On the Classification of Complex Leibniz Superalgebras with Characteristic Sequence
(n-1, 1 | m_1, ..., m_k) and Nilindex n + m

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On the classification of complex Leibniz superalgebras with characteristic sequence

\((n - 1, 1|m_1, \ldots, m_k)\) and nilindex \(n + m\).

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Abstract

In this work we investigate the complex Leibniz superalgebras with characteristic sequence \((n - 1, 1|m_1, \ldots, m_k)\) and with nilindex equal to \(n + m\). We prove that such superalgebras with the condition \(m_2 \neq 0\) have nilindex less than \(n + m\). Therefore the complete classification of Leibniz algebras with characteristic sequence \((n - 1, 1|m_1, \ldots, m_k)\) and with nilindex equal to \(n + m\) is reduced to the classification of filiform Leibniz superalgebras of nilindex equal to \(n + m\), which was provided in \([3]\) and \([7]\).

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1 Introduction

During many years the theory of Lie superalgebras has been actively studied by many mathematicians and physicists. A systematic exposition of basic Lie superalgebras theory can be found in [9]. Many works have been devoted to the study of this topic, but unfortunately most of them do not deal with nilpotent Lie superalgebras. In works [4], [5], [7] the problem of the description of some classes of nilpotent Lie superalgebras has studied. It is well known that Lie superalgebras are a generalization of Lie algebras. In the same way, the notion of Leibniz algebras, which were introduced in [10], can be generalized to Leibniz superalgebras. The elementary properties of Leibniz superalgebras were obtained in [1]. For nilpotent Leibniz superalgebras the description of the case of maximal nilindex (nilpotent Leibniz superalgebras distinguished by the feature of being singly-onegenerated) is not difficult and was done in [1]. However, the next stage (the description of Leibniz superalgebras with dimensions of the even and odd parts equal to n and m, respectively, and of nilindex $n + m$) is a very problematic one. It should be noted that such Lie superalgebras were classified in [7]. Due to the great difficulty of solving in general the problem of description of Leibniz superalgebras of nilindex $n + m$, some restrictions on the characteristic sequence should be added, in particular, since the graded anticommutative identity does not hold in non-Lie Leibniz superalgebras. In the description of the structure of Leibniz superalgebras the crucial task is to prove the existence of a suitable basis (the so-called adapted basis) in which the table of multiplication of the superalgebra has the most convenient form. In the present paper we investigate the Leibniz superalgebras with the characteristic sequence $C(L) = (n-1,1|m_1,m_2,\ldots,m_k)$ and with nilindex equal to $n + m$. Actually, the classification of such superalgebras in the case where $C(L) = (n-1,1|m)$ was obtained in [3] and the main result of the present work consist of the following fact: Leibniz superalgebras with characteristic sequences equal to $C(L) = (n-1,1|m_1,m_2,\ldots,m_k)$ ($m_2 \neq 0$) have nilindex less than $n + m$. Therefore, the classification of Leibniz superalgebras of nilindex $n + m$ and with characteristic sequence equal to $(n-1,1|m_1,m_2,\ldots,m_k)$ is reduced to the case where the characteristic sequence is equal to $(n-1,1|m)$, and this case, as we mentioned above, has already been solved in [3]. In this way we made further step in the solution of the problem of the classification of complex Leibniz superalgebras
Throughout this work we shall consider spaces and (super)algebras over the field of complex numbers.

2 Preliminaries

We recall the notions of Lie and Leibniz superalgebras.

**Definition 2.1** [9] A $\mathbb{Z}_2$-graded vector space $G = G_0 \oplus G_1$ is called a **Lie superalgebra** if it is equipped with a product $[-,-]$ which satisfies the following conditions:

1. $[G_\alpha, G_\beta] \subseteq G_{\alpha+\beta \pmod{2}}$ for any $\alpha, \beta \in \mathbb{Z}_2$,
2. $[x, y] = -(-1)^{\alpha\beta}[y, x]$, for any $x \in G_\alpha$, $y \in G_\beta$,
3. $(-1)^{\alpha\gamma}[x, [y, z]] + (-1)^{\alpha\beta}[y, [z, x]] + (-1)^{\beta\gamma}[z, [x, y]] = 0$ — Jacobi superidentity,

for any $x \in G_\alpha$, $y \in G_\beta$, $z \in G_\gamma$, $\alpha, \beta, \gamma \in \mathbb{Z}_2$.

**Definition 2.2** [1] A $\mathbb{Z}_2$-graded vector space $L = L_0 \oplus L_1$ is called a **Leibniz superalgebra** if it is equipped with a product $[-,-]$ which satisfies the following conditions:

1. $[L_\alpha, L_\beta] \subseteq L_{\alpha+\beta \pmod{2}}$ for any $\alpha, \beta \in \mathbb{Z}_2$,
2. $[x, [y, z]] = [[x, y], z] - (-1)^{\alpha\beta}[[x, z], y]$ — Leibniz superidentity, for any $x \in L$, $y \in L_\alpha$, $z \in L_\beta$.

Evidently, the subspaces $G_0$ and $L_0$ are Lie and Leibniz algebras, respectively.

It should be noted that if in a Leibniz superalgebra $L$ the identity:

$$[x, y] = -(-1)^{\alpha\beta}[y, x],$$

holds for any $x \in L_\alpha$ and $y \in L_\beta$, then the Leibniz superidentity can easily be transformed into the Jacobi superidentity. Thus, Leibniz superalgebras are generalizations of both Lie (super)algebras and Leibniz algebras. For examples of Leibniz superalgebras we refer to [1].

The set of Leibniz superalgebras with dimensions of the even part $L_0$ and the odd part $L_1$, respectively equal to $n$ and $m$, shall be denoted by $\text{Leib}_{n,m}$.

For a given Leibniz superalgebra $L$ we define a descending central sequence in the following way:

$$L^1 = L, \quad L^{k+1} = [L^k, L^1], \quad k \geq 1.$$
Definition 2.3 A Leibniz superalgebra $L$ is called nilpotent, if there exists $s \in \mathbb{N}$ such that $L^s = 0$. The minimal number $s$ with this property is called index of nilpotency (or nilindex) of the superalgebra $L$.

Definition 2.4 The set $\mathcal{R}(L) = \{x \in L \mid [y, x] = 0 \text{ for any } y \in L\}$ is called the right annihilator of a superalgebra $L$.

Using the Leibniz superidentity it is not difficult to see that $\mathcal{R}(L)$ is an ideal of the superalgebra $L$. Moreover, elements of the form $[a, b] + (-1)^{ab}[b, a]$ belong to $\mathcal{R}(L)$.

The following theorem describes nilpotent Leibniz superalgebras with maximal nilindex.

Theorem 2.1 [1] Let $L = L_0 \oplus L_1$ be a Leibniz superalgebra from $\text{Leib}_{n,m}$ with nilindex equal to $n + m + 1$. Then $L$ is isomorphic to one of the following two non isomorphic superalgebras:

\[
\begin{align*}
[e_i, e_1] &= e_{i+1}, \quad 1 \leq i \leq n - 1; \\
[e_i, e_1] &= e_{i+1}, \quad 1 \leq i \leq n + m - 1, \\
[e_i, e_2] &= 2e_{i+2}, \quad 1 \leq i \leq n + m - 2.
\end{align*}
\]

where omitted products are equal to zero and $\{e_1, e_2, \ldots, e_n\}$ is the basis of the superalgebra $L$.

Remark 2.1 From the description of Theorem 2.1 we have that if the odd part $L_1$ of the superalgebra $L$ is non trivial, then either $m = n$ or $m = n + 1$ and the table of multiplication of the second superalgebra in a graded basis $\{x_1, \ldots, x_n, y_1, \ldots, y_m\}$ can be written in the following form:

\[
\begin{align*}
[y_j, x_1] &= y_{j+1}, \quad 1 \leq j \leq m - 1, \\
[x_1, y_1] &= \frac{1}{2}y_2, \\
[x_i, y_1] &= \frac{1}{2}y_i, \quad 2 \leq i \leq m, \\
[y_1, y_1] &= x_1, \\
[y_j, y_1] &= x_{j+1}, \quad 2 \leq j \leq n - 1, \\
[x_i, x_1] &= x_{i+1}, \quad 1 \leq i \leq n - 1.
\end{align*}
\]

Let $L = L_0 \oplus L_1$ be a nilpotent Leibniz superalgebra. For an arbitrary element $x \in L_0$, the operator of right multiplication $R_x$ (defined as $R_x : y \rightarrow [y, x]$) is a nilpotent endomorphism of the space $L_i$, where $i \in \{0, 1\}$. Denote by $C_i(x)$ ($i \in \{0, 1\}$) the descending sequence of the dimensions of Jordan blocks of the operator $R_x$. Consider the lexicographical order on the set $C_i(L_0)$. 

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**Definition 2.5** The sequence

\[ C(L) = \left( \max_{x \in L_0 \setminus \{L_0, L_0\}} C_0(x) \left| \max_{\tilde{x} \in L_0 \setminus \{L_0, L_0\}} C_1(\tilde{x}) \right. \right) \]

is said to be the characteristic sequence of the Leibniz superalgebra \( L \).

Similarly to [5] (corollary 3.0.1) it can be proved that the characteristic sequence is invariant under isomorphisms.

Further we need the following definition.

**Definition 2.6** A Leibniz algebra \( L \) of dimension \( n \) is said to be filiform if \( \dim L_i = n - i \) for \( 2 \leq i \leq n \).

**Lemma 2.1** [2] Let \( L \) be an \( n \)-dimensional Leibniz algebra. Then the following statements are equivalent:

a) \( C(L) = (n - 1, 1) \);

b) \( L \) is a filiform Leibniz algebra;

c) \( L^{n-1} \neq 0 \) and \( L^n = 0 \).

Let \( L \) be a Leibniz superalgebra from \( \text{Leib}_{n,m} \) with characteristic sequence equal to \((n - 1, 1|m_1, m_2, \ldots, m_k)\), (where \( m_1 + m_2 + \ldots + m_k = m \)). Since in [7] the Leibniz superalgebras with characteristic sequence and nilindex equal to \((n - 1, n|m)\) and \( n + m \), respectively, have already been obtained, we shall henceforth reduce our investigation to the case where \( m_2 \neq 0 \).

From Lemma 2.1 we can conclude that the even part \( L_0 \) of \( L \) is a filiform Leibniz algebra. Due to the description of filiform Leibniz algebras in [6], [8], [11] we can obtain the existence of an adapted basis in superalgebra with \( C(L) = (n - 1, 1|m_1, m_2, \ldots, m_k) \), according to the following theorem

**Theorem 2.2** Let \( L = L_0 \oplus L_1 \) be a superalgebra from \( \text{Leib}_{n,m} \) with characteristic sequence equal to \((n - 1, 1|m_1, m_2, \ldots, m_k)\). Then there exists a basis \( \{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m\} \) of \( L \), in which the multiplication satisfies one of the following three conditions:

a) \( [x_1, x_1] = x_3 \).
for some $x \in L_0 \setminus L_0^2$,
\[
[x_1, x_2] = \alpha_4 x_4 + \alpha_5 x_5 + \ldots + \alpha_{n-1} x_{n-1} + \theta x_n,
\]
\[
[x_j, x_2] = \alpha_4 x_{j+2} + \alpha_5 x_{j+3} + \ldots + \alpha_{n+2-j} x_n, \quad 2 \leq j \leq n - 2,
\]
where the omitted products in $L_0$ are equal to zero;

b) $[x_1, x_1] = x_3,$
\[
[x_i, x_1] = x_{i+1}, \quad 3 \leq i \leq n - 1,
\]
\[
[y_j, x] = y_{j+1}, \quad j \notin \{m_1, m_1 + m_2, \ldots, m_1 + m_2 + \ldots + m_k\},
\]
\[
[y_j, x] = 0, \quad j \notin \{m_1, m_1 + m_2, \ldots, m_1 + m_2 + \ldots + m_k\},
\]
for some $x \in L_0 \setminus L_0^2$,
\[
[x_1, x_2] = \beta_4 x_4 + \beta_5 x_5 + \ldots + \beta_n x_n,
\]
\[
[x_2, x_2] = \gamma x_n,
\]
\[
[x_j, x_2] = \beta_4 x_{j+2} + \beta_5 x_{j+3} + \ldots + \beta_{n+2-j} x_n, \quad 3 \leq j \leq n - 2,
\]
where the omitted products in $L_0$ are equal to zero;

c) $[x_i, x_1] = x_{i+1}, \quad 2 \leq i \leq n - 1,$
\[
[x_i, x_1] = -x_{i+1}, \quad 3 \leq i \leq n - 1,
\]
\[
[y_j, x] = y_{j+1}, \quad j \notin \{m_1, m_1 + m_2, \ldots, m_1 + m_2 + \ldots + m_k\},
\]
\[
[y_j, x] = 0, \quad j \notin \{m_1, m_1 + m_2, \ldots, m_1 + m_2 + \ldots + m_k\},
\]
for some $x \in L_0 \setminus L_0^2$,
\[
[x_1, x_1] = \theta_1 x_n,
\]
\[
[x_1, x_2] = -x_3 + \theta_2 x_n,
\]
\[
[x_2, x_2] = \theta_3 x_n,
\]
\[
[x_i, x_j] = -[x_j, x_i] \in \text{lin}\{x_{i+j+1}, x_{i+j+2}, \ldots, x_n\}, \quad 2 \leq i < j \leq n - 2.
\]
3 On the classification of Leibniz superalgebras with characteristic sequence \((n-1, 1|m_1, m_2, \ldots, m_k)\) and nilindex \(n + m\ (m_2 \neq 0)\).

Let \(L\) satisfy to the conditions of Theorem 2.2 and let \(\{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m\}\) be the adapted basis of \(L\). It is not difficult to see that if \(L\) has nilindex equal to \(n + m\), then the superalgebra \(L\) has two generators (due to Theorem 2.1 we have a description of singly-generated Leibniz superalgebras, which have nilindex \(n + m + 1\)) and \(\dim L^i = n + m - 2\) for \(2 \leq i \leq n + m\). It should be noted that the filiform Leibniz algebra \(L_0\) has also two generators, \(x_1\) and \(x_2\).

**Lemma 3.1** In three the classes of superalgebras of Theorem 2.2 instead of the element \(x\) one can choose the element \(x_1\).

**Proof.** Without loss of generality we can assume that \(x\) has the form: \(x = A_1x_1 + A_2x_2\), where \((A_1, A_2) \neq (0, 0)\).

Let us consider the first class of superalgebras of Theorem 2.2 and investigate three cases.

**Case 1.** Let \(A_1(A_1 + A_2) \neq 0\). Then applying the following change of basis:

\[
x_1' = A_1x_1 + A_2x_2, \quad x_2' = (A_1 + A_2)x_2 + A_2(\theta - \alpha_n)x_{n-1},
\]

\[
x_i' = [x_{i-1}', x_1'], \quad 3 \leq i \leq n, \quad y_j' = y_j, \quad 1 \leq j \leq m,
\]

we obtain that the first four multiplications in class a) also do not change.

**Case 2.** Let \(A_1 = 0\). Let us make a change of basis as follows:

\[
x_1' = x_1 + aA_2x_2, \quad \text{where} \quad a(1 + aA_2) \neq 0,
\]

\[
x_2' = (1 + A_2)x_2 + aA_2(\theta - \alpha_n)x_{n-1}, \quad x_i' = [x_{i-1}', x_1'], \quad 3 \leq i \leq n,
\]

\[
y_j' = y_j, \quad j \in \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_{k-1} + 1\},
\]

\[
y_j' = [y_{j-1}', x_1'], \quad j \notin \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_{k-1} + 1\}.
\]

If we choose a sufficiently big value of the parameter \(a\) then we obtain that the first four multiplications in the class a) also do not change. Indeed, the first three
multiplications do not change by the construction and the products $[y'_j, x'_i]$ for $j \in \{m_1, m_1 + m_2, \ldots, m_1 + m_2 + \ldots + m_k\}$ are equal to zero, because otherwise we easily can get a contradiction with the characteristic sequence or nilpotence conditions.

**Case 3.** Let $A_1 \neq 0$ and $A_1 = -A_2$. Then taking the following transformation of basis:

$$x'_1 = A_1 x_1 - A_1 x_2 + a x_2, \quad x'_2 = ax_2 + (a - A_1)(\theta - \alpha_n)x_{n-1}, \quad (a \neq 0),$$

$$x'_i = [x'_{i-1}, x'_1], \quad 3 \leq i \leq n, \quad y'_j = y_j, \quad j \in \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_k + 1\},$$

$$y'_j = [y'_{j-1}, x'_1], \quad j \notin \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_k + 1\},$$

it is not difficult to check that for sufficiently small values of the parameter $a$ the first four multiplications in the class a) are preserved.

Thus, we have shown that in the first case of superalgebras of Theorem 2.2 instead of element $x$ one can choose element $x_1$.

Let us consider the class b) of Theorem 2.2.

If $A_1 \neq 0$, then applying a transformation of basis of the form:

$$x'_1 = A_1 x_1 + A_2 x_2, \quad x'_2 = x_2 - \frac{A_2 \gamma}{A_1} x_{n-1},$$

$$x'_3 = [x'_1, x'_1], \quad x'_i = [x'_{i-1}, x'_1], \quad 4 \leq i \leq n, \quad y'_j = y_j, \quad 1 \leq j \leq m,$$

we obtain that the first four multiplications do not change.

If $A_1 = 0$, then the following change of basis:

$$x'_1 = x_1 + a A_2 x_2, \quad (a \neq 0), \quad x'_2 = x_2 - a A_2 \gamma x_{n-1},$$

$$x'_3 = [x'_1, x'_1], \quad x'_i = [x'_{i-1}, x'_1], \quad 4 \leq i \leq n,$$

$$y'_j = y_j, \quad j \in \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_k + 1\},$$

$$y'_j = [y'_{j-1}, x'_1], \quad j \notin \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_k + 1\},$$

with a sufficiently big value of the parameter $a$ allow to conclude that first four multiplications in class b) do not change.

Now consider the class c) of Theorem 2.2.

If $A_1 \neq 0$, then applying a transformation of basis of the form:

$$x'_1 = A_1 x_1 + A_2 x_2, \quad x'_2 = x_2, \quad x'_i = [x'_{i-1}, x'_1], \quad 3 \leq i \leq n, \quad y'_j = y_j, \quad 1 \leq j \leq m,
we obtain that the first four multiplications are preserved.

If $A_1 = 0$, then take the transformation of basis:

$$x'_1 = x_1 + aA_2 x_2, \ (a \neq 0), \ x'_2 = x_2, \ x'_i = [x'_{i-1}, x'_1], \ 3 \leq i \leq n,$$

$$y'_j = y_j, \quad j \in \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_{k-1} + 1\},$$

$$y'_j = [y'_{j-1}, x'_1], \ j \notin \{1, m_1 + 1, m_1 + m_2 + 1, \ldots, m_1 + m_2 + \ldots + m_{k-1} + 1\}.$$

Then choosing a sufficiently big value of the parameter $a$ allow us to conclude that the first four products in the case c) of Theorem 2.2 do not change.

Thus, we have proven that in the three classes of superalgebras of Theorem 2.2 instead of the element $x$ we can choose element $x_1$. \qed

Since the superalgebra $L = L_0 \oplus L_1$ has two generators the possible cases are as follow: both generators lie in $L_0$; one generator lies in $L_0$ and the another one lies in $L_1$; both generators lie in $L_1$.

We shall not consider the case where both generators lie in the even part (since $m_2 \neq 0$). Firstly we consider the second possible, i.e. the case where one of the generators lies in $L_0$ and the another one lies in $L_1$. It is easy to see that there exist some $m_j, \ 0 \leq j \leq k - 1$ (here $m_0 = 0$), such that $y_{m_1+m_2+\ldots+m_j+1} \notin L^2$. By a shifting of basic elements one can assume that $m_j = m_0$, i.e. the basic element $y_1 \notin L^2$. Of course, by this choice the condition $m_1 \geq m_2 \ldots m_k$, is broken, but we shall not use this condition in our study further. Thus, as generators we can choose the elements $A_1 x_1 + A_2 x_2$ and $y_1$.

Let us introduce the notations

$$[x_i, y_1] = \sum_{j=2}^{m} \alpha_{i,j} y_j, \ 1 \leq i \leq n, \ [y_s, y_1] = \sum_{t=1}^{n} \beta_{s,t} x_t, \ [y_p, x_2] = \sum_{q=2}^{m} \gamma_{p,q} y_q, \ 1 \leq s, p \leq m.$$ 

**Theorem 3.1** Let $L$ be a Leibniz superalgebra from $\text{Leib}_{n,m}$ with characteristic sequence $(n-1, 1|m_1, m_2, \ldots, m_k)$, where $m_1 \geq 2, \ n \geq 4$. Let the elements $A_1 x_1 + A_2 x_2$, $y_1$ be generators and $x_1 \in L^2$. Then $L$ has a nilindex less than $n + m$.

**Proof.** Since $x_1 \in L^2$, then $x_2 \notin L^2$ and therefore as a generator of the $L$ which lies in $L_0$ we can choose $x_2$. Let us assume the contrary, i.e. the nilindex of the superalgebra $L$ is equal to $n + m$. Then we have

$$L = \{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m\}, \ L^2 = \{x_1, x_3, \ldots, x_n, y_2, y_3, \ldots, y_m\}. $$
Since $x_1$ is generator in the filiform Leibniz algebra $L_0$ then it should lie in linear span of products $[y_s,y_1]$, $1 \leq s \leq m$. Therefore, there exists some $s$ ($1 \leq s \leq m$) such that $\beta_{s,1} \neq 0$.

Denote by $y_{m_1+m_2+\ldots+m_0+1}$ the basic element which is the earliest generated among the elements $\{y_{m_1+1},y_{m_1+m_2+1},\ldots,y_{m_1+m_2+\ldots+m_{k-1}+1}\}$, i.e. $y_{m_1+m_2+\ldots+m_0+1}$ is the basic element which first is absent among the elements $\{y_{m_1+1},y_{m_1+m_2+1},\ldots,y_{m_1+m_2+\ldots+m_{k-1}+1}\}$ in the descending central sequence. Therefore, the element $y_{m_1+m_2+\ldots+m_0+1}$ is generated by products of elements of the form either $[x_i,y_1], 2 \leq i \leq n$ or $[y_j,x_2], 1 \leq j \leq m_1$.

Let us show that $x_1 \notin L^3$. Indeed, if $x_1$ lies in $L^3$, then it should be generated by the product $[y_{m_1+m_2+\ldots+m_0+1},y_1]$. From the nilpotence condition we have that $x_1$ is generated by the products $[[x_2,y_1],y_1], [[y_1,x_2],y_1]$ (since the basic elements $x_3,x_4,\ldots,x_n$ and $y_2,y_3,\ldots,y_m$ are obtained by the products involving $x_1$). Hence for generating the element $y_{m_1+m_2+\ldots+m_0+1}$ it is enough to consider the cases where $i = 2$, $j = 1$.

From the equalities
\[
[[x_2,y_1],y_1] = \frac{1}{2} [x_2,[y_1,y_1]] = \frac{1}{2} [x_2, \sum_{s=1,s\neq 2}^{n} \beta_{1,s} x_s] = \sum_{t \geq 3} (*x_t)
\]
(where by the symbol $(*)$ we denote the coefficients of the basic elements $x_t$), we have that the element $x_1$ is not present in the decomposition of the product $[[x_2,y_1],y_1]$. If $x_1$ is generated from the product $[[y_1,x_2],y_1]$, then the expression $[[y_1,x_2],y_1] + [[x_2,y_1],y_1]$ lies in $\mathcal{R}(L)$ and $x_1$ appear in its decomposition. Using the table of multiplication in the algebra $L_0$ from Theorem 2.2 we establish that multiplying the expression $[[y_1,x_2],y_1] + [[x_2,y_1],y_1]$ on the right side by the element $x_1$ sufficiently many times we obtain $x_n \in \mathcal{R}(L)$. Then repeating this procedure finally we obtain $x_1 \in \mathcal{R}(L)$. Thus, we obtain a contradiction, because $x_1 \notin \mathcal{R}(L)$. Therefore, $x_1 \notin L^3$.

Let us consider the cases a) and b). The condition $x_1 \notin L^3$ leads to $\beta_{1,1} \neq 0$.

Since in the cases a) and b) the elements $x_i$, $3 \leq i \leq n$ lie in $\mathcal{R}(L)$, then we can put $x'_i = \beta_{1,i} x_1 + \beta_{1,3} x_3 + \ldots + \beta_{1,n} x_n$ and suppose that $[y_1,y_1] = x_1$.

Consider the subsuperalgebra generated by $y_1 (< y_1 >)$. Then it is easy to check that $< y_1 > = \{x_1,x_3,\ldots,x_n,y_1,y_2,\ldots,y_m\}$. Since this subsuperalgebra is single-generated then from Remark 2.1 we have
\[
[y_j,x_1] = y_{j+1}, \quad 1 \leq j \leq m_1 - 1, \quad [x_1,y_1] = \frac{1}{2} y_2,
\]
\[ [x_i, y_1] = \frac{1}{2} y_i, 3 \leq i \leq m_1, \quad [y_1, y_1] = x_1, \]

\[ [y_j, y_1] = x_{j+1}, \quad 2 \leq j \leq n - 1, \]

where if \( n + m_1 \) is even then \( m_1 = n - 1 \) and if \( n + m_1 \) is odd then \( m_1 = n \).

From above products we have \([y_1, x_1] + [x_1, y_1] = \frac{3}{2} y_2\) which yields \( y_i \in \mathcal{R}(L) \) for \( 2 \leq i \leq m_1 \).

Using the fact that in the cases a) and b) the product \([x_1, x_2]\) belongs to \( \mathcal{R}(L) \) and the following equalities hold

\[ [y_i, x_2] = [[y_{i-1}, x_1], x_2] = [y_{i-1}, [x_1, x_2]] + [[y_{i-1}, x_2], x_1] = [[y_{i-1}, x_2], x_1] = \gamma_1 y_{i+1} + (*y_{i+2} + \ldots + (*y_{m_1} + (*y_{m_1+2} + \ldots + (*y_{m_1+m_2+2} + \ldots + (*y_{m}), \]

for \( 2 \leq i \leq m_1 \), we can conclude that \( y_{m_1+m_2+\ldots+m_0+1} \notin L^3 \), i.e. \( \text{dim} L^3 < n + m - 3 \). Hence we obtain a contradiction with the assumption that the nilindex is equal to \( n + m \).

Now, consider superalgebras from the class b) of Theorem 2.2. We have

\[ L^2 = \{ x_1, x_2, \ldots, x_n, y_1, y_2, y_3, \ldots, y_m \}, \]

\[ L^3 = \{ x_2, \ldots, x_n, y_2, y_3, \ldots, y_m \}. \]

Consider the equalities

\[ [y_1, x_3] = [y_1, [x_2, x_1]] = [[y_1, x_2], x_1] - [[y_1, x_1], x_2] = \sum_{p=2}^{m-2} \gamma_{1,p} y_{p+1} - [y_2, x_2]. \]

From the nilpotence condition of the superalgebra \( L \) it follow that in the decomposition of the product \([y_2, x_2]\) the basic element \( y_2 \) does not participate, i.e. \([y_2, x_2] = \sum_{i=3}^{m} \gamma_{2,i} y_i \). Therefore

\[ [y_1, x_3] = (\gamma_{1,2} - \gamma_{2,3}) y_3 + (\gamma_{1,3} - \gamma_{2,4}) y_4 + \ldots + (\gamma_{1,m-2} - \gamma_{2,m-1}) y_{m-1} - \gamma_{2,m} y_m; \]

In a similar way we obtain

\[ [y_i, x_3] = (\gamma_{i,i+1} - \gamma_{i+1,i+2}) y_{i+2} + \ldots + (\gamma_{i,m-2} - \gamma_{i+1,m-1}) y_{m-1} - \gamma_{i+1,m} y_m, \quad 2 \leq i \leq m - 3. \]

Applying the above arguments for \([y_i, x_j]\), \( 4 \leq j \leq n \) we get that \([y_i, x_j] = \sum_{j \geq i+2} (\ast)y_j \) for \( 4 \leq j \leq n \). Therefore without loss of generality one can assume that the expression
\(\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n\) can be replaced by \(x_1\), i.e. we can suppose \([y_1, y_1] = x_1\).

Consider the equalities

\[
[x_1, y_1] = [[y_1, y_1], y_1] = \frac{1}{2}[y_1, [y_1, y_1]] = \frac{1}{2}[y_1, x_1] = \frac{1}{2}y_2.
\]

Since \([x_1, y_1] + [y_1, x_1] = \frac{3}{2}y_2 \in \mathcal{R}(L)\) and \(y_j = [y_j, x_1]\) for \(1 \leq j \leq m_1 - 1\), then \(y_3, y_4, \ldots, y_{m_1} \in \mathcal{R}(L)\).

Using induction and the following chain of equalities

\[
[y_2, y_1] = [[y_1, x_1], y_1] = [y_1, [x_1, y_1]] + [[y_1, y_1], x_1] = \theta_1 x_1,
\]

\[
[y_i, y_1] = [[y_i-1, x_1], y_1] = [y_i-1, [x_1, y_1]] + [[y_i-1, y_1], x_1] = [[y_i-1, y_1], x_1],
\]

\[
[x_j, y_1] = [[x_j-1, x_1], y_1] = [x_j-1, [x_1, y_1]] + [[x_j-1, y_1], x_1] = [[x_j-1, y_1], x_1],
\]

we establish that

\([y_i, y_1] = 0\) for \(3 \leq i \leq m_1\)

and

\[
x_i, y_1 = \alpha_1 y_1 + \ldots + \alpha_{2,m_1+2-i} y_{m_1} + \alpha_{2,m_1+1} y_{m_1+i-1} + \ldots +
\]

\[
[\alpha_{2,m_1+m_2+2-i} y_{m_1+m_2} + \alpha_{2,m_1+m_2+1} y_{m_1+m_2+i-1} + \ldots + \alpha_{2,m+2-i} y_m, 3 \leq j \leq n].
\]

The obtained products lead to \(y_2 \not\in L^4\) and that the basic element \(y_{m_1+m_2+\ldots+m_n+1}\) is generated by the products \([y_j, x_2], 2 \leq j \leq m_1\).

Since \(y_j \in \mathcal{R}(L)\) for \(2 \leq j \leq m_1\) and the other basic elements \(y_{m_1+1}, \ldots, y_m\) are generated by products of the form \([[y_j, x_2], \ldots]\), where \(2 \leq j \leq m_1\), then \(\{y_2, y_3, \ldots, y_m\} \in \mathcal{R}(L)\). Since \([[y_j, x_2], y_1] = [y_j, [x_2, y_1]] + [[y_j, y_1], x_2] = 0\) and

\[
[[[y_j, x_2], \ldots, x_2], y_1] = [[[y_j, x_2], \ldots], x_2, y_1] +
\]

\[
[[[y_j, x_2], \ldots], y_1, x_2] = [[[y_j, x_2], \ldots], y_1, x_2] = \ldots = 0,
\]

then we easily obtain that

\([y_{m_1+m_2+\ldots+m_1+1}, y_1] = 0, \text{ for } 1 \leq t \leq k - 1\).

Inductively we get

\([y_j, y_1] = 0\) for \(2 \leq j \leq m_1\).
and that $x_3$ does not lie in $L^4$. Thus, we obtain that 

$$L^4 \subseteq \{x_4, \ldots, x_n, y_3, \ldots, y_m\}.$$ 

Hence $\dim L^4 < n + m - 4$, but this contradict the condition that the nilindex is equal to $n + m$. Thus, in the three classes of Theorem 2.2 we obtain a contradiction with the assumption that the superalgebra $L$ has nilindex equal to $n + m$ and therefore the assertion of the theorem is proved. \hfill \Box

From Theorem 3.1 we can assume that $x_1$ and $y_1$ are generators of the superalgebra $L$.

**Theorem 3.2** Let $L$ be a Leibniz superalgebra from $\text{Leib}_{n,m}$ with characteristic sequence equal to $(n - 1, 1 | m_1, m_2, \ldots, m_k)$ and let $\{x_1, y_1\}$ be generators of $L$. Then the superalgebra $L$ has nilindex less than $n + m$.

**Proof.** Let $L$ be a superalgebra satisfying the conditions of the theorem. Then

$$L^2 = \{x_2, x_3, \ldots, x_n, y_2, y_3, \ldots, y_m\}.$$ 

Since $y_{m_1+\ldots+m_t+1} \in L^2$ for any $t$ ($1 \leq t \leq k - 1$), we can conclude that $(\alpha_{1,m_1+\ldots+m_t+1}, \alpha_{2,m_1+\ldots+m_t+1}, \ldots, \alpha_{n,m_1+\ldots+m_t+1}) \neq (0, 0, \ldots, 0)$ for any $t$ ($1 \leq t \leq k - 1$). But this means that for any $t$ ($1 \leq t \leq k - 1$) the basic element $y_{m_1+\ldots+m_t+1}$ is generated by the products $[x_i, y_1]$, $1 \leq i \leq n$.

As in the proof of Theorem 3.1 denote by $y_{m_1+\ldots+m_0+1}$ the basic element which first is absent among the elements

$$\{y_{m_1+1}, y_{m_1+m_2+1}, \ldots, y_{m_1+m_2+\ldots+m_{k-1}+1}\}$$

in descending lower sequence. Then $(\alpha_{1,m_1+\ldots+m_0+1}, \alpha_{2,m_1+\ldots+m_0+1}, \ldots, \alpha_{n,m_1+\ldots+m_0+1}) \neq (0, 0, \ldots, 0)$. Let $f$ be the natural number such that $\alpha_{f,m_1+\ldots+m_0+1} \neq 0$ and $\alpha_{k,m_1+\ldots+m_0+1} = 0$ for $f \leq k \leq n$.

We shall prove that $f = n$. Let us suppose the opposite, i.e. $f < n$. Then for the powers of descending lower sequences we have the following:

$$L^s = \{x_f, \ldots, x_n, y_r, \ldots, y_m, y_{m_1+1}, \ldots, y_{m_1+\ldots+m_0+1}, y_{m_1+\ldots+m_0+1}, \ldots, y_m\},$$

$$L^{s+1} = \{x_{f+1}, \ldots, x_n, y_r, \ldots, y_m, y_{m_1+1}, \ldots, y_{m_1+\ldots+m_0+1}, y_{m_1+\ldots+m_0+1}, \ldots, y_m\}.$$

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\[ L^{s+2} = \{ x_{f+1}, \ldots, x_n, y_r, \ldots, y_{m_1}, y_{m_1+1}, \ldots, y_{m_1+m_0}, y_{m_1+m_0+2}, \ldots, y_m \}. \]

From these we have that the elements \( \{ y_r, \ldots, y_{m_1}, y_{m_1+1}, \ldots, y_{m_1+m_0}, y_{m_1+m_0+2}, \ldots, y_m \} \) are obtained from the products \( [x_i, y_1], \ f + 1 \leq i \leq n. \)

The elements \( \{ y_r, \ldots, y_{m_1}, y_{m_1+1}, \ldots, y_{m_1+m_0}, y_{m_1+m_0+2}, \ldots, y_m \} \) belong to \( L^{s+3} \) (because \( \{ x_{f+1}, \ldots, x_m \} \in L^{s+2} \) and hence \( x_{f+1} \notin L^{s+3} \)). Therefore in the decomposition

\[ [y_{m_1+\ldots+m_0+1}, y_1] = \beta_{m_1+\ldots+m_0+1,f+1} x_{f+1} + \ldots + \beta_{m_1+\ldots+m_0+1,n} x_n \]

we have \( \beta_{m_1+\ldots+m_0+1,f+1} \neq 0. \)

Consider the equalities

\[ [x_f, [y_1, y_1]] = 2[[x_f, y_1], y_1] = 2[\alpha_{f,r} y_r + \alpha_{f,r+1} y_{r+1} + \ldots + \alpha_{f,m_1+1} y_{m_1+1} + \ldots + \beta_{m_1+\ldots+m_0+1,f+1} x_{f+1} + \sum_{i \geq f+2} (\ast) x_i. \]

On the other hand

\[ [x_f, [y_1, y_1]] = [x_f, \beta_{1,2} x_2 + \beta_{1,3} x_3 + \ldots + \beta_{1,n} x_n] = \sum_{j \geq f+2} (\ast) x_j. \]

Comparing the coefficients of the basic elements we obtain

\[ \alpha_{f,m_1+\ldots+m_0+1} \beta_{m_1+\ldots+m_0+1,f+1} = 0, \]

which contradicts the conditions: \( \alpha_{f,m_1+\ldots+m_0+1} \neq 0, \ \beta_{m_1+\ldots+m_0+1,f+1} \neq 0. \) Thus, we get a contradiction with the assumption that \( f < n. \) Now we shall study the case where \( f = n, \) i.e. \( \alpha_{n,m_1+\ldots+m_0+1} \neq 0. \) In this case for some natural number \( p \) we have

\[ L^p = \{ y_{m_1+1}, \ldots, y_{m_1+m_0}, y_{m_1+m_0+1}, \ldots, y_m \}. \]

It is clear that if \( k \geq 3, \) then \( \dim L^{p+1} - \dim L^p \geq 2 \) and we have a contradiction with the nilindex condition. If \( k = 2 \) then the vector space generated by the elements \( < y_{m_1+1}, \ldots, y_{m_1+m_2} > \) forms an ideal of the superalgebra \( L. \) The quotient superalgebra \( \tilde{L} = L/ < y_{m_1+1}, \ldots, y_{m_1+m_2} > \) is also two generated and \( C(\tilde{L}) = (n-1,1|m_1). \) Now applying Lemma 3.4 from [3] we get a contradiction, which completes the proof of the theorem.

Let us investigate the case where both generators lie in odd part of the superalgebra \( L. \) The following theorem clears up the situation in this case.
Theorem 3.3 Let $L = L_0 \oplus L_1$ be a superalgebra from Leibniz with characteristic sequence equal to $(n - 1, 1|m_1, m_2, \ldots, m_k)$, where $m_1 \geq 2$, $n \geq 3$ and let both generators lie in $L_1$. Then the superalgebra $L$ has nilindex less than $n + m$.

Proof. Since both generators of the superalgebra $L$ lie in $L_1$, they are linear combinations of the elements \{\(y_1, y_{m_1+1}, \ldots, y_{m_1+m_2+\ldots+m_{k-1}+1}\)\}. Without loss of generality we may assume that $y_1$ and $y_{m_1+1}$ are generators.

Let $L^{2t} = \{x_1, x_{i+1}, \ldots, x_n, y_j, \ldots, y_m\}$ for some natural number $t$ and let $z$ be an arbitrary element such that $z \in L^{2t} \setminus L^{2t+1}$. Then $z$ is generated by the products of even an number of generators. Hence $z \in L_0$ and $L^{2t+1} = \{x_{i+1}, \ldots, x_n, y_j, \ldots, y_m\}$. In a similar way, having $L^{2t+1} = \{x_{i+1}, \ldots, x_n, y_j, \ldots, y_m\}$ we obtain $L^{2t+2} = \{x_{i+1}, \ldots, x_n, y_{j+1}, \ldots, y_m\}$.

From the above arguments we conclude that $n = m - 1$ or $n = m - 2$.

Let us consider powers of $L$:

\[
L^2 = \{x_1, x_2, \ldots, x_n, y_2, y_3, \ldots, y_{m_1}, y_{m_1+2}, \ldots, y_m\},
\]

\[
L^3 = \{A_1x_1 + A_2x_2, x_3, \ldots, x_n, y_2, y_3, \ldots, y_{m_1}, y_{m_1+2}, \ldots, y_m\},
\]

\[
L^4 \supseteq \{A_1x_1 + A_2x_2, x_3, \ldots, x_n, y_3, y_4, \ldots, y_{m_1}, y_{m_1+3}, \ldots, y_m\}.
\]

Applying the above arguments we get that an element from the set \{\(y_3, y_4, \ldots, y_{m_1}, y_{m_1+3}, \ldots, y_m\)\} disappears in $L^4$. If necessary then by a shifting of basic elements we can suppose that $y_2 \notin L^4$ without loss of the generality. Then

\[
L^4 = \{A_1x_1 + A_2x_2, x_3, \ldots, x_n, y_3, y_4, \ldots, y_{m_1}, y_{m_1+2}, \ldots, y_m\},
\]

\[
L^5 = \{x_3, \ldots, x_n, y_3, y_4, \ldots, y_{m_1}, y_{m_1+2}, \ldots, y_m\}.
\]

From these restrictions on the powers of $L$ in the following products

\[
[y_1, y_1] = \beta_{1,1}x_1 + \beta_{1,2}x_2 + \ldots + \beta_{1,n}x_n,
\]

\[
[y_2, y_1] = \beta_{2,2}(A_1x_1 + A_2x_2) + \beta_{2,3}x_3 + \ldots + \beta_{2,n}x_n,
\]

\[
[y_1, y_{m_1+1}] = \gamma_{1,1}x_1 + \gamma_{1,2}x_2 + \ldots + \gamma_{1,n}x_n,
\]

\[
[y_2, y_{m_1+1}] = \gamma_{2,2}(A_1x_1 + A_2x_2) + \gamma_{2,3}x_3 + \ldots + \gamma_{2,n}x_n,
\]

we obtain the condition \((\beta_{2,2}, \gamma_{2,2}) \neq (0,0)\).
Let us introduce the notations

\[ [x_1, y_1] = \alpha_{1,2}y_2 + \alpha_{1,3}y_3 + \ldots + \alpha_{1,m_1}y_{m_1} + \alpha_{1,m_1+2}y_{m_1+2} + \ldots + \alpha_{1,m_1}y_m, \]

\[ [x_2, y_1] = \alpha_{2,2}y_2 + \alpha_{2,3}y_3 + \ldots + \alpha_{2,m_1}y_{m_1} + \alpha_{2,m_1+2}y_{m_1+2} + \ldots + \alpha_{2,m_1}y_m. \]

Consider the equalities

\[ [x_1, [y_1, y_1]] = [x_1, \beta_{1,1}x_1 + \beta_{1,2}x_2 + \ldots + \beta_{1,n}x_n] = \sum_{j \geq 3} (\ast) x_j. \]

Comparing the coefficients of the basic elements in these equations we obtain

\[ \alpha_{1,2}\beta_{2,2} = 0. \]

Consider the product

\[ [y_1, [y_1, x_1]] = [[y_1, y_1], x_1] - [[y_1, x_1], y_1] = [\beta_{1,1}x_1 + \beta_{1,2}x_2 + \ldots + \beta_{1,n}x_n, x_1] - \]

\[ [y_2, y_1] = -\beta_{2,2}(A_1 x_1 + A_2 x_2) + \sum_{s \geq 3} (\ast) x_s. \]

Since \([y_1, [y_1, x_1]] = [y_1, y_2]\) then \([y_1, y_2] = -\beta_{2,2}(A_1 x_1 + A_2 x_2) + \sum_{s \geq 3} (\ast) x_s.\)

From the following the chain of equalities

\[ [y_1, [x_1, y_1]] = [[y_1, x_1], y_1] - [[y_1, y_1], x_1] = [y_2, y_1] - [\beta_{1,1}x_1 + \beta_{1,2}x_2 + \]

\[ \ldots + \beta_{1,n}x_n, x_1] = \beta_{2,2}(A_1 x_1 + A_2 x_2) + \sum_{t \geq 3} (\ast) x_t \]

and

\[ [y_1, [x_1, y_1]] = [y_1, \alpha_{1,2}y_2 + \alpha_{1,3}y_3 + \ldots + \alpha_{1,m_1}y_{m_1} + \alpha_{1,m_1+2}y_{m_1+2} + \ldots + \alpha_{1,m_1}y_m] = \]

\[ = -\alpha_{1,2}\beta_{2,2}(A_1 x_1 + A_2 x_2) + \sum_{p \geq 3} (\ast) x_p \]

we obtain the restriction \(\beta_{2,2} = -\alpha_{1,2}\beta_{2,2}.\)

Taking into account the condition \(\alpha_{1,2}\beta_{2,2} = 0\) we get \(\beta_{2,2} = 0.\) Consider

\[ [y_1, [y_{m_1+1}, x_1]] = [[y_1, y_{m_1+1}], x_1] - [[y_1, x_1], y_{m_1+1}] = [\gamma_{1,1} x_1 + \gamma_{1,2} x_2 + \ldots \]

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\[ +\gamma_{1,n}x_n, x_1] - [y_2, y_{m_1+1}] = -\gamma_{2,2}(A_1x_1 + A_2x_2) + \sum_{q \geq 3}(*x_q). \]

On the other hand we have \([y_1, [y_{m_1+1}, x_1]] = \sum_{l \geq 3}(*x_l).\) Comparing the coefficients of the basic elements we get \(\gamma_{2,2} = 0,\) which contradicts the condition \((\beta_{2,2}, \gamma_{2,2}) \neq (0,0)\).

Hence, we have \(L^3 = \{x_3, \ldots , x_n, y_2, y_3, \ldots , y_{m_1}, y_{m_1+2}, y_{m_1+3}, \ldots , y_m\},\) i.e. \(A_1x_1 + A_2x_2 \notin L^3.\) Therefore the nilindex of the superalgebra \(L\) is less than \(n + m.\) 

The investigation of the cases where \(L\) is a Leibniz superalgebra with characteristic sequence \((n - 1, 1|m_1, m_2, \ldots , m_k),\) where either \(m_1 < 2\) or \(n < 4,\) give us the same result. Considering these cases consist is a simple routine work, mainly repeating the above technique, hence we omit details for these cases.

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