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Dynamics of quantum correlations for atoms in independent single-mode cavity

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Abstract We investigate the dynamics of two identical atoms resonantly coupled to independent single-mode cavity in zero detuning without rotating wave approximation (RWA). It is shown that for two atoms initially in the ground state, the entanglement (concurrence) and the normalized geometric measure of quantum discord (NGMQD) display similar behavior. There is no sudden death and sudden birth. And the entanglement is always larger than NGMQD in this case. For two atoms initially in excited state, one can see the novel entanglement sudden death (ESD) and sudden birth (ESB) phenomena. The entanglement is not always greater than the NGMQD in this case. Consequently, there is no simple dominance relation between the entanglement and the NGMQD.

Keywords Entanglement · Normalized geometric discord · Double J-C model · ESD and ESB

1 Introduction

Quantum entanglement, originated from nonlocal quantum correlation, is fundamental in quantum physics both for understanding the nonlocality of quantum mechanics [1] and plays an important role in quantum computations and quantum information processing [2–5]. Due to the interactions with the environment in preparation and transmission, the initially entangled states usually

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become mixed ones that are no longer maximally entangled. In Refs. [6–9] the authors investigated the time evolution of entanglement of a bipartite qubit system undergoing various modes of decoherence. It is found that the global entanglement may vanish in finite time, a phenomenon so-called entanglement sudden death (ESD) and have been demonstrated experimentally for optical setups and atomic ensembles [10–12].

Recently, it has been perceived that entanglement is not the only kind of quantum correlation, a new kind of quantum correlation, quantum discord (QD) has attracted a lot of attentions [13, 14] due to its potential to serve as an important resource in the deterministic quantum computation with one pure qubit (DQC1) [15–17] and quantum communication [18]. The quantum discord of a composite system AB is defined by $D_A \equiv \min_{\{E_k^A\}} \sum_k p_k H(\rho_{B/k}) + H(\rho_A) - H(\rho_{AB})$, where $H(\rho_{AB}) = Tr(\rho_{AB} \log_2 \rho_{AB})$ is the von Neumann entropy and the minimum is taken over all positive operator valued measures (POVMs) $\{E_k^A\}$ on the subsystem A with $p_k = Tr(E_k^A \rho_{AB})$ being the probability of the kth outcome and $\rho_{B/k} = Tr_A(E_k^A \rho_{AB})/p_k$ being the conditional state of subsystem B.

Because of the minimization taken over all possible POVM, or von Neumann measurements, is is generally difficult to calculate measurement based discord. In order to overcome this difficulty geometric measure of quantum discord (GMQD) has been introduced by Dakic et al [19]. Recently, Dakic et al [18] show that the GMQD is related to the fidelity of remote state preparation which provides an operational meaning to GMQD.

Many works have been devoted to entanglement in various systems with different entanglement measures and the local decoherence influence on the entanglement evolution . Recently, a more general quantum correlation, geometric measure of quantum discord (GMQD), has also received a great deal of attention [20, 21]. The comparisons with entanglement dynamics have been also performed [22, 23]. However, to the best of our knowledge, the comparisons between entanglement and NGMQD of two-level atoms coupled to independent single-mode cavities without rotating wave approximation (RWA) has not been found in the literature. We believe that the dynamics of entanglement and NGMQD in the framework of our model is also fundamental interest. In addition, some novel property which different from previous results has also been discussed.

2 MODEL AND EFFECTIVE HAMILTONIAN WITHOUT ROTATING WAVE APPROXIMATION

We generally consider a system consisting of two two-level atoms interacting with independent single-mode cavities with annihilation (resp. creation) operator a_k (resp. a_k^+) for the kth mode with frequency ω_k. The total system is described by the Hamiltonian $H = H_0 + H_I$:

$$H_0 = \frac{1}{2}\Omega_1 \sigma_z^1 + \frac{1}{2}\Omega_2 \sigma_z^2 + \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 \tag{1}$$

$$H_I = g_1(\sigma_1^+ + \sigma_1^-)(a_1^+ + a_1) + g_2(\sigma_2^+ + \sigma_2^-)(a_2^+ + a_2) \tag{2}$$

Here, Ω_k is the transition frequency of the kth atom, $k = 1, 2$, $\sigma_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli operator with $|e\rangle$ and $|g\rangle$ the atomic excited and ground states, respectively, $\sigma^+ = (\sigma^-)^\dagger \equiv |e\rangle\langle g|$ are the raising and lowing operators, $\dagger$ stands for the transpose an conjugation. g_1 and g_2 are the coupling constants.

As the interaction term H_I contains the counter-rotating terms, that is, the high-frequency terms with frequencies $\pm(\omega_k + \Omega_k)$ like

$$V = a_k^+ \sigma_k^+ e^{i(\omega_k+\Omega_k)} + H.C.$$

in the interaction picture, the Hamiltonian H is not exactly solvable even for the simple cases of single mode or single excitation.We use the generalized version [24] of the Fröhlich-Nakajima transformation [25,26] $exp(S)$ to eliminate the high-frequency terms in the effective Hamiltonian. Here

$$S = \sum_{k=1,2} A_k(a_k^+ \sigma_k^+ - a_k \sigma_k^-) \tag{3}$$

with

$$A_k = \frac{g_k}{\omega_k + \Omega_k}. \tag{4}$$

Up to the second order, the effect Hamiltonian $H_{eff} = exp(S)Hexp(-S)$ is given by

$$H_{eff} \backsimeq H_0 + H_1 + \frac{1}{2}[S, H_1] + \frac{1}{2}[S, H_I], \tag{5}$$

where $H_1 = H_I + [S, H_0]$ is the first order term. It is direct to show that

$$[S, H_1] = \sum_{k=1,2} A_k g_k (a_k a_k + a_k^+ a_k^+)\sigma_z^k, \tag{6}$$

$$[S, H_I] = [S, H_1] + 2 \sum_{k=1,2} A_k g_k (a_k^+ a_k \sigma_z^k - \sigma_k^- \sigma_k^+). \tag{7}$$

Since the total excitation number operator of the qubit-cavity system in the transformed Hamiltonian is a conserved observable, one may focus on the single-particle excitation subspace and omit the high-frequency terms including $a_k a_k$ and $a_k^+ a_k^+$ [27],

$$\begin{aligned} H_{eff} = & \sum_{k=1,2} \omega_k a_k^+ a_k + \sum_{k=1,2} g_k (a_k \sigma_k^+ + a_k^+ \sigma_k^-) \\ & + \sum_{k=1,2} \frac{1}{2} \Omega_k \sigma_z^k + \sum_{k=1,2} A_k g_k (a_k^+ a_k \sigma_z^k - \sigma_k^- \sigma_k^+). \end{aligned} \tag{8}$$

The above effective Hamiltonian is different from the one derived from the RWA and is exactly solvable. For simplicity, in the following discussion, we consider the case that the two-level atoms couple to the adjacent cavity with the same coupling strength $g_1 = g_2 \equiv g$ and the zero detuning $\delta = \Omega - \omega \equiv 0$ $(\Omega_1 = \Omega_2 = \omega_1 = \omega_2 \equiv \Omega)$.

3 DYNAMICS OF CORRELATIONS FOR TWO-LEVEL SYSTEMS

We use concurrence as the measure to characterize the quantum entanglement of a two-qubit state ρ [28,29],

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

where λ_i are the eigenvalues, in decreasing order, of the matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, ρ^* denotes the complex conjugation of ρ and σ_y is the Pauli matrix. For a density matrix ρ of the form,

$$\rho = \begin{pmatrix} a & 0 & 0 & \omega \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ \omega^* & 0 & 0 & d \end{pmatrix}, \tag{9}$$

the concurrence is given by

$$C(\rho) = 2\max\{0, |z| - \sqrt{ad}, |\omega| - \sqrt{bc}\}. \tag{10}$$

The geometric measure of quantum discord (GMQD) is defined by [19]

$$D_A^g(\rho) = \min_{\chi \in \Omega_0} \|\rho - \chi\|^2, \tag{11}$$

where Ω_0 denotes the set of zero-discord states and $\|X - Y\|^2 = Tr(X - Y)^2$ is the square norm in the Hilbert-Schmidt space. An arbitrary two-qubit state can be written in Bloch representation:

$$\rho = \frac{1}{4}[I \otimes I + \sum_{i=1}^{3}(x_i\sigma_i \otimes I + y_i I \otimes \sigma_i) + \sum_{i,j=1}^{3} R_{ij}\sigma_i \otimes \sigma_j],$$

where $x_i = Tr\rho(\sigma_i \otimes I)$, $y_i = Tr\rho(I \otimes \sigma_i)$ are components of the local Bloch vectors, $\sigma_i, i \in \{1,2,3\}$ are the three Pauli matrices, and R_{ij} are components of the correlation tensor. The GMQD of a two-qubit state is given by [19]

$$D_A^g(\rho) = \frac{1}{4}(\|x\|^2 + \|R\|^2 - k_{max}), \tag{12}$$

where $x = (x_1, x_2, x_3)^T$, $k_{\max}$ is the largest eigenvalue of the matrix $K = xx^T + RR^T$. By introducing a matric $\Re$ defined by

$$\Re = \begin{pmatrix} 1 & y^T \\ x & R \end{pmatrix}, \tag{13}$$

and a 3 by 4 matrix $\Re'$ by deleting the first row of $\Re$. Then the analytical expression of GMQD can be further rewritten as [30]

$$D_A^g(\rho) = \frac{1}{4}\left[\left(\sum_k \lambda_k^2\right) - \max_k \lambda_k^2\right], \tag{14}$$

where λ_k are the singular values of $\Re'$. The maximum value of $D_A^g(\rho)$ is $\frac{1}{2}$ for two-qubit states. It is natural to consider $2D_A^g(\rho)$ as a properly normalized measure.

3.0.1 Atoms initially in ground state

We first consider the case that cavities are initially maximal entangled while the two atoms are in the (separable) ground state,

$$|\psi(0)>=\frac{\sqrt{2}}{2}|\downarrow\downarrow 01>+\frac{\sqrt{2}}{2}|\downarrow\downarrow 10>. \tag{15}$$

The time dependent wave function can be generally expressed as

$$\begin{aligned}|\psi(t)>= &\ x_1(t)|\downarrow\downarrow 01>+x_2(t)|\downarrow\downarrow 10>\\ &+x_3(t)|\downarrow\uparrow 00>+x_4(t)|\uparrow\downarrow 00>,\end{aligned} \tag{16}$$

where

$$\begin{aligned}x_1(t)=x_2(t)&=\frac{e^{-\frac{gt(-2ig+y)}{2\Omega}}\left[i\left(-1+e^{\frac{gty}{\Omega}}\right)g+\left(1+e^{\frac{gty}{\Omega}}\right)y\right]}{2\sqrt{2}y},\\ x_3(t)=x_4(t)&=-\frac{ie^{-\frac{gt(-2ig+y)}{2\Omega}}\left(-1+e^{\frac{gty}{\Omega}}\right)\Omega}{\sqrt{2}y},\quad y=i\sqrt{g^2+4\Omega^2}.\end{aligned} \tag{17}$$

The reduced density matrix ρ of two atoms can be obtained by tracing out the photonic part of $|\psi(t)><\psi(t)|$. In the basis $|\uparrow\uparrow>$, $|\uparrow\downarrow>$, $|\downarrow\uparrow>$ and $|\downarrow\downarrow>$, it is of the form

$$\rho=\begin{pmatrix}0&0&0&0\\0&|x_4|^2&x_4x_3^*&0\\0&x_3x_4^*&|x_3|^2&0\\0&0&0&|x_1|^2+|x_2|^2\end{pmatrix}. \tag{18}$$

From Eq. (17), we see that the state depends on the transition frequency Ω, which is different from the results in [31,32], where the correlation of the systems is only influenced by the coupling strength g and the frequency ω in the given initial states in the RWA. Such dependence on the transition frequency Ω will influence the evolution of correlation in quantum systems. We denote $\lambda\equiv\frac{\Omega}{g}$, a parameter representing the relationship between the transition frequency Ω of atoms and the coupling strength g. The concurrence of the state (18) is given by

$$C(\rho)=\frac{4\lambda^2\sin^2(\frac{\sqrt{1+4\lambda^2}t\Omega}{2\lambda^2})}{1+4\lambda^2}. \tag{19}$$

We consider $\lambda>1$ ($\Omega>g$), which is compatible with the approximation in Eq. (5). The dynamical evolution of the entanglement in Eq. (19) is shown in Fig. 1, the concurrence as a function of the time interval t and the transition frequency Ω. One can see that concurrence for two atoms varies periodically with t and Ω. There is no entanglement sudden death. The atoms remain disentangled when $\frac{\sqrt{1+4\lambda^2}}{2\lambda^2}t\Omega=k\pi$, $k=0,1,\cdots$.

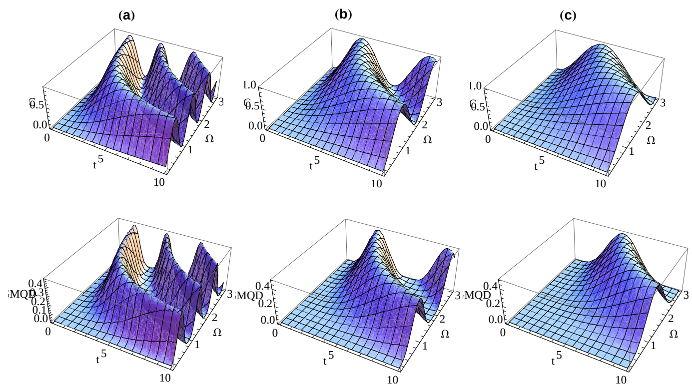

Fig. 1 (Color online) The dynamical evolution of the concurrence and GMQD as a function of time interval t and the transition frequency Ω in the case of zero detuning, with $g_1 = g_2 \equiv g$, $\Omega = \omega$ and $\lambda \equiv \frac{\Omega}{g}$, for the case of (a) $\lambda = 3$, (b) $\lambda = 6$, (c) $\lambda = 9$.

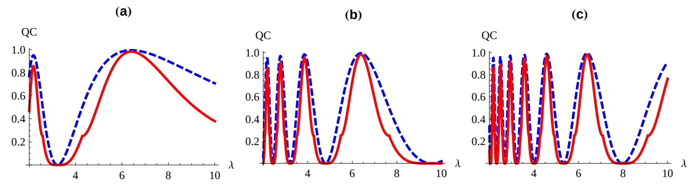

Fig. 2 (Color online) The evolution of quantum correlation(QC) as a function of λ in the case of zero detuning, with $g_1 = g_2 \equiv g$, $\Omega = \omega$, $\lambda \equiv \frac{\Omega}{g}$ and $T = \Omega t$, for (a) $T = 10$, (b) $T = 30$, (c) $T = 50$. The blue dashed line and the red solid line are corresponding to concurrence and NGMQD respectively.

According to Eq. (14), the GMQD of the state (18) is given by

$$D_A^g(\rho) = \frac{1 + 2\lambda^2 + 13\lambda^4 + (6\lambda^2 - 4\lambda^4)\cos(\frac{\sqrt{z}t\Omega}{\lambda^2}) + 7\lambda^4\cos(\frac{2\sqrt{z}t\Omega}{\lambda^2})}{2(z)^2}$$
$$- \mathrm{Max}\left[\frac{1 + 2\lambda^2 + 7\lambda^4 + 2\lambda^2(3 + 2\lambda^2)\cos(\frac{\sqrt{z}t\Omega}{\lambda^2}) + 5\lambda^4\cos(\frac{2\sqrt{z}t\Omega}{\lambda^2})}{2(z)^2}, \frac{4\lambda^4\sin^4(\frac{\sqrt{z}t\Omega}{2\lambda^2})}{(z)^2}\right], \quad (20)$$

where $z = 1 + 4\lambda^2$.

For GMQD in Eq. (20), it follows the similar tendency as concurrence showing in Fig. 1. Based on the intuitive observations from Fig. 1, we can see the maximum value of GMQD is approaching to $\frac{1}{2}$, it is easy to be explained according to the analytic expression of Eq. (21). In order to further compare the GMQD with the concurrence, we consider a normalized geometric measure of quantum discord (NGMQD) $2D_A^g(\rho)$ as a proper measurement for quantum correlations, in the case that cavities are initially maximal entangled while the two atoms are in the (separable) ground state, for zero detunnings. The results are collected in Fig. 2. The evolution of both NGMQD and concurrence display similar behavior, we notice that the amplitude of oscillation of concurrence and NGMQD as a function of λ is gradually increasing until to maximal value one. In fact, we can prove that entanglement is always larger than NGMQD in this case. It is interesting to note from Fig. 2 that zero-discord (that is so-called classical-quantum states, according to the definition of Eq. (11), an arbitrarily state $\rho \in \Omega_0$ if and only if $D_A^g(\rho) \equiv 0$, so zero-discord states are equivalent to zero-GMQD states) states are appearing when entanglement is zero, it means that separable states with the form of Eq. (18) have no non-classical correlations (NGMQD), which is different from the previous literature [33].

3.0.2 Atoms initially in excited state

Now we consider the case that the atoms are initially in excited state,

$$|\varphi(0)> = \frac{\sqrt{2}}{2}|\uparrow\uparrow 01> + \frac{\sqrt{2}}{2}|\uparrow\uparrow 10> . \tag{21}$$

The state of the system at time t is given by

$$\begin{aligned} |\varphi(t)> = \;& y_1(t)|\uparrow\uparrow 01> + y_2(t)|\uparrow\uparrow 10> \\ & + y_3(t)|\downarrow\uparrow 11> + y_4(t)|\uparrow\downarrow 02> \\ & + y_5(t)|\downarrow\downarrow 12> + y_6(t)|\downarrow\uparrow 20> \\ & + y_7(t)|\uparrow\downarrow 11> + y_8(t)|\downarrow\downarrow 21> . \end{aligned} \tag{22}$$

According to Schrödinger equation and the initial condition in Eq. (21), we have

$$\begin{aligned} |y_1(t)|^2 =\;& \tfrac{1}{8(\Theta\Xi)^2\Lambda^2}\Big\{(\Theta\Xi)^2\left[(-1+\Lambda)\cos\left(\tfrac{\Theta T}{2\lambda^2}\right)+(1+\Lambda)\cos\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2 \\ & +\left[\left(3+8\lambda^2-3\Lambda\right)\Xi\sin\left(\tfrac{\Theta T}{2\lambda^2}\right)-\left(3+8\lambda^2+3\Lambda\right)\Theta\sin\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2\Big\}, \\ |y_3(t)|^2 =\;& \tfrac{\lambda^2}{2(\Theta\Xi)^2\Lambda^2}\Big\{(\Theta\Xi)^2\left[\cos\left(\tfrac{\Theta T}{2\lambda^2}\right)-\cos\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2 \\ & +\left[\left(-2-4\lambda^2+\Lambda\right)\Xi\sin\left(\tfrac{\Theta T}{2\lambda^2}\right)+\left(2+4\lambda^2+\Lambda\right)\Theta\sin\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2\Big\}, \\ |y_4(t)|^2 =\;& \tfrac{\lambda^2}{4(\Theta\Xi)^2\Lambda^2}\Big\{(\Theta\Xi)^2\left[\cos\left(\tfrac{\Theta T}{2\lambda^2}\right)-\cos\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2 \\ & +\left[\left(-1-4\lambda^2+2\Lambda\right)\Xi\sin\left(\tfrac{\Theta T}{2\lambda^2}\right)+\left(1+4\lambda^2+2\Lambda\right)\Theta\sin\left(\tfrac{\Xi T}{2\lambda^2}\right)\right]^2\Big\}, \\ |y_5(t)|^2 =\;& \lambda^4\left[\cos\left(\tfrac{\Xi T}{2\lambda^2}\right)-\cos\left(\tfrac{\Theta T}{2\lambda^2}\right)\right]^2/\Lambda^2, \end{aligned} \tag{23}$$

where, $T = \Omega t$, $\Lambda = \sqrt{1+6\lambda^2+8\lambda^4}$, $\Theta = \sqrt{5+12\lambda^2-4\sqrt{1+6\lambda^2+8\lambda^4}}$, and $\Xi = \sqrt{5+12\lambda^2+4\sqrt{1+6\lambda^2+8\lambda^4}}$. While $y_2(t) = y_1(t)$, $y_7(t) = y_3(t)$, $y_6(t) = y_4(t)$ and $y_8(t) = y_5(t)$.

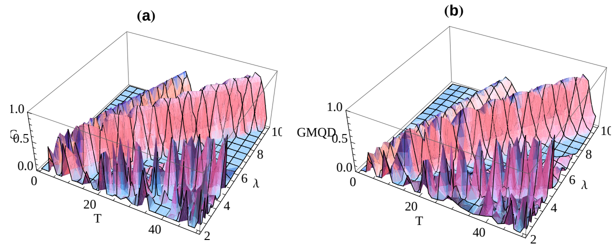

Fig. 3 (Color online) The evolution of quantum correlation(QC) as a function of λ and T in the case of zero detuning, with $g_1 = g_2 \equiv g$, $\Omega = \omega$, $\lambda \equiv \frac{\Omega}{g}$ and $T = \Omega t$, for (a) concurrence, (b) NGMQD.

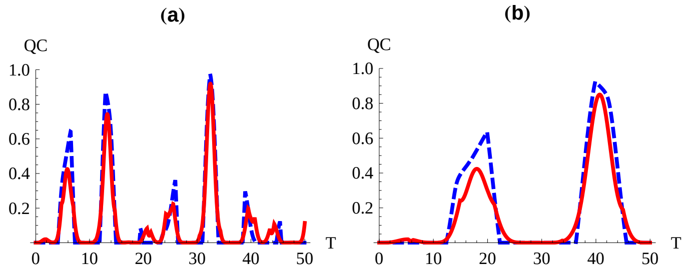

Fig. 4 (Color online) The evolution of quantum correlation(QC) as a function of T in the case of zero detuning, with $g_1 = g_2 \equiv g$, $\Omega = \omega$, $\lambda \equiv \frac{\Omega}{g}$ and $T = \Omega t$, for (a) $\lambda = 3$, (b) $\lambda = 9$. The blue dashed line and the red solid line correspond to the concurrence and NGMQD respectively. In Fig. 4b, the NGMQD has sudden changes at T=14.85 and T=21.25. The NGMQD is greater than the concurrence for $21.56 < T < 24.5$ and $33.5 < T < 37$.

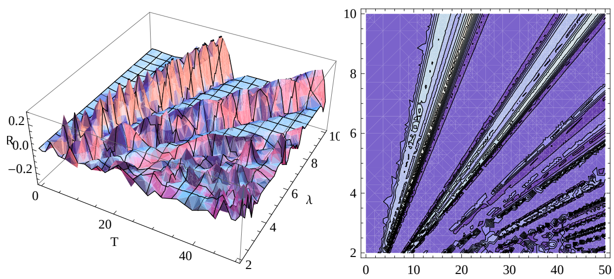

Fig. 5 (Color online) The evolution of difference between concurrence and the NGMQD as a function of λ and T in the case of zero detuning, with $g_1 = g_2 \equiv g$, $\Omega = \omega$, $\lambda \equiv \frac{\Omega}{g}$ and $T = \Omega t$.

The reduced density matrix of the two atoms is given by

$$\rho = \begin{pmatrix} |y_1|^2 + |y_2|^2 & 0 & 0 & 0 \\ 0 & |y_4|^2 + |y_7|^2 & x_7 y_3^* & 0 \\ 0 & y_3 y_7^* & |y_3|^2 + |y_6|^2 & 0 \\ 0 & 0 & 0 & |y_5|^2 + |y_8|^2 \end{pmatrix}. \tag{24}$$

According to Eq. (10), the concurrence in this case has the form

$$C(\rho) = 2\mathrm{Max}[0, |y_7 y_3^*| - \sqrt{(|y_1|^2 + |y_2|^2)(|y_5|^2 + |y_8|^2)}]. \tag{25}$$

The GMQD for the states Eq. (24) is given by

$$D_A^g(\rho) = \frac{1}{4}\left(2\lambda_1{}^2 + \lambda_3{}^2 - \mathrm{Max}\left[\lambda_1{}^2, \lambda_3{}^2\right]\right), \tag{26}$$

where $\lambda_1^2 = 4|y_3|^4$, $\lambda_3^2 = 2[4|y_1|^4 + 2(|y_3|^2 + |y_4|^2)^2 - 4(|y_1|^2 + |y_5|^2)(|y_3|^2 + |y_4|^2) + 4|y_5|^4]$.

The dynamical evolution of the concurrence and NGMQD is shown in Fig. 3 for the case that cavities are initially maximal entangled while the two atoms are in the (separable) excited state, in zero detunnings. In this case one sees both entanglement sudden death and sudden birth phenomena, as well as NGMQD sudden death and sudden birth, Fig. 3b. It is also clearly shown in Fig. 4 for some fixed λ. Moreover, the entanglement is not always greater than the NGMQD, which is different from the case in Fig. 2. We give an intuitive comparison of the relationships between $C(\rho)$ and $2D_A^g(\rho)$ by studying the quantity

$$R(\rho) = C(\rho) - 2D_A^g(\rho) \tag{27}$$

for the states defined by Eq. (24). Without loss of generality, we consider $R(\rho)$ as a function of time T and the parameter λ for zero detunnings. From Fig. 5 we can see that there exist states ρ_1 and ρ_2 such that $C(\rho_1) > 2D_A^g(\rho_1)$, while $C(\rho_2) < 2D_A^g(\rho_2)$. Even for a given state ρ (24), it appears that $C(\rho_{t_1}) > 2D_A^g(\rho_{t_1})$ at time t_1, nevertheless, $C(\rho_{t_2}) < 2D_A^g(\rho_{t_2})$ at time t_2.

4 CONCLUSION

In summary, we have systematically studied the dynamics of two identical atoms resonantly coupled to independent single-mode cavity in zero detuning without rotating wave approximation. The evolutions are investigated in detail for two different initial states. For two atoms are initially in the ground state, concurrence and the NGMQD increase in the first period and then decreases, after it decreases to zero, it immediately increases again, there is no entanglement sudden death, the NGMQD vanishes only at some discrete times. We show that entanglement is always larger than NGMQD in this case. For two atoms are initially in the excited state, we have shown the novel entanglement and NGMQD sudden death and sudden birth phenomena. It is illustrated that entanglement is not always greater than the NGMQD in this case. They are different not only quantitatively, but also qualitatively.

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