

## **Rainbow over Paris**

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# RAINBOW OVER PARIS

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**ABSTRACT.** Rainbow barrier options are a well-known form of barrier options where the option is written on one underlying asset while the knock-out (or knock-in) barrier is linked to a second asset. In the present paper we extend the existing literature on Parisian options to price Parisian rainbow barrier options.

**Keywords:** Exotic Options, Parisian Options, Rainbow Barrier Options

## 1. INTRODUCTION

In an outside barrier option, also known as two-asset barrier option, contingent option or rainbow barrier option, one of the underlying assets,  $S_1$ , determines how much the option is in- or out-of-the-money and the other asset,  $S_2$ , is linked to the barrier trigger. Originally, according to Heynen and Kat (1994), these options were designed by Bankers Trust International in 1993 as a call option on a basket of Belgian stocks which was knocked-out when the Belgian Franc appreciated by more than 3.5%. First, Heynen and Kat (1994), and subsequently Zhang (1995a) and Carr (1996), have developed closed-form solutions in terms of the cumulative bivariate normal distribution function. In analogy to Carr (1996), another example for a useful application for these options is for a US oil producer which is considering to extend its production into Canada. The exploration of a new oil field necessitates an initial investment in Canadian dollars. Hence, the producer is exposed to the risk of a rise of the Canadian dollar against the US dollar. To be hedged against such a rise, the oil producer can either buy Canadian dollars forward or go long a call option on Canadian dollars, i.e. purchase a fixed amount of Canadian dollars at a fixed exchange rate with respect to the US dollar. Since the US producer is not yet sure that it will expand its exploration operations, it decides that the flexibility of the option outweighs its higher up-front premium payment. However, to reduce the magnitude of this premium, the producer might be interested in alternatively buying an outside barrier option which

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knocks out if the oil price rises. The rationale is that if oil prices rise, the producer's enhanced revenues, both current and potential future, will be more than the compensation for the lost option to fix Canadian dollar costs.

One big disadvantage of standard barrier options and hence of rainbow barrier options is that the barrier trigger only depends on a single touching of the barrier by the underlying price process. This exposes the holder and the writer of these options to the risk that the counterparty may manipulate the underlying for a short time period such that the payoff of the barrier option is manipulated in a for the counterparty favorable way, i.e. the barrier event is triggered by a price fluctuation intentionally caused by the counterparty. In order to hinder such manipulative behavior, barrier options have been invented where the trigger event does not depend on a single hit of the barrier but on the time period spent beyond the barrier. Thus, the longer this time period is, the more costly a manipulation would be. Various forms of these contracts have appeared in the literature. Chesney et al. (1997) were the first to price standard Parisian barrier options and more recently these options and their extensions with exotic payoffs have regained attention in the literature, see e.g. Haber et al. (1999), Bernard et al. (2005), Bernard and Boyle (2010), Chen and Suchanecski (2011), Anderluh (2009), Anderluh and van der Weide (2009), Chesney and Gauthier (2006), Dassios and Wu (2008a, 2008b, 2008c), Labart and Lelong (2009a, 2009b) and the books by Detemple (2006) and Jeanblanc et al. (2009).

The aim of the present paper is to merge the above mentioned types of options in order to develop a new type of option which we call Parisian rainbow barrier options. These options knock out or knock in if the value of the second asset remains below or above the barrier for a certain period of time. Our valuation formulas encompass all of the existing valuation formulas for rainbow barrier options and Parisian options as special cases.

The remainder of the paper is organized as follows. In Section 2, we briefly review the valuation formulas of rainbow barrier options and we then derive the valuation formulas of Parisian rainbow options for standard Parisian barrier triggers in Section 3. In Section 4, we present some numerical analyses, especially with regard to the price impact of the correlation between the two assets in connection with the Parisian barrier features. After outlining a number of useful applications for Parisian rainbow barrier options in Section 5, the results of this paper are summarized in the conclusion.

## 2. MODEL FRAMEWORK AND VALUATION OF RAINBOW BARRIER OPTIONS

In order to value Parisian rainbow barrier options, we place ourselves in the framework of Black and Scholes (1973) and Merton (1973). We assume that there are two assets which follow geometric Brownian motions under the equivalent martingale measure  $P$

Type	Payoff	Condition
rainbow down-and-in call	$(S_1(T) - K)^+$	$\inf_{t \in [0, T]} S_2(t) \leq H$
rainbow up-and-out put	$(K - S_1(T))^+$	$\sup_{t \in [0, T]} S_2(t) < H$

TABLE 1. Payoff structure and barrier condition for a rainbow down-and-in call and a rainbow up-and-out put option.

which makes the discounted asset price processes be martingales:

$$\begin{aligned}
dS_1(t) &= (r - q_1)S_1(t)dt + \sigma_1 S_1(t) \left( \sqrt{1 - \rho^2} dW_1(t) + \rho dW_2(t) \right), \quad S_1(0) = S_1^0, \\
dS_2(t) &= (r - q_2)S_2(t) dt + \sigma_2 S_2(t) dW_2(t), \quad S_2(0) = S_2^0,
\end{aligned} \tag{1}$$

where  $W_1(t)$  and  $W_2(t)$  are two independent Brownian motions under the risk-neutral martingale measure  $P$ ,  $\rho \in [-1, +1]$  indicates the correlation between these two assets,  $S_1^0, S_2^0$  are today's asset values, i.e. positive constants, and as usual  $r, q_1, q_2, \sigma_1$  and  $\sigma_2$  denote the constant risk-free interest rate, constant dividend yields and volatilities for assets  $S_1$  and  $S_2$ , respectively.

In contrast to standard barrier options (also known as inside barrier options) where knock-in or knock-out is controlled by the behavior of the asset price underlying the option contract, rainbow barrier options (also known as outside barrier option) are options where a second asset determines whether the option is knocked in or knocked out, respectively. Here, we assume that  $S_1$  is the underlying asset determining the payoff of the option contract and that  $S_2$  is the asset which triggers the knock-in or knock-out event. For illustration, the payoff structures of a rainbow knock-in call and a rainbow knock-out put option are given in Table 1. Here,  $H$  is used to denote the barrier level. Going forward, for simplicity, we assume that the barrier level  $H$  is constant. The extension to exponential barriers and other special cases can be accommodated easily. The payoff structures of the other types of rainbow barrier options can be described similarly.

The pricing formulas for rainbow barrier knock-out call or put options with constant barrier  $H$  can be expressed in terms of the cumulative bivariate normal distribution function with correlation coefficient  $\rho$  as follows (see Heynen and Kat (1994), Zhang (1995a), Carr (1996) or Haug (2007)):

$$RBO = \eta S_1^0 e^{-q_1 T} \left[ M(\eta d_1, \phi e_1; -\eta \phi \rho) - e^{\frac{2(r - q_2 - \frac{1}{2}\sigma_2^2 + \rho\sigma_1\sigma_2)\ln(H/S_2^0)}{\sigma_2^2}} M(\eta d_3, \phi e_3; -\eta \phi \rho) \right] \\ - \eta K e^{-rT} \left[ M(\eta d_2, \phi e_2; -\eta \phi \rho) - e^{\frac{2(r - q_2 - \frac{1}{2}\sigma_2^2)\ln(H/S_2^0)}{\sigma_2^2}} M(\eta d_4, \phi e_4; -\eta \phi \rho) \right]$$

where

$$d_1 = \frac{\ln(S_1^0/K) + (r_1 - q_1 + \frac{1}{2}\sigma_1^2) T}{\sigma_1 \sqrt{T}} \quad d_2 = d_1 - \sigma_1 \sqrt{T} \\ d_3 = d_1 + \frac{2\rho \ln(H/S_2^0)}{\sigma_2 \sqrt{T}} \quad d_4 = d_2 + \frac{2\rho \ln(H/S_2^0)}{\sigma_2 \sqrt{T}} \\ e_1 = \frac{\ln(H/S_2^0) - (r_2 - q_2 - \frac{1}{2}\sigma_2^2 + \rho\sigma_1\sigma_2) T}{\sigma_2 \sqrt{T}} \quad e_2 = e_1 + \rho\sigma_1 \sqrt{T} \\ e_3 = e_1 - \frac{2 \ln(H/S_2^0)}{\sigma_2 \sqrt{T}} \quad e_4 = e_2 - \frac{2 \ln(H/S_2^0)}{\sigma_2 \sqrt{T}}$$

with  $M(a, b; \rho)$  denoting the cumulative bivariate standard normal distribution function defined as

$$M(a, b; \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^a \int_{-\infty}^b \exp \left\{ -\frac{x^2 - 2\rho xy + y^2}{2(1 - \rho^2)} \right\} dx dy$$

and where

$$\begin{aligned} \eta = 1 \text{ and } \phi = -1 & \quad \text{for a down-and-out call,} \\ \eta = 1 \text{ and } \phi = 1 & \quad \text{for an up-and-out call,} \\ \eta = -1 \text{ and } \phi = -1 & \quad \text{for a down-and-out put,} \\ \eta = -1 \text{ and } \phi = 1 & \quad \text{for an up-and-out put.} \end{aligned}$$

Similar to standard barrier options, here too, an in-out parity applies and therefore the formulas for rainbow knock-in options are omitted for brevity. Correlation digital options have the payout of a digital option instead of that of a plain-vanilla option and are therefore a special case to the above presented formulas (see Zhang (1995b)). Kwok et al. (1998) extended the concept of rainbow barrier options to multi-asset options (e.g. basket options) which knock out if a distinct external asset hits a barrier.

### 3. VALUATION OF PARISIAN RAINBOW BARRIER OPTIONS

In this section, we introduce a new type of option which we call Parisian rainbow barrier options. More specifically, we add the Parisian feature to rainbow barrier options, i.e. the

option contract (with underlying asset  $S_1$ ) will be knocked in or knocked out, respectively, when the second asset  $S_2$  satisfies the Parisian barrier knock-in or knock-out condition, i.e. the option contract is knocked in or knocked out if the underlying asset value hits the barrier stays consecutively below it for a time longer than some predetermined time period  $d$  before the maturity date.

**3.1. Parisian rainbow up-and-in call.** In the standard Parisian rainbow *up-and-in* call option framework, the call option  $(S_1(T) - K)^+$  is only valid at the maturity date  $T$  if the following technical condition is satisfied:

$$T_H^+ = \inf\{t > 0 \mid (t - g_{H,t}^{S_2}) \mathbf{1}_{\{S_2(t) \geq H\}} > d\} \leq T, \quad (2)$$

where  $g_{H,t}^{S_2}$  denotes the last time before  $t$  at which the value of  $S_2$  hits the barrier  $H$ :

$$g_{H,t}^{S_2} = \sup\left\{s \leq t \mid S_2(s) = H\right\}; \sup\{\emptyset\} = 0.$$

That is, the option written on the asset  $S_1$  becomes valuable only if the excursion of the second asset  $S_2$  lasts consecutively longer than  $d$  units of time. We use  $T_H^+$  to denote the first time at which an excursion above  $H$  lasts longer than  $d$  under the convention that  $\inf\{\emptyset\} = \infty$ . Note that

$$\begin{aligned} g_{H,t}^{S_2} &= \sup\left\{s \leq t \mid S_2(s) = H\right\} \\ &= \sup\{s \leq t \mid Z_s = b\} = g_{b,t}^Z, \text{ with } Z_t := W_2(t) + \mu_2 t, \end{aligned} \quad (3)$$

$b := \frac{1}{\sigma_2} \ln\left(\frac{H}{S_2^0}\right)$  and  $\mu_2 := \frac{1}{\sigma_2} \left(r - q_2 - \frac{1}{2}\sigma_2^2\right)$ . Therefore, the condition in (2) is equivalent to

$$T_b^+ := \inf\{t > 0 \mid (t - g_{b,t}^Z) \mathbf{1}_{\{Z_t \geq b\}} > d\} \leq T.$$

In order to simplify the valuation procedure we introduce a new probability measure  $P^*$ , which is defined by the Radon–Nikodym density<sup>1</sup>

$$\left. \frac{dP}{dP^*} \right|_{\mathcal{F}_T} = \exp\left\{\mu_2 Z_T - \frac{\mu_2^2}{2} T\right\}.$$

Under this measure  $(Z_t)_{0 \leq t \leq T}$  is a martingale and a driftless Brownian motion. Thereby, we transform the event of “the excursion of the value of the asset  $S_2$  above the constant barrier  $H$ ” to the event of “the excursion of the Brownian motion  $Z_t$  above a constant barrier  $b = \frac{1}{\sigma_2} \ln\left(\frac{H}{S_2^0}\right)$ ”.

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<sup>1</sup>This formula is just the inverse of the Radon–Nikodym density  $\left. \frac{dP^*}{dP} \right|_{\mathcal{F}_T} = \exp\left\{-\mu_2 Z_T - \frac{\mu_2^2}{2} T\right\}$ , which transforms  $Z$  into a martingale under  $P^*$ .

To proceed with the valuation, firstly, we derive the log-asset prices under the risk-neutral measure:

$$\begin{aligned} dX_t &:= d\left(\ln \frac{S_1(t)}{S_1^0}\right) = \mu_1 dt + \sigma_1 \left(\sqrt{1-\rho^2} dW_1(t) + \rho dW_2(t)\right), \quad X_0 = 0 \\ dZ_t &:= \frac{1}{\sigma_2} d\left(\ln \frac{S_2(t)}{S_2^0}\right) = \mu_2 dt + dW_2(t), \quad Z_0 = 0, \end{aligned} \quad (4)$$

where  $\mu_1 = r - q_1 - \frac{\sigma_1^2}{2}$ . Secondly, we introduce the following stochastic process

$$M_t = X_t - \rho\sigma_1 Z_t = \mu t + \sigma_1 \sqrt{1-\rho^2} W_1(t), \quad (5)$$

where  $\mu = \mu_1 - \rho\sigma_1\mu_2$ . By construction  $M_t$  is independent of  $Z_t$  and its density at time  $t$  is given by  $f_{M_t}(m) = \frac{1}{\sqrt{2\pi\sigma_1^2 t(1-\rho^2)}} \exp\left\{-\frac{1}{2}\left(\frac{m-\mu t}{\sigma_1\sqrt{t(1-\rho^2)}}\right)^2\right\}$  where  $m \in \mathbb{R}$ . The value of the option is calculated as

$$\begin{aligned} & E\left[e^{-rT}(S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}\right] \\ &= e^{-rT} E\left[(S_1^0 e^{X_T} - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}\right] \\ &= e^{-rT} E\left[(S_1^0 e^{M_T + \rho\sigma_1 Z_T} - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}\right] \\ &= e^{-rT} E^*\left[(S_1^0 e^{M_T + \rho\sigma_1 Z_T} - K)^+ \exp\left(\mu_2 Z_T - \frac{\mu_2^2}{2} T\right) \mathbf{1}_{\{T_b^+ \leq T\}}\right] \\ &= e^{-(r+\frac{\mu_2^2}{2})T} E^*\left[(S_1^0 e^{M_T + \rho\sigma_1 Z_T} - K)^+ e^{\mu_2 Z_T} \mathbf{1}_{\{T_b^+ \leq T\}}\right]. \end{aligned}$$

In the above derivation, the first two steps are straightforward substitutions. In the third step, we switch from the measure  $P$  to  $P^*$ . The change to the probability measure  $P^*$  is needed in order to use the results of Chesney et al. (1997), where a closed-form representation for the joint density of  $(Z, T_b^+)$  under  $P^*$  is provided.

Up to this point we have rewritten the option valuation formula as a function of the independent processes  $M_T$  and  $Z_T$ . Now we need to distinguish between the case of  $H > S_2^0$  and  $H < S_2^0$ .

For  $H > S_2^0$ , from Chesney et al. (1997), the valuation formula is rewritten as follows:

$$\begin{aligned} & E\left[e^{-rT}(S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}\right] \\ &= e^{-(r+\frac{\mu_2^2}{2})T} \left\{ \int_{-\infty}^b \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_1(T, z, b) dz \right. \\ &+ \left. \int_b^{+\infty} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_2(T, z, b) dz \right\}. \end{aligned}$$

where the density  $h_1(T, y, b)$  is uniquely determined by inverting the Laplace transform (c.f. Chesney et al. (1997))

$$\hat{h}_1(\lambda, y, b) = \frac{e^{(y-2b)\sqrt{2\lambda}}\psi(-\sqrt{2\lambda d})}{\sqrt{2\lambda}\psi(\sqrt{2\lambda d})}$$

with  $\psi(z) = \int_0^\infty x \exp\left(-\frac{x^2}{2} + zx\right) dx = 1 + z\sqrt{2\pi}e^{\frac{z^2}{2}}N(z),$

where  $\lambda$  denotes the parameter of the Laplace transform. Similarly  $h_2(T, y, b)$  is determined by inverting the following Laplace transform:

$$\begin{aligned} \hat{h}_2(\lambda, y, b) &= \frac{e^{-y\sqrt{2\lambda}}}{\sqrt{2\lambda}\psi(\sqrt{2\lambda d})} + \frac{\sqrt{2\lambda d}e^{\lambda d}}{\psi(\sqrt{2\lambda d})} \left( e^{-y\sqrt{2\lambda}} \left( N\left(-\sqrt{2\lambda d} + \frac{y-b}{\sqrt{d}}\right) - N(-\sqrt{2\lambda d}) \right) \right. \\ &\quad \left. - e^{(y-2b)\sqrt{2\lambda}} N\left(-\sqrt{2\lambda d} - \frac{y-b}{\sqrt{d}}\right) \right). \end{aligned}$$

For  $H < S_2^0$ , from Chesney et al. (1997), the valuation is rewritten as follows:

$$\begin{aligned} &E \left[ e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}} \right] \\ &= e^{-(r+\frac{\mu_2^2}{2})T} \left\{ \int_{-\infty}^{-b} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_4(T, z, b) dz \right. \\ &\quad \left. + \int_{-b}^{+\infty} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_3(T, z, b) dz \right\}. \end{aligned}$$

The densities  $h_3(T, y, b)$  and  $h_4(T, y, b)$  are determined by the inverse Laplace transforms of

$$\hat{h}_3(\lambda, y, b) = \frac{e^{(2b-y)\sqrt{2\lambda}}\psi(-\sqrt{2\lambda d})}{\sqrt{2\lambda}\psi(\sqrt{2\lambda d})}$$

and

$$\begin{aligned} \hat{h}_4(\lambda, y, b) &= \frac{e^{y\sqrt{2\lambda}}}{\sqrt{2\lambda}\psi(\sqrt{2\lambda d})} + \frac{\sqrt{2\lambda d}e^{\lambda d}}{\psi(\sqrt{2\lambda d})} \left( e^{y\sqrt{2\lambda}} \left( N\left(-\sqrt{2\lambda d} - \frac{y-b}{\sqrt{d}}\right) - N(-\sqrt{2\lambda d}) \right) \right. \\ &\quad \left. - e^{(2b-y)\sqrt{2\lambda}} N\left(-\sqrt{2\lambda d} + \frac{y-b}{\sqrt{d}}\right) \right), \end{aligned}$$

respectively.

Note that the integral  $\int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm$ , independent of the relation between  $S_2^0$  and  $H$ , is equal, up to a special discount factor, to the price of a European



call option with strike price  $Ke^{\rho\sigma_1 z}$  and it is easily shown that

$$\begin{aligned} & \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm \\ &= S_1^0 e^{\mu T + \frac{1}{2}\sigma_1^2(1-\rho^2)T + \rho\sigma_1 z} N\left(\frac{\ln \frac{S_1^0}{K} + \mu T + \sigma_1^2(1-\rho^2)T + \rho\sigma_1 z}{\sigma_1 \sqrt{(1-\rho^2)T}}\right) - KN\left(\frac{\ln \frac{S_1^0}{K} + \mu T + \rho\sigma_1 z}{\sigma_1 \sqrt{(1-\rho^2)T}}\right). \end{aligned}$$

More specifically if  $\rho = 0$ , the previous equation can be reduced to

$$\begin{aligned} & \int_{\ln \frac{K}{S_1^0}}^{+\infty} (S_1^0 e^m - K) f_{M_T}(m) dm \\ &= S_1^0 e^{(r-q_1)T} N\left(\frac{\ln \frac{S_1^0}{K} + (r-q_1 + \frac{1}{2}\sigma_1^2)T}{\sigma_1 \sqrt{T}}\right) - KN\left(\frac{\ln \frac{S_1^0}{K} + (r-q_1 - \frac{1}{2}\sigma_1^2)T}{\sigma_1 \sqrt{T}}\right), \end{aligned}$$

which corresponds to the undiscounted Black–Scholes call option formula. As a result, we retrieve what is expected for the case of independence between the two assets, i.e.

$$E\left[e^{-rT}(S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}\right] = BSC(S_0^1, T, K, \sigma_1, r, q_1) E[\mathbf{1}_{\{T_b^+ \leq T\}}],$$

where  $BSC(S_0^1, T, K, \sigma_1, r, q_1)$  denotes the standard Black–Scholes call formula with initial value  $S_0^1$ , strike price  $K$ , expiry date  $T$ , volatility  $\sigma_1$ , risk–free interest rate  $r$  and dividend yield  $q_1$ . Furthermore, the term  $E[\mathbf{1}_{\{T_b^+ \leq T\}}]$  represents the probability that the Parisian knock–in event is triggered.

**3.2. Parisian rainbow down–and–in put.** In the standard Parisian rainbow *down–and–in* put option framework, the option  $(K - S_1(T))^+$  is only valid at the maturity date  $T$  if the following technical condition is satisfied:

$$T_H^- = \inf\{t > 0 | (t - g_{H,t}^{S_2}) \mathbf{1}_{\{S_2(t) \leq H\}} > d\} \leq T. \quad (6)$$

Again  $g_{H,t}^{S_2}$  is the last time before  $t$  at which the value of  $S_2$  hits the barrier  $H$ . That is, the option written on the asset  $S_1$  becomes valuable if the excursion of the second asset  $S_2$  lasts consecutively longer than  $d$  units of time. We use  $T_H^-$  to denote the first time at which an excursion below  $H$  lasts longer than  $d$ . Using the fact  $g_{H,t}^{S_2} = g_{b,t}^Z$  the condition in (6) is equivalent to

$$T_b^- := \inf\{t > 0 | (t - g_{b,t}^Z) \mathbf{1}_{\{Z_t \leq b\}} > d\} \leq T.$$

Introducing the probability measure  $P^*$ , as in Subsection 3.1, we transform the event “the excursion of the value of the asset  $S_2$  below the constant barrier  $H$ ” to the event “the excursion of the Brownian motion  $Z_t$  below a constant barrier  $b = \frac{1}{\sigma_2} \ln\left(\frac{H}{S_2^0}\right)$ ”.

By introducing the process  $M_t$ , as in (5), the value of the option is as follows:

$$\begin{aligned} & E\left[e^{-rT}(K - S_1(T))^+ \mathbf{1}_{\{T_b^- \leq T\}}\right] \\ &= e^{-(r+\frac{\mu_2}{2})T} E^*\left[\left(K - S_1 e^{M_T + \rho\sigma_1 Z_T}\right)^+ e^{\mu_2 Z_T} \mathbf{1}_{\{T_b^- \leq T\}}\right]. \end{aligned}$$

We need, again, to distinguish between  $S_2^0 < H$  and  $S_2^0 > H$ . For the former case, it holds that

$$\begin{aligned} & E \left[ e^{-rT} (K - S_1(T))^+ \mathbf{1}_{\{T_b^- \leq T\}} \right] \\ &= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^b \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_2(T, z, b) dz \right. \\ & \quad \left. + \int_b^{\infty} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_1(T, z, b) dz \right\}. \end{aligned}$$

For  $H < S_2^0$ , the value of the Parisian rainbow down-and-in put option is given by

$$\begin{aligned} & E \left[ e^{-rT} (K - S_1(T))^+ \mathbf{1}_{\{T_b^- \leq T\}} \right] \\ &= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^{-b} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_4(T, z, b) dz \right. \\ & \quad \left. + \int_{-b}^{+\infty} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_3(T, z, b) dz \right\}. \end{aligned}$$

Analogously to the case of the Parisian rainbow up-and-in call, the integral

$\int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm$  is equal, up to a special discount factor, to the price of a European put option with strike price  $K e^{\rho\sigma_1 z}$ . It is easily shown that

$$\begin{aligned} & \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm \\ &= KN \left( -\frac{\ln \frac{S_2^0}{K} + \mu T + \rho\sigma_1 z}{\sigma_1 \sqrt{(1-\rho^2)T}} \right) - S_1^0 e^{\mu T + \frac{1}{2}\sigma_1^2(1-\rho^2)T + \rho\sigma_1 z} N \left( -\frac{\ln \frac{S_2^0}{K} + \mu T + \sigma_1^2(1-\rho^2)T + \rho\sigma_1 z}{\sigma_1 \sqrt{(1-\rho^2)T}} \right). \end{aligned}$$

For  $\rho = 0$  the analysis follows the case of the up-and-in call and the Black-Scholes put option formula is obtained.

**3.3. Other types of Parisian rainbow options.** Performing the same steps as in Subsection 3.1, the value of the Parisian rainbow *up-and-in* put option with payoff function  $(K - S_1(T))^+ \mathbf{1}_{\{T_b^+ \leq T\}}$  at time  $T$  for  $S_2^0 < H$  is

$$\begin{aligned} & E \left[ e^{-rT} (K - S_1(T))^+ \mathbf{1}_{\{T_b^+ \leq T\}} \right] \\ &= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^b \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_2(T, z, b) dz \right. \\ & \quad \left. + \int_b^{+\infty} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_1(T, z, b) dz \right\} \end{aligned}$$

and for  $S_2^0 > H$  it is

$$\begin{aligned}
& E \left[ e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}} \right] \\
&= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^{-b} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_4(T, z, b) dz \right. \\
&+ \left. \int_{-b}^{+\infty} \int_{-\infty}^{\ln \frac{K}{S_1^0} - \rho\sigma_1 z} (K - S_1^0 e^{m+\rho\sigma_1 z}) f_{M_T}(m) dm e^{\mu_2 z} h_3(T, z, b) dz \right\}.
\end{aligned}$$

For the valuation of the Parisian rainbow *down-and-in* call option with payoff function  $(S_1(T) - K)^+ \mathbf{1}_{\{T_b^- \leq T\}}$  at time  $T$ , we follow the same steps performed in 3.2, and for  $S_2^0 < H$ , we reach the **integral representation**

$$\begin{aligned}
& E \left[ e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^- \leq T\}} \right] \\
&= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^b \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_2(T, z, b) dz \right. \\
&+ \left. \int_b^{+\infty} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_1(T, z, b) dz \right\}
\end{aligned}$$

and for  $S_2^0 > H$ , we have

$$\begin{aligned}
& E[e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^- \leq T\}}] \\
&= e^{-(r+\frac{\mu_2}{2})T} \left\{ \int_{-\infty}^{-b} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_4(T, z, b) dz \right. \\
&+ \left. \int_{-b}^{+\infty} \int_{\ln \frac{K}{S_1^0} - \rho\sigma_1 z}^{+\infty} (S_1^0 e^{m+\rho\sigma_1 z} - K) f_{M_T}(m) dm e^{\mu_2 z} h_3(T, z, b) dz \right\}.
\end{aligned}$$

The last formula concludes the valuation of all possible “in” type of Parisian rainbow options. In order to value the corresponding “out” type, we only need to apply the in–out parity, as the next example shows.

The Parisian rainbow *up-and-out* call with payoff  $(S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ > T\}}$  at  $T$  is calculated by using the in–out parity as follows:

$$E[e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ > T\}}] = BSC(S_1^0, T, K, \sigma_1, r, q_1) - E[e^{-rT} (S_1(T) - K)^+ \mathbf{1}_{\{T_b^+ \leq T\}}]$$

i.e. the difference between the value of the plain–vanilla call option and that of the Parisian rainbow up–and–in call option. Similarly, the Parisian rainbow *up-and-out* put value is the difference between the value of the plain–vanilla put option and that of the Parisian rainbow up–and–in put option.

The Parisian rainbow *down-and-out* call value is the difference between the value of the plain–vanilla call option and that of the Parisian rainbow down–and–in call option.

$\rho$	-0.75	-0.50	-0.25	0	0.25	0.50	0.75
U&I call ( $\sigma_2 = 0.20$ )	3.371	4.701	6.043	7.411	8.811	10.255	11.760
U&I call ( $\sigma_2 = 0.25$ )	3.882	5.225	6.569	7.926	9.305	10.716	12.176
U&I call ( $\sigma_2 = 0.30$ )	4.245	5.586	6.921	8.262	9.619	10.999	12.423

TABLE 2. Parisian rainbow up-and-in call as a function of  $\rho$  where we have used  $S_1^0 = 100$ ,  $S_2^0 = 100$ ,  $K = 90$ ,  $r = 0.05$ ,  $\sigma_1 = 0.2$ ,  $H = 110$ ,  $T = 1$ ,  $d = 1/12$ ,  $q_1 = 0$ ,  $q_2 = 0$ .

$\rho$	-0.75	-0.50	-0.25	0	0.25	0.50	0.75
U&O call ( $\sigma_2 = 0.20$ )	13.329	11.999	10.656	9.288	7.888	6.445	4.939
U&O call ( $\sigma_2 = 0.25$ )	12.818	11.474	10.131	8.773	7.394	5.984	4.524
U&O call ( $\sigma_2 = 0.30$ )	12.454	11.113	9.779	8.437	7.081	5.700	4.277

TABLE 3. Parisian rainbow up-and-out call as a function of  $\rho$  where we have used  $S_1^0 = 100$ ,  $S_2^0 = 100$ ,  $K = 90$ ,  $r = 0.05$ ,  $\sigma_1 = 0.2$ ,  $H = 110$ ,  $T = 1$ ,  $d = 1/12$ ,  $q_1 = 0$ ,  $q_2 = 0$ .

Finally, the Parisian rainbow *down-and-out* put option value is the difference between the value of the plain-vanilla put option and that of the Parisian rainbow down-and-in put option.

#### 4. NUMERICAL RESULTS

Up to this point we have adopted the Laplace transform approach initiated by Chesney et al. (1997). In the following numerical analysis we adopt the approach of Bernard et al. (2005) for the inversion of the Laplace transforms. The advantage of this approach is that the Laplace transforms, needed to value standard Parisian barrier options, are approximated by a linear combination of fractional power functions in the Laplace parameter, which have well-known inverse Laplace transforms. Alternatively, the numerical inversion algorithm of Abate and Whitt (1995) might be used.

In what follows, we focus on demonstrating numerical results for Parisian rainbow up-and-in and up-and-out call options as illustrative examples.

Table 2 illustrates the price of a Parisian rainbow up-and-in call as a function of the correlation coefficient  $\rho$  of the two assets for a varying volatility level  $\sigma_2$ . There are two observations. One the one hand, for a given  $\rho$ , the price rises in the volatility  $\sigma_2$ . This is due to the fact that a higher  $\sigma_2$  makes it more likely the the barrier level is crossed by the asset  $S_2$  from below. Although at the same time it is more likely that asset  $S_2$  performs worse for a higher  $\sigma_2$ , the Parisian knock-in condition is more likely satisfied for a higher  $\sigma_2$ , if the required Parisian time (length of excursion) is short enough. On the

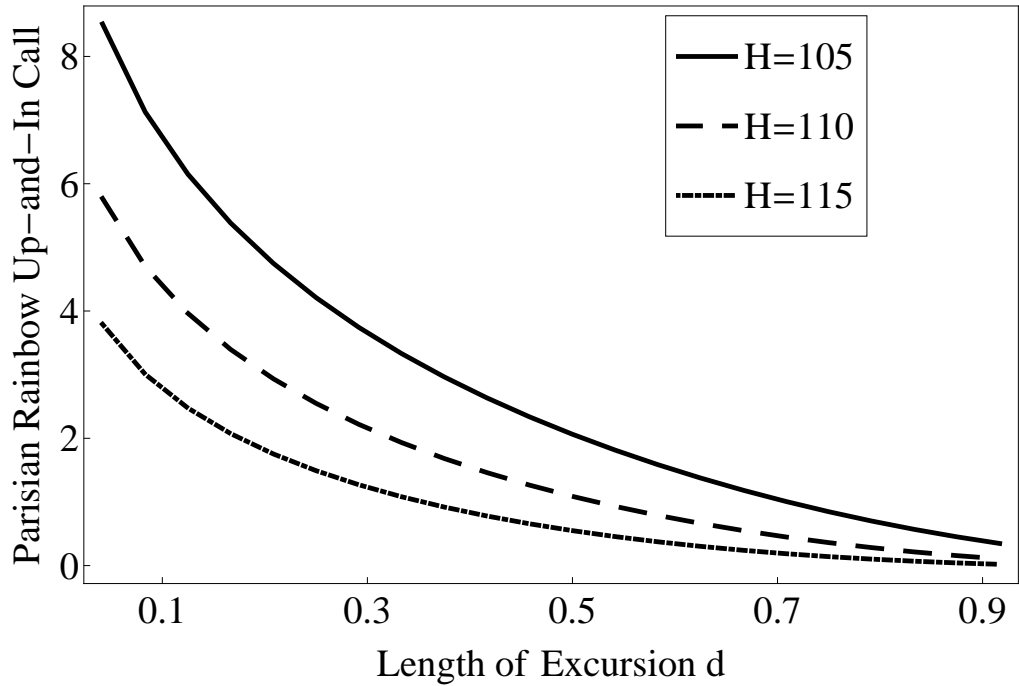


FIGURE 1. Parisian rainbow up-and-in call as a function of  $d$  where we have used  $S_1^0 = 100$ ,  $S_2^0 = 100$ ,  $K = 90$ ,  $r = 0.05$ ,  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.2$ ,  $\rho = -0.5$ ,  $T = 1$ .

other hand, for a given  $\sigma_2$ , the price increases in the correlation coefficient. Since we deal with a Parisian rainbow up-and-in rainbow call option, the option is only valid if asset  $S_2$  outperforms the barrier at least for a time period of  $d$  and simultaneously the asset  $S_1$ 's terminal value is bigger than the strike level. The more correlated the two assets are, the more likely it is that both of the assets perform well at the same time and consequently the knock-in condition is met while the option is in-the-money. When a Parisian rainbow up-and-out call option is considered, we shall expect the opposite effect, i.e. a negative effect of the correlation on the price (c.f. Table 3). In addition, there is a negative relation between the volatility  $\sigma_2$  and the price of a Parisian rainbow up-and-out call option.

The influence of the length of excursion  $d$  on the prices of Parisian rainbow up-and-in and a Parisian rainbow up-and-out calls for different barrier levels  $H$  is exhibited in Figures 1 and 2. The longer the length of excursion, the more difficult the knock-in condition will be satisfied (reversely, the easier the knock-out condition). As a consequence, the price of a Parisian rainbow up-and-in call decreases in  $d$ , while the price of a Parisian rainbow up-and-out call increases in  $d$ . Similar effects result for the barrier level  $H$ . A higher barrier level indicates a lower price for the Parisian rainbow up-and-in call, but a higher price for a Parisian rainbow up-and-out call.

In Table 4, we illustrate several values for Parisian rainbow up-and-in and up-and-out call options as a function of dividend yield  $q_1$ . A higher dividend yield implies that the net rate of return from asset  $S_1$  is lowered. Consequently, the price of both options decreases, because call options are considered.

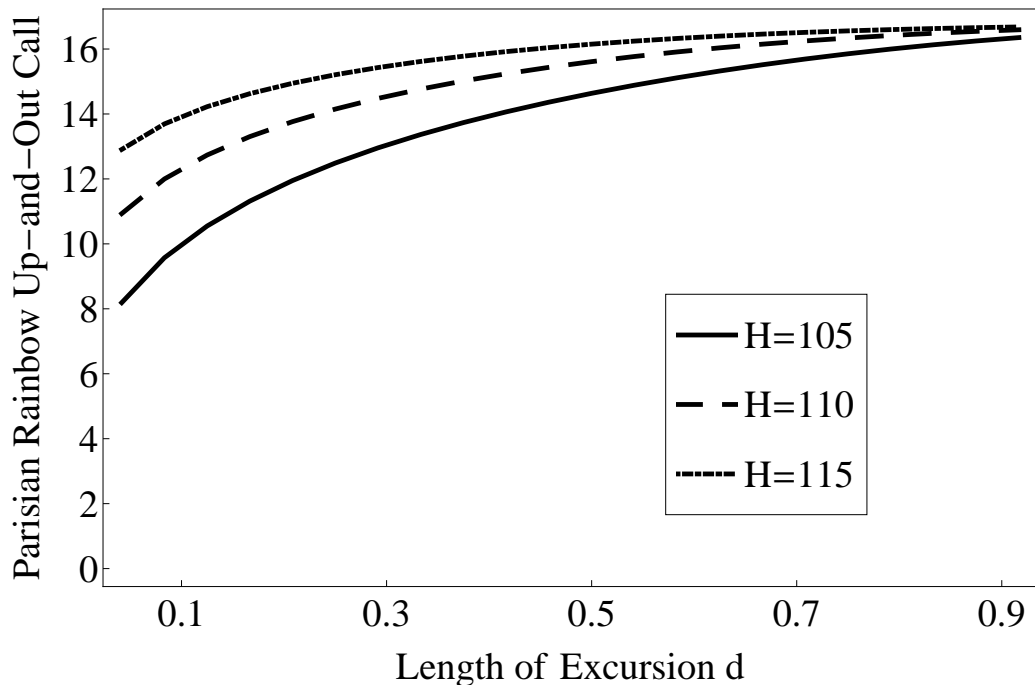


FIGURE 2. Parisian rainbow up-and-out call as a function of  $d$  where we have used  $S_1^0 = 100$ ,  $S_2^0 = 100$ ,  $K = 90$ ,  $r = 0.05$ ,  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.2$ ,  $\rho = -0.5$ ,  $T = 1$ .

$q_1$	0	0.01	0.02	0.03	0.04
U&I call	4.458	4.193	3.938	3.693	3.458
U&O call	12.242	11.708	11.186	10.676	10.179

TABLE 4. Parisian rainbow up-and-in and up-and-out calls as a function of  $q_1$  where we have used  $S_1^0 = 100$ ,  $S_2^0 = 100$ ,  $K = 90$ ,  $r = 0.05$ ,  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.2$ ;  $H = 110$ ,  $T = 1$ ,  $d = 1/12$ ,  $q_2 = 0.01$ ,  $\rho = -0.5$ .

## 5. APPLICATIONS

Parisian rainbow barrier options, valuation formulas for which are derived in the present paper, have a number of possible useful applications in various areas of financial derivatives and corporate finance.

Firstly, as already outlined in the introduction, rainbow barrier options have been traded for a long time in the OTC markets with a stock as underlying asset and an FX rate as barrier asset, but also with both assets being distinct FX rates or two stocks. These options may be traded as stand-alone OTC options or they may be included in some structured product. If the model is slightly adapted, even non-traded assets such as the Harmonized Index of Consumer Prices (HICP) or a precipitation or temperature index may serve as

barrier assets for inflation-linked or weather-linked instruments. Parisian rainbow barrier options are then a natural extension of these options which avoids barrier events caused by sudden spikes in the barrier assets. In the case of inflation-linked Parisian barrier options, an option on a stock in the consumer sector may be knocked out if the inflation rate in a country falls below a certain level for given period. Similarly in the case of a weather-linked Parisian barrier option, the option on the stock of an orange juice producer may be knocked out if the precipitation or the temperature level exceeds a threshold level for a given time period.

Secondly, Parisian rainbow barrier options of the knock-out type are a straightforward generalization of the vulnerable options presented in Hull and White (1995), Klein (1996) and Klein and Inglis (2001). These authors analyze the pricing of options and other derivative securities if the unilateral counterparty credit risk that the issuer of these products defaults before maturity and possible correlation effects are taken into account. Here, the barrier asset usually represents the value of the assets of the writer of the derivative contract and default occurs if this value hits the default barrier which may be represented by the value of the issuer's liabilities. By using the Parisian barrier trigger, the issuer is given the chance to recover if he suffers a liquidity squeeze. Furthermore, in analogy to the literature on the contingent claim approach to modelling a firm's capital structure, these options can be used to represent the various components of a firm's balance sheet and thanks to the Parisian barrier trigger a Chapter 11 bankruptcy procedure can be easily incorporated, just similar to Chen and Suchaneki (2007).

Thirdly, Parisian rainbow barrier options may be used to model a certain kind of executive stock options. The option is written on the stock of the company, but the barrier trigger is linked to the performance index of the employee. Here, the Parisian barrier trigger is used to knock-in the stock option only if the performance of the employee has been sufficiently good for a given period of time, thus giving the employee an incentive to perform in a sustainable way.

Finally, Parisian barrier options have been repeatedly used to represent the conversion feature of convertible bonds. With standard Parisian barrier options, the conversion is linked to the underlying asset. In contrast, with Parisian rainbow barrier options, the conversion event is not linked to the price of the underlying stock but to some other trigger, e.g. to a capital ratio or to the performance of an equity index or a sector index. More recently, during the financial crisis, as a result of lack of opportunities for banks to raise equity capital, the issuance of contingent capital, also known as CoCos, has been proposed by regulators in order to facilitate the banks' recapitalization (see Maes and Schoutens (2010) and Pitt et al. (2011)). The contingent capital is issued as contingent convertible bond which is automatically converted into equity when the trigger level has been reached.

If the trigger is linked to the price of the underlying stock, then an approach is to price to CoCos simply as a down-and-in put on the stock price. This represents a market-based conversion trigger. However, the barrier trigger can also be linked to regulatory capital ratios or other accounting-based key numbers. Furthermore, especially in times of crisis, it may not be desirable to have a conversion trigger being too sensitive to exaggerations. A possible solution for taking these effects into account would be to use Parisian down-and-in rainbow options for the pricing of CoCos where the trigger depends on a regulatory capital ratio or the banks CDS spread exceeding a certain threshold level.

## 6. CONCLUSION

The present paper introduces Parisian rainbow barrier options which incorporate the Parisian barrier trigger in outside barrier options. We derive analytical valuation formulae for all the eight types of Parisian rainbow barrier options. To illustrate the sensitivities of these new options, numerical analyses are carried out based on the newly derived formulas.

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