Fractional Differentiability for the Stress Velocities to the Solution of the Prandtl-Reuss Problem

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Abstract

We consider the loading of an elastic perfectly plastic body governed by the Prandtl-Reuss law. It is shown that the stress velocities of the body have fractional derivatives of order $1/2 - \delta$ up to the boundary in the direction of the loading parameter, and of order $1/3 - \delta$ in the interior of the body in direction of the space variables.

Key words: Prandtl-Reuss-law, elastic plastic deformation, regularity, fractional differentiability
AMS classification 49N60, 35B65,74C05,74G65

1 Introduction

The Prandtl-Reuss problem describes the deformation of an elastic perfectly plastic body occupying a bounded domain Ω. We prove a regularity result for solutions to the associated variational inequality.
For the proper formulation we need to fix some notations. Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$ be a bounded domain in $\mathbb{R}^n$ with Lipschitz boundary $\partial\Omega$, the main application naturally is the case $n = 3$. By $\mathbb{R}^{n\times n}_{sym}$, we denote the set of all symmetric $n \times n$ matrices $\tau \in \mathbb{R}^{n\times n}$ with unit matrix $\mathbb{I}$. For $\tau, \sigma \in \mathbb{R}^{n\times n}$, the scalar product $\sigma : \tau$, Euclidean norm $|\sigma|$, the trace $\operatorname{tr}\sigma$ and the deviator σ_D are given by:

$$\sigma : \tau = \sum_{i,k=1}^{n} \sigma_{ik}\tau_{ik}, \quad |\sigma| = (\sigma : \sigma)^{1/2}, \quad \operatorname{tr}\sigma = \sum_i \sigma_{ii}, \quad \sigma_D = \sigma - \frac{\operatorname{tr}\sigma}{n}\mathbb{I}.$$

The symbol $L^p(\Omega)$, with $1 \leq p \leq \infty$, denotes the usual Lebesgue-space, where we do not distinguish between scalar-, vector- , or tensor-valued functions as long as no confusion arises, in all cases we indicate the $L^2(\Omega)$-scalar product with brackets $(\,\cdot\,,\,\cdot\,)$. Let $I = [0, T]$, $T > 0$ be a fixed interval, then for a Banach space X (which is always a function space in the sequel), the symbol $L^p(I, X)$ stands for measurable and p-summable functions defined on the interval I with values in X. For $X = L^q(\Omega)$, we frequently shorten the notation to $L^p(L^q)$, if no confusion arises. Now let $f \in L^2(I, L^2(\Omega))$, $p_0 \in L^2(I, L^2(\partial\Omega))$ be given vector fields representing volume forces and external loading.

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We introduce the class of *admissible* stresses σ i.e. functions defined on the interval $I = [0,T]$ with values in the set of symmetric matrices,

$$\sigma : \ [0,T] \times \Omega \longrightarrow \mathbb{R}^{n\times n}_{sym}, \quad \sigma \in L^2(I; L^2(\Omega)),$$

satisfying the balance of forces (in the weak formulation) for almost every $t \in I$

$$(\sigma(t), \nabla\varphi) = (f(t), \varphi) + \int_{\partial\Omega\setminus\Gamma} p_0(t)\varphi\, do \quad \text{for all } \varphi \in H^1_\Gamma(\Omega). \tag{1.1}$$

Here, Γ is either void or a relatively open subset of $\partial\Omega$, and $H^1_\Gamma(\Omega)$ is the Sobolev space containing all functions $\varphi : \ \Omega \to \mathbb{R}^n$, such that $\nabla\varphi \in L^2(\Omega)$, and $\varphi|_\Gamma = 0$ in the sense of traces. Neither $\partial\Omega$ nor Γ need to be connected. Note that all derivatives that arise are to be understood in the distributional sense. The variable t is interpreted as the 'loading' parameter, we use the notation $\dot{g} = \frac{\partial}{\partial t} g$ (for any function g under consideration), and, with some abuse of notation, we refer to it also as time derivative.
In addition, the formulation of the Prandtl-Reuss law involves a yield condition and the compliance tensor A. We confine ourselves to the von Mises yield condition

$$|\sigma_D| \leq \kappa \tag{1.2}$$

where $\kappa > 0$ is the so-called yield constant. Furthermore, the compliance tensor $A = (a^{\nu\mu}_{ik})$, a given symmetric tensor of rank four, must satisfy the usual positivity or *ellipticity condition*

$$B : AB \ \geq \ c_0|B|^2 \ \text{ for all } B \in \mathbb{R}^{n\times n}_{sym} \text{ with some } c_0 > 0,$$

for simplicity, we assume that the entries $a^{\nu\mu}_{ik}$ are constant.
We introduce the convex set $\mathbb{K}(t)$:

$$\mathbb{K}(t) = \{\tau \in L^2(\Omega; \mathbb{R}^{n\times n}_{sym}) \text{ s.t. } |\tau_D(x)| \leq \kappa \text{ a.e. in } x \in \Omega, \text{ and } \tau \text{ satisfies (1.1)}\} \tag{1.3}$$

where σ has to be replaced by τ in (1.1), of course. For the data f and p_0 we assume the following additional regularity properties

$$f, \dot{f} \in \ L^\infty(0,T; L^\infty(\Omega)), \tag{1.4}$$
$$p_0, \dot{p}_0 \in \ L^\infty(0,T; L^\infty(\partial\Omega)), \tag{1.5}$$

i.e. these functions are essentially bounded. In addition we fix an initial value $\sigma_0 \in \mathbb{K}(0)$. Now we consider the following variational inequality (*Prandtl-Reuss law*): Find

$$\sigma \in L^2(0,T; L^2(\Omega, \mathbb{R}^{n\times n}_{sym})) \text{ with } \dot{\sigma} \in L^2(0,T; L^2), \text{ s.t.} \tag{1.6}$$

$$\sigma(t) := \sigma(t,\cdot) \in \mathbb{K}(t) \qquad \text{for a.e. } t \in [0,T], \tag{1.7}$$
$$(A\dot{\sigma}(t), \sigma(t) - \tau) \leq 0 \qquad \text{for all } \tau \in \mathbb{K}(t), \text{ a.e. in } t \in [0,T], \tag{1.8}$$
$$\sigma(0,\cdot) = \sigma_0. \tag{1.9}$$

The inclusion $\dot{\sigma} \in L^2(L^2)$ ensures that $\sigma(0)$ is defined. In fact, there holds the stronger regularity property [6, 16, 2]

$$\dot{\sigma} \in L^\infty(0,T; L^2). \tag{1.10}$$

It is well known, that the Prandtl-Reuss problem has a unique solution, i.e. there there exists an admissible stress function σ satisfying (1.6) – (1.9) [10, 16, 2, 6].
For the reconstruction of strains, and more specifically, the displacement field from the stresses, additional assumptions are needed which are known as safe load condition:

There exists a function $\hat\sigma \in L^2(0,T;L^2(\Omega;\mathbb{R}^{n\times n}_{sym}))$ and a $\delta_0 > 0$ such that

$$\dot{\hat\sigma} \in L^\infty(I, L^\infty(\Omega)),\ \ddot{\hat\sigma} \in L^1(I, L^\infty(\Omega)), \tag{1.11}$$

for all t, $\hat\sigma(t)$ satisfies the balance of forces (1.1) *together with the initial condition $\hat\sigma(0) = \sigma_0$, and*

$$|\hat\sigma_D| \le \kappa - \delta_0 \tag{1.12}$$

almost everywhere.

If the safe load condition is fulfilled a vector field $u \in L^\infty(I, L^{\frac{n}{n-1}}(\Omega,\mathbb{R}^n))$ exists with $\dot u \in L^\infty(I, L^{\frac{n}{n-1}}(\Omega))$ (cf. [16]),

$$\begin{aligned} &\varepsilon(\dot u) := \frac{1}{2}(\nabla \dot u + \nabla \dot u^\top) \ \in \ L^\infty(I, \mathcal{C}(\overline\Omega)^*), \\ &\langle A\dot\sigma - \varepsilon(\dot u), \sigma - \tau\rangle \le 0 \text{ for a.e. } t \in I, \end{aligned} \tag{1.13}$$

where, as usual, $\mathcal{C}(\overline\Omega)$, $\mathcal{C}(\overline\Omega)^*$ stand for the space of functions continuous on the closure of the underlying domain and its dual space, respectively. The inequality (1.13) holds for all $\tau \in \sigma + \mathcal{C}(\overline\Omega;\mathbb{R}^{n\times n}_{sym})$ with $|\tau_D| \le \kappa$. Here the brackets $\langle\ ,\ \rangle$ have to be understood as L^2-scalar product, if both arguments are in $L^2(\Omega)$, and in general in the duality pairing $\langle \mathcal{C}^*, \mathcal{C}\rangle$. However, it is not allowed to split up the sums: since σ is not known to be in $\mathcal{C}(\overline\Omega)$, e.g. the term $\langle \varepsilon(\dot u), \sigma\rangle$ may have no sense and has to be defined in an appropriate way [15, 1].
In the sequel we will not dwell upon these results; however, for describing the physical situation, the relation (1.13) is useful: If there were more smoothness, in particular the quantity $\dot\varepsilon$ an L^1-function (which is not the case), (1.13) would imply

$$A\dot\sigma(t,x) = \varepsilon(\dot u)(t,x) \text{ in points } (t,x) \text{ where } |\sigma_D| < \kappa \text{ in a neighborhood } U(t,x). \tag{1.14}$$

In general we have

$$\varepsilon(\dot u) - A\dot\sigma = \dot\lambda \tag{1.15}$$

with $\dot\lambda \in \mathcal{C}(\overline\Omega, \mathbb{R}^{n\times n}_{sym})^*$ and, if $\dot\lambda$ were smooth enough,

$$\dot\lambda \cdot (\sigma_D - \tau_D) \ge 0 \quad \text{for all } \tau \in \mathbb{K}(t), \tag{1.16}$$

i.e. $\dot\lambda$ is an outer normal to the yield surface $|\tau_D| = \kappa$.
The quantity $\dot u$ can be interpreted as displacement velocitiy and $\varepsilon(\dot u)$ as the strain velocity. Thus, if the stress satisfies $|\sigma_D| < \kappa$ in a neighborhood of a point (t,x), the stress and strain velocity $\dot\sigma$ and $\varepsilon(\dot u)$ satisfy the relations of linear elasticity (differentiated with respect to t). If the boundary of the yield surface is touched, then the additional plastic deformation λ together with the corresponding velocity $\dot\lambda$ appears such that (1.15) and (1.16) hold.
Due to the bad behaviour of u and $\dot u$, not many regularity results for the Prandtl-Reuss law are available. In [2] the so called Norton-Hoff approximation is used to prove that the stresses σ satisfiy

$$\sigma \in L^\infty(0,T;H^1_{\text{loc}}(\Omega)),$$

i.e. the spatial derivatives of σ are locally in L^2. This result´was also shown in [5] by means of a different approximation.

The situation of regularity for Prandtl-Reuss's law is very similar to that of the Hencky model. In [14], Seregin constructed approximations to the Hencky model which indicate that the normal derivatives of σ are NOT in L^2 near the boundary $\partial\Omega$. For special geometric situations, in [7] and [4] it was shown that the tangential derivatives of σ are in L^2 up to the boundary.
One can prove analogous results to [4] as for the Prandtl-Reuss problem. Furthermore, the improved L^p-property $u \in L^{\frac{n}{n-1}+\delta}(\Omega)$ for Hencky's law due to Hardt Kinderlehrer [9] can be done for the Prandtl-Reuss law in the sense that $\dot{u} \in L^\infty(L^{\frac{n}{n-1}+\delta}(\Omega))$ for some $\delta > 0$. Up to now, no regularity results are known for the stress velocities $\dot{\sigma}$.
The purpose of the present paper is to prove that $\dot{\sigma}$ has fractional derivatives in time direction of order $\frac{1}{2} - \delta$, for all $\delta > 0$, up to the boundary. Using the aforementioned H^1_{loc}-regularity result for σ, cross interpolation implies that $\dot{\sigma}$ has local fractional derivatives of order $\frac{1}{3} - \delta$ in spatial direction. An imbedding theorem for anisotropic fractional Sobolev spaces applied in the case $n = 3$ then leads to $\dot{\sigma} \in L^q(L^q)$, with $q = \frac{22}{9} - \delta'$ for all $\delta' > 0$. The classical theory gives $\dot{\sigma} \in L^\infty(L^2)$, see also estimate (2.10) below. Our technique can also be applied to problems with elastic plastic deformation with hardening. In the latter case, interior $H^{\frac{1}{2}-\delta}$-regularity can be achieved also for the fractional derivatives in space direction, but we do not elaborate this here.

2 Penalty approximation

We introduce a common approximation of the Prandtl-Reuss problem via a penalty potential. To this end we define for $\tau \in \mathbb{R}^n_{sym}$ the function

$$\beta_\mu(\tau) = \frac{1}{2\mu}\left[|\tau| - \kappa\right]_+^2, \quad \mu > 0, \tag{2.1}$$

where for any real function $[\xi]_+ := \max\{\xi, 0\}$ is the positive part of ξ, and get

$$\beta'_\mu(\tau) = \frac{\partial}{\partial\tau}\beta_\mu(\tau) = \begin{cases} \mu^{-1}\left[|\tau| - \kappa\right]_+ \tau|\tau|^{-1} & \text{for } |\tau| \neq 0 \\ 0 & \text{for } |\tau| = 0. \end{cases} \tag{2.2}$$

Now we want to find $\sigma = \sigma_\mu \in L^2(0,T; L^2(\Omega, \mathbb{R}^{n\times n}_{sym}))$ and $u_\mu \in L^2(0,T; H^1_\Gamma(\Omega, \mathbb{R}^n))$ such that

$$\dot{\sigma_\mu} \in L^2(L^2), \quad \dot{u}_\mu \in L^2(H^1_\Gamma) \tag{2.3}$$

$$(\sigma_\mu, \nabla\varphi) = (f, \varphi) + \int_{\partial\Omega} p_0\varphi \, do \qquad \text{for all } \varphi \in H^1_\Gamma(\Omega, \mathbb{R}^n), \text{ a.e. in } t \in [0,T] \tag{2.4}$$

$$\varepsilon(\dot{u}_\mu) = A\dot{\sigma_\mu} + \beta'_\mu(\sigma_{\mu D}) \qquad \text{for a.e. } (t,x) \in [0,T] \times \Omega, \tag{2.5}$$

$$\sigma_\mu(0) = \sigma_0 \tag{2.6}$$

Note that the penalty potential ist just $\beta_\mu(\sigma_{\mu D})$. It is well known that the penalized problem (2.3)-(2.6) has a solution cf. the methods worked out in [15]. To prove this (2.5) is turned into the equivalent weak formulation

$$\begin{aligned} &(A\dot{\sigma_\mu}, \tau) + (\beta'_\mu(\sigma_{\mu D}), \tau_D) = 0 \\ &\text{for all } \tau \in L^2(L^2) \text{ with } (\tau, \nabla\varphi) = 0 \text{ for all } \varphi \in H^1_\Gamma(\Omega). \end{aligned} \tag{2.7}$$

Now the problem can be solved using a Rothe-Approximation. Thereby the 'time' -derivative $\dot{\sigma}(t,x)$ is approximated by the difference quotient

$$D^{-h}\sigma(t,x) = \frac{1}{h}\left(\sigma(t,x) - \sigma(t-h,x)\right),$$

considered on a discrete set of time steps $t_k = kh$, $k = 1, ..., N$, $kN = T$. The discretized problem can be solved successively at each 'time' step t_k via a minimization argument. Thereby the existence of an admissible stress has to be assumed, while the safe load condition is not yet needed for this step. The details can be done following the arguments in [2, 13]. Essentially the proof consists in using $\sigma - \hat{\sigma}$ and $D^h(\sigma - \hat{\sigma})$ as testfunctions and then performing the energy-approach. Extending the Rothe solutions by piecewise linear interpolation to σ^h routine energy estimates lead to the following uniform bounds

$$\|\sigma_\mu^h\|_{L^2(L^\infty)} \;\leq\; K_\mu, \quad \|\dot{\sigma}_\mu^h\|_{L^2(L^2)} \;\leq\; K_\mu, \quad h \to 0,$$

here the condition $|\sigma_{0D}| \leq \kappa$ is needed. Since σ_0 satisfies the equation (1.1) for the balance of forces, we get the additional estimate

$$\|\dot{\sigma}_\mu^h\|_{L^\infty(L^2)} \;\leq\; K_\mu.$$

Passing to the limit $h \to 0$ for a subsequence and using monotonicity arguments imply the convergence of the Rothe-approximations to the solution σ_μ of (2.3), (2.4), (2.6), (2.7) together with the estimates

$$\|\sigma_\mu\|_{L^\infty(L^2)} \;\leq\; K, \;\mu \to 0, \quad \|\dot{\sigma}_\mu\|_{L^\infty(L^2)} \;\leq\; K_\mu, \tag{2.8}$$

A decomposition argument involving Korn's inequality gives the existence of $v_\mu(t) \in H^1_\Gamma(\Omega, \mathbb{R}^n)$ such that

$$\varepsilon(v_\mu) = \frac{1}{2}\left(\nabla v_\mu + \nabla v_\mu^\top\right) = A\dot{\sigma}_\mu + \beta'_\mu(\sigma_{\mu D}) \qquad \text{a.e. in } (t,x).$$

Setting $v_\mu = \dot{u}_\mu$ establishes equation (2.5). With the additional assumption of the safe load condition there holds the uniform estimate

$$\|\beta'_\mu(\sigma_{\mu D})\|_{L^\infty(L^1)} + \|\varepsilon(\dot{u}_\mu)\|_{L^\infty(L^1)} \;\leq\; K, \quad \mu \to 0. \tag{2.9}$$

and we may conclude

$$\|\dot{\sigma}_\mu\|_{L^\infty(L^2)} \;\leq\; K, \mu \to 0. \tag{2.10}$$

Finally, any weak $L^2(L^2)$ - limit (up to the choice of subsequences) $\sigma = \lim\limits_{\mu\to 0} \sigma_\mu$ satisfies (1.7)-(1.10). The variational inequality (1.8) follows from (2.4), passing to the limit and the monotonicity of β_μ.
Unfortunately, the estimate (2.9) cannot be improved substantially. Thus (after passing to subsequences), the strain velocities $\varepsilon(\dot{u}_\mu)$ weakly converge in the space $\mathcal{C}^*$ only as $\mu \to 0$. By Temam's imbedding theorem, (2.9) implies

$$\|\dot{u}\|_{L^\infty(L^{n/(n-1)})} \;\leq\; K, \quad \mu \to 0 \tag{2.11}$$

uniformly. As already mentioned, using techniques based on reverse Hölder inequality, (2.11) can be improved with $L^\infty(L^{n/(n-1)})$ replaced by $L^\infty(L^q)$, $q > \frac{n}{n-1}$, for some q, but we shall not need this.

Other penalty approximations give similar uniform estimates. One possibility is the Norton Hoff approximation where $\beta_\mu(\sigma_D)$ is replaced by $\frac{1}{p}\frac{1}{\kappa^p}|\sigma_D|^p$ with $p \to \infty$ (see [16, 2]). The penalization with $\beta_\mu(\sigma_D)$ as in the present paper has the advantage that $\beta'_\mu(\sigma_D)$ grows linearly in σ_D, and then some steps in the proof are simpler, for example the existence of u with (2.5) is derived in an L^2-setting while for the Norton-Hoff approximation L^p-theory has to be used. The advantage of the Norton Hoff model is, that we immediately have an L^{p_0}-estimate for σ_D (thereafter, for σ), p_0 arbitrarily large, uniformly in $p \geq p_0$. This is important for the proof of local $L^\infty(H^1_{\text{loc}})$-regularity of σ, in the limit $\mu \to 0$ or $p \to \infty$, cf [2, 5].
If one uses the penalty approximation (2.3) - (2.6) with $\beta'_\mu(\sigma_D)$ up to now the proof of the $L^\infty(H^1_{\text{loc}})$-regularity of σ works only in dimensions $n = 2, 3, 4$, while the approach via the Norton-Hoff approximation works in arbitrary dimension.
In the present paper, we use $\beta_\mu(\sigma_D)$ for the approximation (2.5) since it is convenient, however, for fractional differentiability of $\dot\sigma$ in space direction we need $\sigma \in L^\infty(H^1_{\text{loc}})$. Because of the uniqueness of the stresses in Prandtl-Reuss problem fortunately it does not matter by which approximation the differentiability of σ is obtained. We do not need a uniform bound of σ_μ in $L^\infty(H^1_{\text{loc}})$ as $\mu \to 0$.

3 An asymptotic property of the penalty potential and its derivative

For optimization problems and their penalization it is a rather common and simply proved fact, that the penalty term tends to zero provided the admissible set is not empty. We need the analogue statement also for the Prandtl-Reuss problem and its penalization, but since we are not dealing with an optimization problem here, so we have to prove it.

Lemma 1 *Let σ_μ be the solution of* (2.3)-(2.6) *and assume $f \in L^\infty(L^2)$, $p_0 \in L^\infty(L^2(\partial\Omega))$ and $\sigma_0 \in \mathbb{K}(0)$ (see* (1.3)*). Then*

$$\int_0^T \int_\Omega \beta_\mu(\sigma_{\mu D}) dx\, dt = \frac{1}{2\mu}\int_0^T \int_\Omega [|\sigma_{\mu D}| - \kappa]_+^2 \; dx\, dt \to 0 \quad \textit{as } \mu \to 0.$$

Proof. We may assume that σ_μ converges to σ weakly in $L^2(L^2)$, and the uniform estimates (2.8) are valid. We obtain from (2.7)

$$\int_0^T \left\{ (A\dot\sigma_\mu, \sigma_\mu - \sigma) + \frac{1}{\mu}\left([|\sigma_{\mu D}| - \kappa]_+ \sigma_{\mu D}|\sigma_{\mu D}|^{-1}, \sigma_{\mu D} - \sigma_D \right) \right\} dt = 0.$$

Since $|\sigma_D| \leq \kappa$, we estimate

$$\begin{aligned} [|\sigma_{\mu D}| - \kappa]_+ \frac{\sigma_{\mu D}}{|\sigma_{\mu D}|^{-1}} : (\sigma_{\mu D} - \sigma_D) &= [|\sigma_{\mu D}| - \kappa]_+ \left(|\sigma_{\mu D}| - \frac{\sigma_{\mu D}}{|\sigma_{\mu D}|^{-1}} : \sigma_D \right) \\ &\geq [|\sigma_{\mu D}| - \kappa]_+ (|\sigma_{\mu D}| - \kappa)\,. \end{aligned}$$

Hence

$$\frac{1}{2}\int_0^T \frac{d}{dt}\left(A(\sigma_\mu - \sigma), \sigma_\mu - \sigma\right) dt + \frac{1}{\mu}\int_0^T \int_\Omega [|\sigma_{\mu D}| - \kappa]_+^2 \;\leq\; -\int_0^T (A\dot\sigma, \sigma_\mu - \sigma)\; dt. \quad (3.1)$$

The right hand side of (3.1) tends to zero due to the weak convergence of σ_μ, the left hand side consists of two definite terms, since $\sigma_\mu(0) = \sigma_0$. and the Lemma is proved. $\square$

The next considerations are crucial for the proof of the main result. Recalling the definitions (2.1) and (2.2) for β_μ and β'_μ, the convexity of β_μ implies

$$\begin{aligned} \beta_\mu(\sigma_{\mu D}(t+h,\cdot)) - \beta_\mu(\sigma_\mu(t,\cdot)) &\geq \beta'_\mu(\sigma_{\mu D}(t,\cdot)) : (\sigma_{\mu D}(t+h,\cdot) - \sigma_\mu(t,\cdot)) \\ &= \beta'_\mu(\sigma_{\mu D}(t,\cdot)) : \Delta_h \sigma_\mu(t,\cdot) \end{aligned} \tag{3.2}$$

where $\Delta_h \sigma_\mu(t,x) = \sigma_\mu(t+h,x) - \sigma_\mu(t,x)$. Now we deal with the quantity

$$\mathcal{T} := \frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} (\beta'_\mu(\sigma_{\mu D}), \Delta_s \dot{\sigma_\mu}) dt\, ds. \tag{3.3}$$

We decompose $\mathcal{T} = \mathcal{T}_1 - \mathcal{T}_2$ where

$$\begin{aligned} \mathcal{T}_1 &= \frac{1}{h^2} \int_{t_1}^{t_2-h} \left(\beta'_\mu(\sigma_{\mu D}), \int_0^h \dot{\sigma_\mu}(t+s,\cdot)\; ds \right)\; dt, \\ \mathcal{T}_2 &= \frac{1}{h} \int_{t_1}^{t_2-h} \left(\beta'_\mu(\sigma_{\mu D}), \dot{\sigma_\mu} \right)\; dt = \frac{1}{h} \int_\Omega \beta_\mu(\sigma_{\mu D})\; dx \Big|_{t_1}^{t_2-h}. \end{aligned}$$

Due to Lemma 1,

$$\mathcal{T}_2 \to 0 \;\text{ as } \mu \to 0,\; \text{ for fixed } h > 0,\; \text{ and a. e. } t_1 < t_2 < T - h. \tag{3.4}$$

Next we rewrite $\mathcal{T}_1$, for $t_1 < t_2 - h$, $t_2 < T - h$ we use (3.2) to estimate

$$\begin{aligned} \mathcal{T}_1 &= \frac{1}{h^2} \int_{t_1}^{t_2-h} \left(\beta'_\mu(\sigma_{\mu D}), \Delta_h \sigma_\mu \right) dt \leq \frac{1}{h^2} \int_{t_1}^{t_2-h} \int_\Omega \Delta_h \beta_\mu(\sigma_{\mu D}) dx\, dt \\ &= \frac{1}{h^2} \int_{t_1+h}^{t_2} \int_\Omega \beta_\mu(\sigma_{\mu D}) dx\, dt - \frac{1}{h^2} \int_{t_1}^{t_2-h} \int_\Omega \beta_\mu(\sigma_{\mu D}) dx\, dt. \end{aligned}$$

Hence, in view of lemma 1, we have for fixed $h > 0$

$$\limsup_{\mu \to 0} \mathcal{T}_1 \leq 0 \qquad \text{a.e. with respect to } t_1, t_2, \quad t_1 \leq t_2 - h.$$

Together with (3.3) this gives the following result:

Lemma 2 *For almost all $t_1, t_2 \in [0,T]$ with $t_1 < t_2 - h$ there holds the inequality:*

$$\limsup_{\mu \to 0} \left\{ \frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} \left(\beta'_\mu(\sigma_{\mu D}), \Delta_s \dot{\sigma_\mu} \right)\; dt\; ds \right\} \;\leq\; 0.$$

4 Fractional time-differentiability of the stress velocity

Recalling the notation for the difference in 'time' direction $\Delta_s w(t,x) = w(t+s,x) - w(t,x)$. we now formulate our main result.

Theorem 1 *Let σ be the solution of the Prandtl-Reuss-law* (1.6) – (1.9)*, where the data f, p_0 satisfy the regularity conditions* (1.4) *and* (1.5)*, respectively. Assume further the existence of a safe load $\widehat{\sigma}$ such that* (1.12) *together with the regularity assumptions* (1.11) *hold. Then*

$$\sup_{0<h<T} \frac{1}{h^2} \int_0^h \int_0^{T-h} \int_\Omega |\Delta_s \dot{\sigma}|^2 dx\, dt\, ds < \infty. \tag{4.5}$$

Remark 1 This estimate has not to be confused with the Nikolski-space inclusion

$$\sup_{0<h<T} \frac{1}{h} \int_0^{T-h} \int_\Omega |\Delta_h \dot{\sigma}|^2 \; dx\, dt < \infty \tag{4.6}$$

which states that $\dot{\sigma}$ has the fractional derivative of order $\frac{1}{2}$ in t-direction in the sense of Nikolski-spaces.

Our result is slightly weaker. However, if we define the periodic extension $\tilde{\sigma}$ of σ by

$$\tilde{\sigma}(t,\cdot) = \begin{cases} \sigma(t,\cdot), & t \in [0,T] \\ \sigma(-t,\cdot), & t \in [-T,0] \end{cases}, \quad \tilde{\sigma}(t+2kT,\cdot) = \tilde{\sigma}(t,\cdot) \text{ for } k \in \mathbb{Z},$$

then

$$\tilde{\sigma}(t,x) = \sum_{m=-\infty}^{\infty} c_m(x) \exp(im\pi/T), \tag{4.7}$$

and by simple Fourier analysis, we obtain the following conclusion, which follows from Theorem 1 and Lemma A.1 in the appendix. Note that for differentiability up to order 3/2 in t this extension is acceptable though not for higher order derivatives.

Corollary 1 *Under the hypotheses of Theorem 1 there exists for all small $\delta > 0$ a bound K_δ, depending on the length T of the "time interval" with*

$$\sum_{m=-\infty}^{\infty} \int_\Omega |m|^{3-\delta} |c_m(x)|^2 \; dx \;\leq K_\delta. \tag{4.8}$$

Thus (4.5) is almost equivalent to (4.6). Function spaces with derivatives defined via Fourier transformation are sometimes called Liouville spaces. Hence Corollary 1 states that the stress velocity $\dot{\sigma}$ has fractional derivatives with respect to t of order $\frac{1}{2} - \delta'$ for any $\delta' > 0$ in the sense of Liouville spaces.

Proof of Theorem 1. We fix $h > 0$, $h < T$ and choose $\Delta_s \dot{\sigma_\mu} = \dot{\sigma_\mu}(\cdot + s, \cdot) - \dot{\sigma_\mu}$ as test function in the penalty equation (2.5), integrate the variable s from 0 to h and the variable t from t_1 to $t_2 - h$, $0 \leq t_1 \leq t_2 \leq T$, $t_2 \geq t_1 + h$. Recalling that $\varepsilon(\dot{u}_\mu) : \tau = \nabla \dot{u}_\mu : \tau$, if τ is symmetric, this yields

$$\begin{aligned} \mathcal{L}_\mu := \frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} (\nabla \dot{u}_\mu, \Delta_s \dot{\sigma}_\mu) \; dt\, ds \; = \\ = \underbrace{\frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} (A \dot{\sigma}_\mu, \Delta_s \dot{\sigma}_\mu) \; dt\, ds}_{=: \mathcal{R}_{\mu 1}} + \underbrace{\frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} \big(\beta'_\mu(\sigma_{\mu D}), \Delta_s \dot{\sigma}_\mu\big) \; dt\, ds}_{=: \mathcal{R}_{\mu 2}} \end{aligned} \tag{4.9}$$

We analyze $\mathcal{L}_\mu$ using the safe load condition:

$$\begin{aligned}\mathcal{L}_\mu =&\frac{1}{h^2}\int_0^h\int_{t_1}^{t_2-h}\left(\nabla \dot u_\mu, \Delta_s\left(\dot\sigma_\mu-\dot{\widehat{\sigma}}\right)\right)\,dt\,ds\\ &+\frac{1}{2}\frac{1}{h^2}\int_0^h\int_{t_1}^{t_2-h}\left(\nabla \dot u_\mu+\nabla \dot u_\mu^\top, \Delta_s\dot{\widehat{\sigma}}\right)\,dt\,ds\end{aligned}$$

The first term vanishes since $\dot\sigma_\mu$ and $\dot{\widehat{\sigma}}$ satisfy the equation for balance of forces, differentiated with respect to t, we used the symmetry of $\widehat{\sigma}$ in order to resume the term $\frac{1}{2}\left(\nabla \dot u_\mu+\nabla \dot u_\mu^\top\right)$ in the last equality. Thus

$$|\mathcal{L}_\mu| \;\le\; \frac{1}{h^2}\int_0^h\left\|\nabla \dot u_\mu+\nabla \dot u_\mu^\top\right\|_{L^\infty(L^1)}\left\|\Delta_s\dot{\widehat{\sigma}}\right\|_{L^1(L^\infty)}ds$$

Due to the safe load condition we may use the estimate (2.9) which means that $\|\nabla \dot u_\mu+\nabla \dot u_\mu^\top\|_{L^\infty(L^1)}\le K$ uniformly as $\mu\to 0$. Furthermore we observe

$$\frac{1}{h^2}\int_0^h\|\Delta_s\dot{\widehat{\sigma}}\|_{L^1(L^\infty)}ds\le\frac{1}{h^2}\int_0^h s\|D^s\dot{\widehat{\sigma}}\|_{L^1(L^\infty)}\;ds\;\le\;\|\ddot{\widehat{\sigma}}\|_{L^1(L^\infty)}$$

The latter quantity is bounded according to the assumption (1.11) on the safe load stress. Hence

$$|\mathcal{L}_\mu|\;\le\;K\qquad\text{uniformly as }\mu\to 0\tag{4.10}$$

On the other hand, we have $\mathcal{L}_\mu=\mathcal{R}_{\mu 1}+\mathcal{R}_{\mu 2}$ where $\limsup_{\mu\to 0}\mathcal{R}_{\mu 2}\le 0$ due to Lemma 2. Thus by (4.10) and (4.9) we get

$$\liminf_{\mu\to 0}\mathcal{R}_{\mu 1}=\liminf_{\mu\to 0}(\mathcal{L}_\mu-\mathcal{R}_{2\mu}\ge K_1\tag{4.11}$$

We rewrite

$$A\dot\sigma_\mu:\Delta_s\dot\sigma_\mu=-\frac{1}{2}\Delta_s\dot\sigma_\mu:A\Delta_s\dot\sigma_\mu+\frac{1}{2}\Delta_s\left(\dot\sigma_\mu:A\dot\sigma_\mu\right),$$

which implies

$$\begin{aligned}&\frac{1}{2h^2}\int_0^h\int_{t_1}^{t_2-h}\int_\Omega\Delta_s\dot\sigma_\mu:A\Delta_s\dot\sigma_\mu dx\,dt\,ds=\\ &\quad-\frac{1}{h^2}\int_0^s\int_{t_1}^{t_2-h}\int_\Omega A\dot\sigma_\mu:\Delta_s\dot\sigma_\mu dx\,dt\,ds+\\ &\quad+\frac{1}{2h^2}\int_0^h\int_{t_1}^{t_2-h}\int_\Omega\Delta_s\left(\dot\sigma_\mu:A\dot\sigma_\mu\right)dx\,dt\,ds=:-\mathcal{R}_{\mu 1}+\mathcal{R}_{\mu 3}\end{aligned}$$

Inspecting the last term we find

$$\mathcal{R}_{\mu 3}=\frac{1}{2h^2}\int_0^h\left\{\int_{t_2-h}^{t_2-h+s}\int_\Omega\dot\sigma_\mu:A\dot\sigma_\mu dx\,dt-\int_{t_1}^{t_1+s}\int_\Omega\dot\sigma_\mu:A\dot\sigma_\mu\,dx\,dt\right\}ds\tag{4.12}$$

and with $\|\dot{\sigma}_\mu\|_{L^\infty(L^2)} \leq K_2$ and $|s| \leq h$ we arrive at the following estimate, which holds uniformly as $\mu \to 0$

$$|\mathcal{R}_3| \leq \frac{1}{2h^2} \int_0^h s|A| \|\dot{\sigma}_\mu\|^2_{L^\infty(L^2)} ds \leq K \|\dot{\sigma}_\mu\|^2_{L^\infty(L^2)} \leq K_3 \tag{4.13}$$

Thus we derive from (4.12) and (4.11) that

$$\limsup_{\mu\to 0} h^{-2} \int_0^h \int_{t_1}^{t_2-h} (\Delta_s \dot{\sigma}_\mu, A\Delta_s \dot{\sigma}_\mu) \, dt \, ds \ \leq \ \limsup \mu \to 0(-\mathcal{R}_{\mu 1} + \mathcal{R}_{mu3}) \leq K_1 + K_3.$$

and we obtain from the lower semi-continuity of positively definite quadratic forms that

$$\frac{1}{h^2} \int_0^h \int_{t_1}^{t_2-h} (\Delta_s \dot{\sigma}, A\Delta_s \dot{\sigma}) \, dt \, ds \ \leq \ K \tag{4.14}$$

for the weak $L^2(L^2)$-limit $\dot{\sigma} = \lim_{\mu\to 0} \dot{\sigma}_\mu$, a.e. with respect to t_1, t_2, $0 \leq t_1 < t_2 - h \leq T$. K does not depend on t_1, t_2. From the absolute continuity of Lebesgue's integral we conclude from (4.14)

$$\frac{1}{h^2} \int_0^h \int_0^{T-h} (\Delta_s \dot{\sigma}, A\Delta_s \dot{\sigma}) \, dt \, ds \ \leq \ K$$

which implies Theorem 1.

Remark 2 If the requirements of Theorem 1 are met the solution σ of the Prandtl-Reuss problem satisfies $\sigma \in L^\infty(0,T; H^1_{loc}(\Omega))$ (see [2, 5]). We can combine this result with our main theorem to gain that $\dot{\sigma}$ has local spatial derivatives of order $1/3 - \delta$ for any positive δ. To be more precise, let $Q \subset [0,T] \times \Omega$ be a closed cube with edge length R. We extend each component σ_{ij} by symmetric reflection into a periodic function S_{ij} defined on a cube $\hat{Q} \supset Q$, where $\hat{Q}$ has edge length $2R$ and center (t_0, x_0), then (we omit the subscript $_{ij}$ for simplicity)

$$S \in L^\infty_{per}(0,T; H^1_{per}(\hat{Q})), \tag{4.15}$$

and we have the Fourier expansion

$$S = \sum_m c_m \exp(im'(x - x_0)) \exp(im_0(t - t_0)),$$

where the summation is taken over all multi-indices $(m_0, m') \in \mathbb{Z}^{n+1}$. From (4.15) we have

$$\sum_m |m'|^2 |c_m|^2 < \infty,$$

while Theorem 1 and Lemma A.1 from the appendix give

$$\sum_m |m_0|^{3-\delta} |c_m|^2 < \infty.$$

Now Hölder's inequality implies

$$\sum_m |m'|^{\frac{2}{3}-\delta'} |m_0|^2 |c_m|^2 < \infty.$$

In other words, if $b_k(t)$ are the Fourier coefficients of $\dot\sigma$ belonging to the expansion in spatial direction, we get

$$\sum_{k\in\mathbb{Z}^n} |k|^{\frac{2}{3}-\delta'} \int |b_k|^2 dt < \infty.$$

For $n = 3$, e.g., embedding theorems (e.g. [12, p. 390]) lead to

$$\dot\sigma \in L^q(L^q),\ q = \frac{4\cdot 2}{4-2r},\ r = \left[\frac{1}{4}\left(3\cdot(\frac{1}{3})^{-1} + (\frac{1}{2}-\delta)^{-1}\right)\right]^{-1} = (\frac{11}{4}+\delta')^{-1} = \frac{4}{11}-\delta$$

Note that we only used the fact that $\sigma \in L^2(H^1_{loc})$, not $\sigma \in L^\infty(H^1_{loc})$. The latter slightly stronger fact could be used to establish an additional Morrey condition for the spatial derivatives of $\dot\sigma$ of order $\frac{1}{3}-\delta$. In some cases it can be shown that the tangential derivatives (near the boundary) of σ are in L^2 up to the boundary. In [7, 4] this was shown for Hencky's law, but the proof works also for the Prandtl-Reuss law. Thus one obtains the existence of fractional tangential derivatives for $\dot\sigma$ of order $\frac{1}{3}-\delta$ for positive δ. Furthermore, since the existence of spatial fractional derivatives of order $\frac{1}{2}-\delta$ for σ are known to exist up to the boundary in case of dimension 2 (see the proof for Hencky's model in [11]), we obtain in a similar manner that the fractional derivatives of order $\frac{1}{6}-\delta$ for $\dot\sigma$ exist in L^2 up to the boundary.

Appendix. Quasi-equivalence of norms describing fractional derivatives

As mentioned above, we work out the correspondence between the quantity defined in (4.5) and fractional time derivatives defined by means of Fourier transforms. Recall that any $S \in L^2_{per}([-T,T],L^2)$ can be expanded into a Fourier series

$$S(t,x) = \sum_{m=-\infty}^{\infty} c_m(x)\exp\frac{im\pi t}{T}, \quad c_m \in L^2(\Omega,\mathbb{C}), \quad \sum_{m=-\infty}^{\infty}\int_\Omega |c_m|^2 dx < \infty. \tag{A.1}$$

If in addition $\dot S \in L^2_{per}([-T,T],L^2)$, then for a.e. (t,x),

$$\dot S(t,x) = \sum_{m=-\infty}^{\infty} \frac{im\pi}{T} c_m(x)\exp\frac{im\pi t}{T}, \text{ and } \sum_{m=-\infty}^{\infty} m^2 \int_\Omega |c_m|^2 dx < \infty. \tag{A.2}$$

Lemma A.1 *Let $S, \dot S \in L^2_{per}([-T,T],L^2)$, and assume that*

$$I(S)^2 := \sup_{0<h<T} \frac{1}{h^2}\int_0^h \int_{-T}^T \int_\Omega |\Delta_s \dot S|^2\,dx\,dt\,ds < \infty. \tag{A.3}$$

Then, for any $\delta > 0$,

$$\sum_{m=-\infty}^{\infty} |m|^3 |\ln(1+|m|)|^{-1-\delta} \int_\Omega |c_m|^2 dx \,\le\, C_\delta I(S)^2.$$

In particular, this implies that S has fractional derivatives with respect to t of order $\frac{3}{2}-\delta'$, if the fractional derivatives of a function are defined via the Fourier series.

Proof. The relation (A.2) together with (A.3) implies

$$\frac{1}{h^2}\sum_{m=-\infty}^{\infty} m^2 \int_0^h |\exp\frac{im\pi s}{T} - 1|^2 ds \int_\Omega |c_m(x)|^2 dx \le \frac{T^2}{\pi^2} I(S)^2. \tag{A.4}$$

With

$$\int_0^h |\exp\frac{im\pi s}{T} - 1|^2 ds = 2\left(h - \frac{T}{m\pi}\sin(\frac{m\pi h}{T})\right)$$

we deduce from (A.4) for all $0 < h < T$:

$$\frac{1}{h}\sum_{m=-\infty,\, m\neq 0}^{\infty} 2m^2\left(1 - \frac{T}{mh\pi}\sin(\frac{m\pi h}{T})\right)\int_\Omega |c_m|^2 dx \le \frac{T^2}{\pi^2} I(S)^2. \tag{A.5}$$

Now put $h_j = 2^{-j}$, and $M_j = \{m \in \mathbb{Z} \,|\, \frac{1}{8}2^j T \le |m| \le \frac{1}{4}2^j T\}$, which basically means $|m| \sim h_j^{-1} = 2^j$ for $m \in M_j$. Then, for $m \in M_j$, we have

$$2\left(1 - \frac{T}{mh_j\pi}\sin(\frac{m\pi h_j}{T})\right) \ge c_0 > 0 \tag{A.6}$$

with some universal constant $c_0 = c_0(\sin)$. We choose $h = h_j$ in (A.3), multiply with $\frac{1}{j^{1+\delta}}$ and sum with respect to $j \ge j_0$, where j_0 is the minimal exponent fulfilling the condition $2^{-j_0} \le T$. This implies

$$\sum_{j\ge j_0} 2^j j^{-1-\delta} \sum_{m=-\infty,\, m\neq 0}^{\infty}\left(1 - \frac{T}{mh_j\pi}\sin(\frac{m\pi h_j}{T})\right)\int_\Omega m^2|c_m|^2 dx \le \frac{T^2 I(S)^2}{\pi^2}\sum_{j\ge j_0} j^{-1-\delta} =: K' \tag{A.7}$$

Using (A.6), the last inequality leads to the estimate

$$\begin{aligned} &c_0 \sum_{j\ge j_0} 2^j j^{-1-\delta} \sum_{m\in M_j} \int_\Omega m^2 |c_m|^2 dx \\ &\le \sum_{j\ge j_0} 2^j j^{-1-\delta} \sum_{m\in M_j,\, m\neq 0}\left(1 - \frac{T}{mh_j\pi}\sin(\frac{m\pi h_j}{T})\right)\int_\Omega m^2|c_m|^2 dx \le K'. \end{aligned}$$

Since $|m| \sim 2^j$ for $m \in M_j$, it follows $|m|\,(\log(1+|m|))^{-1-\delta} \le K 2^j j^{-1-\delta}$. Hence we conclude

$$\sum_{j\ge j_0}\sum_{m\in M_j} |m|^3 \,(\log(1+|m|))^{-1-\delta}\int_\Omega |c_m|^2 dx \le C I(S)^2.$$

Finally we have to observe that the union $\bigcup_{j=j^0}^\infty M_j$ contains all $m \in \mathbb{Z}$ except those m with $|m| < T2^{j_0-3}$. But for this finite number of m's we find a constant C such that $|m|\ln(1+|m|)^{-1-\delta} \le C(1 - \frac{T}{mh_0\pi}\sin(\frac{m\pi h_0}{T})$, where $h_0 = 2^{-j_0}$, and then use (A.5), which finishes the proof of the lemma. □

Under the hypothesis of Theorem 1 we know for the solution σ of the Prandtl-Reuss-problem that for fixed positive $h_0 < T$,

$$|\sigma|_{\frac{3}{2},[0,T]} = \left(\sup_{0<h<h_o} \frac{1}{h^2}\int_0^h \int_0^{T-h_0} \int_\Omega |\Delta_s \dot{S}|^2 \, dx\, dt\, ds\right)^{\frac{1}{2}} < \infty \tag{A.8}$$

Now S be the even extension (with respect to t) of the function σ to the interval $[-T,T]$ and denote the periodic extension from $[-T,T]$ again by S Since σ, $\dot{\sigma} \in L^\infty([0,T],L^2)$, we have $S, \dot{S} \in L^\infty_{per}([-T,T],L^2(\Omega))$. In addition, for $I(S)$ defined as in (A.3), we get

$$I(S) \le C(|\sigma|_{\frac{3}{2},[0,T]} + \|\dot{\sigma}\|_{L^\infty(L^2)}). \tag{A.9}$$

In fact, since $\dot{S} = -\dot{\sigma}$ a.e. in $[-T,0]$ and $\dot{S} = \dot{\sigma}$ a.e. in $[0,T]$, and since (A.8) holds, (A.9) is proved once the 'transition integrals'

$$J = \frac{1}{h^2}\int_0^h \int_{E_s} \int_\Omega |\Delta_s \dot{S}|^2 dx\, dt\, ds, \quad E_s = \{t < 0 \,|\, t+s \ge 0\}, \text{ or } E_s = \{t < T \,|\, t+s \ge T\}$$

are bounded uniformly with respect to $h \in (0,h_0]$, which is obvious since $\Delta_s \dot{S} \in L^\infty(L^2)$ and E_s is of length at most h since $s \in (0,h)$.

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