human activity and the correlation strengths between stations reflect the intensity of human migration from
one station to another (i.e. the transfer function of human movement from one place to another). Hence, our present work provides some perspective and tools to study human dynamics on spatial networks.

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REFERENCES