Addendum to the Preprint 514
Fractional Interior Differentiability of the Stress Velocities to Elastic Plastic Problems with Hardening

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It turned out the result for the interior space regularity holds for both kinematic and isotropic hardening. Theorem 2.4 can be formulated now as

Theorem 2.4 (Local regularity in space) *Assuming the requirements of Theorem 2.1 (σ, ξ) be again the solution pair of the hardening problems formulated in Section 1. Then the velocities $\dot\sigma$, $\dot\xi$ have local fractional derivatives of order $1/2$ in space direction, in the following sense*

$$\sup_{0\le h\le h_0} h^{-1} \int_0^{T-h} \int_{\Omega_0} |\Delta_i^h \dot\sigma|^2 + |\Delta_i^h \dot\xi|^2 \, dx\, dt \le C,\ \ i = 1, \dots, n \tag{2.3}$$

for any domain Ω_0 such that $\overline{\Omega}_0 \subset \Omega$ and $h_0 \le dist(\partial\Omega, \partial\Omega_0)$.

The proof only needs a modification - a simplification as a matter of fact - of subsection 5.2.

5.2 Testing the strain velocity

We recall the formulation of the lemma and present the different proof. References to previous numbers refer to the main text.

Lemma 5.3 *Let ζ be a localization function as introduced in Section 5.1, and let $h_0 > 0$ be fixed such that $h_0 < dist\,(supp\,\zeta, \partial\Omega)$. Then*

$$\left| \int_0^h \int_{t_1}^{t_2-h} \big(\nabla \dot u\, ,\, \zeta^2 (E_t^s E_i^h - I)\dot\sigma\big)_\Omega \, dt \right| \le C\, h^2 \tag{5.7}$$

with h, t_1, t_2 as in Lemma 5.2 and C again independent of these parameters.

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Proof. We denote

$$\mathcal{S} = \int_0^h \int_{t_1}^{t_2-h} \big(\nabla\dot u\,,\, \zeta^2(E_t^s E_i^h - I)\dot\sigma\big)_\Omega \,dt\,ds = \mathcal{S}^1 + \mathcal{S}^2,$$

where

$$\begin{aligned}
\mathcal{S}^1 :&= \int_0^h \int_{t_1}^{t_2-h} \big(\nabla\dot u\,,\, \zeta^2 E_t^s(E_i^h - I)\dot\sigma\big)_\Omega \,dt\,ds = \int_0^h \int_{t_1}^{t_2-h} \big(\nabla\dot u\,,\, \zeta^2 E_t^s\Delta_i^h\dot\sigma\big)_\Omega \,dt\,ds,\\
\mathcal{S}^2 :&= \int_0^h \int_{t_1}^{t_2-h} \big(\nabla\dot u\,,\, \zeta^2(E_t^s - I)\dot\sigma\big)_\Omega \,dt\,ds = \int_0^h \int_{t_1}^{t_2-h} \big(\nabla\dot u\,,\, \zeta^2\Delta_t^s\dot\sigma\big)_\Omega \,dt\,ds.
\end{aligned} \tag{5.8}$$

Step 1. Estimates for $|\mathcal{S}^1|$.
To this end, we integrate by parts in the term $\mathcal{S}^1$, then use the relation $-\operatorname{div}\sigma = f$, end up with

$$\mathcal{S}^1 = -\int_0^h \int_{t_1}^{t_2-h} \big(\dot u\zeta^2\,,\, E_t^s\Delta_i^h\dot f\big)_\Omega \,dt\,ds - \int_0^h \int_{t_1}^{t_2-h} \big(\dot u\nabla\zeta^2\,,\, \Delta_t^s\dot\sigma\big)_\Omega \,dt\,ds =: \mathcal{S}^{1a} + \mathcal{S}^{1b}.$$

Moving the operator Δ_i^h from $\dot f$ to $\dot u\zeta^2$ yields

$$\mathcal{S}^{1a} = -\int_0^h \int_{t_1}^{t_2-h} \big(\Delta_i^{-h}(\dot u\zeta^2)\,,\, E_t^s\dot f\big)_\Omega \,dt\,ds.$$

Since

$$\|\Delta_i^{-h}(\dot u\zeta^2)\|_{L^\infty(L^2)} = h\|D_j^{-h}\big(\dot u\zeta^2\big)\|_{L^\infty(L^2)} \le C\big(\|\dot u\|_{L^\infty(L^2)} + \|\nabla\dot u\|_{L^\infty(L^2)}\big)h,$$

the uniform estimates (3.6), (3.8) together with the assumption $\dot f \in L^\infty(L^2)$ (cf (1.4)) lead to

$$|\mathcal{S}^{1a}| \le h\int_0^h \|\dot f\|_{L^1(L^2)}\|D_i^{-h}(\dot u\zeta^2)\|_{L^\infty(L^2)}\,ds \le C_T h^2,$$

where K_T is independent of $0 < \mu \le \mu_0$, and $0 < h \le h_0$. A similar argument works for the summand $\mathcal{S}^{1b}$, hence, again with (3.6) and (3.8)

$$\begin{aligned}
|\mathcal{S}^{1b}| &= \left|\int_0^h \int_{t_1}^{t_2-h} \big(\Delta_i^{-h}(\dot u\nabla\zeta^2)\,,\, E_t^s\dot\sigma\big)_\Omega \,dt\,ds\right|\\
&\le C\ h\int_0^h \|\dot\sigma\|_{L^1(L^2)}\|D_i^{-h}(\dot u\nabla\zeta^2)\|_{L^\infty(L^2)}\,ds \le C_T\ h^2.
\end{aligned}$$

Step 2. Estimates for $|\mathcal{S}^2|$.
To show that this quantity is bounded by Ch^2, it is not enough to use $\nabla\dot{u} \in L^\infty(L^2)$ together with (2.2), because then we only get the bound $C\,h^{3/2}$. Instead we go back to the solutions of the penalized problem. Unfortunately the presence of the localization term ζ^2 prohibits to argue with the safe load as in the proof of Theorem 2.1, nevertheless the estimate for the term $|\mathcal{I}|$ (cf (4.4)) gives already the desired estimate in the case $\zeta \equiv 1$. Recall that the system (3.4) and (3.5) leads to

$$\int_0^h \int_{t_1}^{t_2-h} (\nabla\dot{u}_\mu\,,\,\zeta^2\Delta_t^s\sigma_\mu)_\Omega\,dt\,ds = \int_0^h \int_{t_1}^{t_2-h} \big(A\dot{\sigma}_\mu\,,\,\zeta^2\Delta_t^s\dot{\sigma}_\mu\big)_\Omega + \big(H\dot{\xi}_\mu\,,\,\zeta^2\Delta_t^s\dot{\xi}_\mu\big)_\Omega\,dt\,ds\,+$$

$$\int_0^h \int_{t_1}^{t_2-h} \big(G_{1\mu}\,,\,\zeta^2\Delta_t^s\dot{\sigma}_\mu\big)_\Omega + \big(G_{2\mu}\,,\,\zeta^2\Delta_t^s\dot{\xi}_\mu\big)_\Omega\,dt\,ds =: \mathcal{S}_\mu^{2a} + \mathcal{T}_{0\mu}, \tag{5.9}$$

where $\mathcal{T}_{0\mu}$ was defined in (4.2). Using again "the product-rule" (4.5) we obtain

$$\mathcal{S}_\mu^{2a} = -\frac{1}{2}\int_0^h \int_{t_1}^{t_2-h} \big(\zeta^2 A\Delta_t^s\dot{\sigma}_\mu\,,\,\Delta_t^s\dot{\sigma}_\mu\big)_\Omega + \big(\zeta^2 H\Delta_t^s\dot{\xi}_\mu\,,\,\Delta_t^s\dot{\xi}_\mu\big)_\Omega\,dt\,ds+$$

$$+\frac{1}{2}\int_0^h \int_{t_1}^{t_2-h} \int_\Omega \zeta^2\Delta_t^s\,\big(\,A\dot{\sigma}_\mu : \dot{\sigma}_\mu + H\dot{\xi}_\mu\,:\,\dot{\xi}_\mu\,\big)\;dx\,dt\,ds.$$

Note that $\lim_{\mu\to 0}\mathcal{S}_\mu^{2a}$ as well as the limits for both summands on the right hand side exist due to (3.9). The limit of the first integral is bounded by Ch^2 due to Theorem 2.1 while for the second integral we get this bound following the same arguments as in the proof of Theorem 2.1, in particular the arguments after (4.7), hence we have

$$\Big|\int_0^h \int_{t_1}^{t_2-h} \big(A\dot{\sigma}\,,\,\zeta^2\Delta_t^s\dot{\sigma}\big)_\Omega + \big(H\dot{\xi}\,,\,\zeta^2\Delta_t^s\dot{\xi}\big)_\Omega\,dt\,ds\,\Big| = \lim_{\mu\to 0}|\mathcal{S}_\mu^{2a}| \le Ch^2. \tag{5.10}$$

Since the limits of the other two terms in the equation (5.9) exist, we obtain that even $\lim_{\mu\to 0}\mathcal{T}_{0\mu}$ exists. In particular, the representation (5.9) for $\zeta = 1$ (compare (4.4)) together with the estimate for $|\mathcal{I}|$ in the proof of Theorem 2.1 then gives

$$\lim_{\mu\to 0}\Big|\int_0^h \int_{t_1}^{t_2-h} \big(G_{1\mu}\,,\,\Delta_t^s\dot{\sigma}_\mu\big)_\Omega + \big(G_{2\mu}\,,\,\Delta_t^s\dot{\xi}_\mu\big)_\Omega\,dt\,ds\,\Big| \le Ch^2 \tag{5.11}$$

To extend this to the case where ζ is a proper localization function we use similar calculations as in Lemma 4.3, in particular the convexity of G_μ and Lemma 4.1. For fixed h, we get

$$
\begin{aligned}
\lim_{\mu\to 0} |\mathcal{T}_{0\mu}| &= \lim_{\mu\to 0} \Big| \int_{t_1}^{t_2-h} (\zeta^2 G_{1\mu}, \Delta_t^h \sigma_\mu)_\Omega + (\zeta^2 G_{2\mu}, \Delta_t^h \xi_\mu)_\Omega \, dt \,\Big| \\
&= \lim_{\mu\to 0} \Big| \int_{t_1}^{t_2-h} \int_\Omega \zeta^2 \,\Big(\, \Delta_t^h G_\mu - (G_{1\mu} : \Delta_t^h \sigma_\mu + G_{2\mu} : \Delta_t^h \xi_\mu) \,\Big) \; dx\, dt \,\Big| \\
&= \lim_{\mu\to 0} \int_{t_1}^{t_2-h} \int_\Omega \zeta^2 \,\Big(\, \Delta_t^h G_\mu - (G_{1\mu} : \Delta_t^h \sigma_\mu + G_{2\mu} : \Delta_t^h \xi_\mu) \,\Big) \; dx\, dt \\
&\le \max \zeta^2 \lim_{\mu\to 0} \int_{t_1}^{t_2-h} \int_\Omega \Delta_t^h G_\mu - (G_{1\mu} : \Delta_t^h \sigma_\mu + G_{2\mu} : \Delta_t^h \xi_\mu) \, dx\, dt \\
&= C(\zeta) \lim_{\mu\to 0} \Big| \int_{t_1}^{t_2-h} \int_\Omega (G_{1\mu} : \Delta_t^h \sigma + G_{2\mu} : \Delta_t^h \xi_\mu) \, dx\, dt \,\Big| \\
&= C(\zeta) \lim_{\mu\to 0} \Big| \int_0^h \int_{t_1}^{t_2-h} \big(G_{1\mu}\,,\, \Delta_t^s \dot\sigma_\mu\big)_\Omega + \big(G_{2\mu}\,,\, \Delta_t^s \dot\xi_\mu\big)_\Omega \, dt\, ds \,\Big| \le Ch^2,
\end{aligned}
$$

observe, that the third equality and the following inquality are true because the integrand is non-negative almost everywhere due to the convexity of G_μ, while the last inequality follows from (5.11). Together with (5.10) this gives the bound for $|\mathcal{S}^2|$. □

Now the proof of Theorem 2.4 runs exactly in the same way, if we observe that in the case of isotropic hardening we only have $\nabla \dot u_\mu \rightharpoonup \nabla \dot u$ but this is sufficient for the arguments used in (5.15) and (5.16). [1]

[1] The revised preprint can be found on the webpage http://www.mathematik.uni-kassel.de/%7Especovi/Publications.html

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