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Monitoring and Privacy in Automobile
Insurance Markets with Moral Hazard

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Abstract

This paper considers moral hazard in insurance markets when voluntary monitoring technologies are available and insureds may choose the precision of monitoring. Also privacy costs incurred thereby are taken into account. Two alternative contract schemes are compared in terms of welfare: (i) monitoring conditional on the loss with only the insurance indemnities based on the monitoring data, and (ii) unrestricted monitoring with both the premiums and the indemnities depending on the data. With any contract scheme some monitoring will be optimal unless the privacy costs increase too fast in relation to the precision of the monitoring signal. In the benchmark situation (without privacy costs) relying completely on both signals (monitoring and the outcome) informative of effort (ii) maximizes welfare. In the presence of privacy costs, the contract with conditional monitoring (i) might dominate the contract which fully includes the outcome and the monitoring signal into the sharing rule (ii). Apart from the direct effect of restricting privacy costs only to the state of loss, there are also an additional indirect incentive and a risk-sharing effect with this contract. Letting the individuals choose the precision of the monitoring technology at the time they reveal the data (ex post) is inefficient with either contract scheme.

Keywords: moral hazard, conditional monitoring, value of information, privacy
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1 Introduction

As monitoring technologies have become cheaper in the last years, an increasing number of automobile insurance companies have started offering innovative contracts, which monitor the driving behavior of the insureds. The data are used to better calculate individual risk and to adjust to it the contract of a particular insured. Since 2004 the US insurer Progressive has been offering its "TripSense" contract, by which a monitoring device in the car records mileage, speed, acceleration and braking. At the end of a billing period, the insured can review the collected data, compare with other customers of the insurer and decide whether to upload the data to the company. Depending on the records, the insurer promises reductions of the insurance premiums for the next period. Britain's largest insurer, Norwich Union, introduced a similar contract, aimed at young drivers, which put more weight on which time of the day the car is used. In the meanwhile the company has extended its offer to general customers with its "pay-as-you-drive" scheme that monitors the type of road among other things. In contrast to Progressive's contract scheme, the data are transferred by GPS regularly within a billing period to the insurer, so that insureds can observe the continuously updated insurance premium. Other insurers testing similar products with their customers are Siwss Re, Zürich Schweiz, WGV. The Swiss insurer Winterthur is considering launching a project with a black box, which stores among other things data about the longitudinal and cross acceleration of the vehicle, and which would be opened only in the case of an accident. Future technologies will not only allow for the collection of an increasing variety of data concerning the driver's own behavior, but also combine them with data related to the traffic situations and to the immediate environment of the driver at any particular point in time.[1]

Generally, monitoring helps reduce information asymmetries and thus raises the efficiency of the contracts. However, a potential drawback of monitoring is the loss of privacy. Moreover, future monitoring technologies will also support entertainment and maintenance services, which will require even more data connectivity. Such otherwise attractive services might be perceived by the consumers as an increased threat to their privacy. This explains why some insurers are concerned that monitoring would not be accepted by their customers.

This paper analyzes the effects of monitoring on the contracts and also on welfare when insurance markets are characterized by moral hazard[2] and privacy costs are taken into account. Loss prevention being the subject of the analysis, a model with a fixed loss is used. This assumption takes into account that, when it comes to ex ante moral hazard, the drivers' care mainly influences the number of claims and, to a much smaller extent, the value of a damage[3]. Although the number of insurers who have already started or are considering the launch of monitoring contracts

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[1]Even currently the monitoring technology of the German insurer WGV allows for comparing the driver's actual speed with the speed limits for the particular streets.

[2]A previous paper analyzed, in a similar setting, the problem of adverse selection (see http://www.bgpe.de/, DP No.5).

[3]Lemaire (1998) argues that, in practice, the cost of an accident is regarded as being beyond the control of a policyholder.

is continuously increasing, such contracts are still a relatively new phenomenon. The dynamics of this market and the wide variety in particular terms of existing contracts, may indicate that, by trial and error, insurers are still searching for the optimal contract. Because the monitoring contracts which are already offered by the insurance companies differ in their specific design, and bearing in mind that even more contract variations are conceivable, several different scenarios are analyzed and compared with respect to the resulting welfare.

Depending on the specific technology used for collecting, correlating and analyzing the data, the precision of data and its informativeness of true risk may vary. In the existing real world applications, the accuracy and the range of collected data are restricted by the particular monitoring technology of the insurer and insureds are only free to decide whether or not to reveal the data. However, as monitoring technologies evolve and become more sophisticated, in the future it might be possible to let the insureds decide how much and how precise should be the information they wish to reveal. In view of the potential occurrence of privacy costs, it therefore seems reasonable to consider such contracts that allow the insureds to determine the precision of monitoring.

Specifically, two alternative contract schemes are analyzed. In the first one, monitoring takes place conditional on the loss and only the indemnities, which are paid in case of an accident to the insured, are based on the information collected by a black box. In contrast, the second contract scheme fully relies both on monitoring and on the outcome in that it provides monitoring in either state of nature, and both the premiums and the indemnities can depend on the monitoring data. Within a contract scheme, the following scenarios are analyzed: (a) the precision of information is exogenous and there are no privacy costs (benchmark situation); (b) in the presence of privacy costs the precision of information is agreed upon ex ante, i.e., when the contract is signed; and (c) insureds decide ex post, i.e., at the time they reveal the information, how precise it should be.

It turns out that, in any contract scheme or scenario, some monitoring will be optimal unless privacy costs increase too fast in relation to the precision of the monitoring signal. If there are no privacy costs (benchmark situation), the second contract scheme with unrestricted monitoring is welfare enhancing. This is because it applies the principle, which was derived by Holmström (1979), Shavell (1979a) and Harris and Raviv (1979), that the contract should depend on any signal which is informative of effort.[4]

The main result of this paper is that, with privacy costs, the contract scheme with

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[4]It can be shown that, even though this contract scheme approaches full damage insurance when the precision of monitoring becomes arbitrarily close to perfect, the outcome-dependency of the payments should not be suppressed. That is, in practice monitoring should not replace partial insurance however close to perfect the technology is. This follows from a comparison of this contract scheme with exogenously prescribed full damage insurance, where only the insurance premiums depend on the monitoring signal. It turns out that (the analysis can be provided upon request from the author), as the level of precision grows, the deficiency in terms of welfare of the latter contract scheme as compared to the former one will grow, even though the former contract approaches full damage insurance. The reason for this result is that using both the outcome and the monitoring signal allows for more efficient penalties for deviating from high effort.

conditional monitoring may outperform the contract which fully relies on both signals. This result is attained not only because privacy costs are in expectation smaller (direct effect), but also because privacy costs produce an additional incentive effect on effort and also a positive risk-sharing effect (indirect effects). This applies all the more, the higher is the efficiency of effort, the faster privacy costs grow in precision and the less efficient is the monitoring technology.

Another result is that, for any contract scheme, determining ex post the level of precision introduces an additional information asymmetry into the contractual relationship. It generates an undesired additional risk from the viewpoint of the insured and thus it decreases welfare. Therefore such contracts with ex post flexibility of monitoring precision, or more generally with ex post flexibility concerning the decision whether or not to reveal information, cannot be expected to persist in the long run.

Although automobile insurance markets are used as a primary example in this paper, the results may be applied equally well to other types of insurance. Health insurers or life insurers, for instance, also provide different schemes to monitor the behavior of their customers. Even some public health insurers offer rebates, or higher coverage, if the insureds prove to have attended regular medical examinations[5] or various nutrition and fitness training courses. Private health insurers put even more effort in detecting the health behavior of their customers (e.g. smoking, or the engagement in risky sports). Still, the amount of information which health insurers actually retrieve about their customers' health behavior is probably far below what is technically feasible through more detailed medical reports or the assignment of additional medical tests. Undoubtedly, privacy is a sensitive issue in health insurance, and probably individuals are more concerned about the privacy of their health behavior rather than that of their driving behavior. Differing privacy costs for different spheres of activity may help to explain why different levels of monitoring in different insurance markets are applied even if monitoring may be equally feasible in all of them. Moreover, with conditional monitoring, inferring the efficiency of the contracts, which are offered in different insurance markets, by simply comparing their observable features might be misleading. If a contract provides more insurance coverage, and therefore better risk sharing than another, it need not imply that the efficiency of the monitoring technology, and therefore the welfare of the consumers in this market, is higher. Better risk sharing in a market may as well be the result of a lower efficiency of monitoring, however, combined with a greater concern about privacy in this sphere of activity. This paper demonstrates that the overall welfare of individuals in this market will be lower than in a market with lower privacy costs, although the contracts seemingly suggest the opposite.

There is a comprehensive literature on the role of imperfect monitoring in the presence of moral hazard. The seminal papers by Holmström (1979), Shavell (1979a), and Harris / Raviv (1979), which were mentioned above, analyze the value of additional information (besides the outcome) when it is costless and exogenous for

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[5]In dental health insurance, for instance, the coverage of certain costs depends on the regularity of previous dental examinations.

the contractual relationship. They find that, no matter how noisy the additional signal is, its value is always positive as long as it is informative of effort and the agent is risk averse. Milgrom (1981) establishes the relationship between first-order stochastic dominance as a criterion for the favorableness of signals for the contractual relationship and the marginal likelihood ratio property. He finds that signals with this property result in an increasing contract sharing rule. Gjesdal (1982) and Grossman / Hart (1983) also perform a comparison of different information systems, however, applying Blackwell's criterion (see also Laffont (1989), 62-66). Kim (1995) introduces the mean preserving spread criterion for ranking information systems, which can be applied more broadly than Homström's informativeness criterion or Blackwell's criterion.

For the special case of insurance markets, Shavell (1979b) considers two forms of monitoring - ex ante and ex post. He finds that, if monitoring is imperfect, ex ante observations, where both the insurance premium and indemnity depend on the information, are more valuable than ex post observations, which take place conditional on the loss, and with which only the indemnity depends on the observation. This result is derived also in this paper by the comparison of the first and second contract schemes for the scenario of no privacy costs. Also Shavell's finding that, for the same quality of observation and if the costs of ex ante monitoring are sufficiently low, ex ante observation will be preferred no matter how low is the cost of ex post observation, is confirmed in this paper. The focus of this paper, however, is to find out under what conditions which contract scheme will be efficient when the quality of monitoring, and hence also the costs of monitoring, are endogenous.

A number of articles on endogenous monitoring deal with random sampling (see e.g. Townsend (1979), Lambert (1985), Dye (1986)), which, however, is not the subject of this paper. Singh (1985) considers monitoring with endogenous precision of the signal, the model for which resembles the one in this paper. However, his focus lies mainly on the marginal value of information at the point of zero information and the results are applied in a context rather different from this paper. Meth (1996) also studies a principal-agent model in which the agent can affect the precision of additional information. In contrast to this paper, in which the outcome distribution is independent of precision, in his model precision is understood as a reduction of the outcome variance, such that it improves the principal's ability to infer the agent's effort from the outcome. Meth finds that, if the reduction of the outcome variance is unobservable, an additional problem of moral hazard emerges. An analogous result is obtained in this paper for the case that the level of precision is determined ex post.

As in this paper, Demougin / Fluet (2001) consider a binary signal on effort, the precision of which may endogenously vary. The focus of their paper is to characterize the optimal incentive and monitoring mix in a principal-agent relationship where both parties are risk-neutral and agents face a limited-liability constraint. This paper, in contrast, considers risk-averse agents. It is closely related to Kim / Suh (1992) as it models the monitoring technology in a similar fashion - the distribution of a binary signal depends both on the exerted effort and on the comprehensiveness of the monitoring data. In their article Kim / Suh study a principal-agent

relationship, in which the principal can choose the precision of the monitoring technology, which incurs constant marginal costs. Specifically, the principal chooses the level of the monitoring investment after he observes the outcome, which in fact permits conditional monitoring. The main result of their article is that, for a concave monitoring technology, the level of precision decreases in the observed outcome. In contrast, this paper considers a situation, in which the agents (i.e. the insureds) are those who may choose the amount of monitoring data they supply and, thus, the level of precision. This is the reason why an additional information asymmetry enters the contract when precision is determined ex post. The special case of monitoring taking place conditional on the loss is included by explicitly setting up the respective framework. Although letting the precision of monitoring be dependent on the outcome (which would make sense only for the second contract scheme) is not considered explicitly, it can be verified that allowing for it would result in a larger level of precision being optimal for the loss state, which corresponds to Kim / Suh's result. In their article, Kim / Suh (1992) use a square root function to illustrate the incentive and risk-sharing effects of precision. In this paper, the same function is applied but rather for the purpose of comparing the differing contract schemes in terms of welfare.

The paper is organized as follows: the next section outlines the main setting of the model, section 3. considers the first contract scheme with monitoring conditional on the loss and basing only the indemnities on the monitoring signal. Within this contract scheme, a benchmark situation with exogenous precision and no privacy costs (3.1), in the presence of privacy costs, endogenous precision ex ante (3.2), and endogenous precision ex post (3.3) are addressed. The second contract scheme (section 4) envisages monitoring in both states of nature and allows for both the premiums and indemnities to depend on the monitoring signal. Both the outcome and the monitoring signal are fully involved into the sharing rule. Section 5 compares the results in terms of welfare, section 6 discusses the results and concludes.

2 The model

It is assumed that individuals have an initial wealth of W and there are two possible outcomes: loss $(L < W)$ and no loss, and two possible levels of effort $(e^H > e^L)$ with corresponding probabilities of no loss $p^H > p^L$. Insurers, who compete perfectly, offer contracts with premium r and indemnity d. Depending on the precision of the collected information $i \in [0, 1]$ and on the effort chosen by the individual, a binary signal s^j, $j \in \{H, L\}$ is generated by the monitoring technology, where $P(s^H|e^H, i)$ is the probability of observing the good signal if effort was high and given that the precision of information is i, where $\frac{\partial P(s^H|e^H,i)}{\partial i} > 0$, $\frac{\partial^2 P(s^H|e^H,i)}{\partial i^2} < 0$, $\forall i$, $P(s^L|e^H, i) = 1 - P(s^H|e^H, i)$, and for simplicity $P(s^H|e^H, i) = P(s^L|e^L, i) = P(i)$. If no information is revealed, the signal is not informative of effort and $P(0) = \frac{1}{2}$. Further it holds that $\frac{\partial P(i)}{\partial i}|_{i=1} = 0$, i.e. when all the feasible precision is applied (all collected data are submitted), no further increase of the probability of observing the good signal is possible, but it need not mean perfect information, $P(1) < 1$.

Individuals' utility function is given by $U(w,e,i) = u(w) - v(e) - g(i)$, where $u(w)$ is the Von Neumann-Morgenstern utility function of net wealth, $v(e)$ is the disutility of effort,and $g(i)$ is the loss of privacy, which is increasing and convex in the precision of information, i.e. $u'(w) > 0, u''(w) < 0, v(e^H) > v(e^L), g'(i) > 0, g''(i) > 0$. In addition, the marginal privacy loss from increasing precision when precision is zero is also zero, $g'(0) = 0$.

3   Conditional monitoring

The terms of this contract scheme provide that monitoring as a signal of exerted effort is restricted only to the loss state. As is depicted in Fig.1, the insured pays the premium immediately after accepting the contract, so that it cannot be based on the collected data. After accepting the contract, the insured decides which effort level to choose. If a loss occurs, the insured decides whether, and how precise information to reveal to the insurer. Based on this information, a signal is generated and the insurer pays an indemnity depending on that signal.
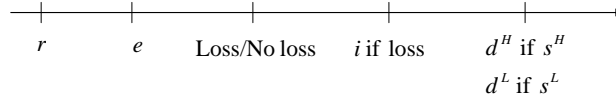

Figure 1: time structure - basing indemnities on the signal

3.1   Benchmark situation

Due to perfect competition, in equilibrium those contracts will result by which the individuals attain the highest expected utility. In case that individuals prefer to implement e^L, no incentives and no monitoring are necessary.
In case that individuals wish to implement the high effort level e^H, the optimal contract is found by

$$\max_{r,d^H,d^L} p^H u(w_N) + (1 - p^H)[P(i)u(w_H) + (1 - P(i))u(w_L)] - v(e^H) \tag{1}$$

s.t.

$$p^H u(w_N) + (1 - p^H)[P(i)u(w_H) + (1 - P(i))u(w_L)] - v(e^H) \geq$$
$$p^L u(w_N) + (1 - p^L)[(1 - P(i))u(w_H) + P(i)u(w_L)] - v(e^L)$$
$$r - (1 - p^H)[P(i)d^H + (1 - P(i))d^L] \geq 0$$

where $w_L = W - L - r + d^L$, $w_H = W - L - r + d^H$, $w_N = W - r$ are the net wealths in the respective states of nature. Let λ_1 and λ_2 be the Lagrange multipliers for the

incentive constraint and zero-profit constraint respectively, which are both positive (see A1 in the appendix). From the first order conditions one gets for an interior solution the following expressions[6]:

$$\frac{1}{u'(w_H)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{1-p^L}{1-p^H}\frac{1-P(i)}{P(i)}}_{\Psi_H}) \tag{2}$$

$$\frac{1}{u'(w_L)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{1-p^L}{1-p^H}\frac{P(i)}{1-P(i)}}_{\Psi_L}) \tag{3}$$

$$\frac{1}{u'(w_N)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{p^L}{p^H}}_{\Psi_N}) \tag{4}$$

For $i = 0$ the standard non-monitoring solution results, where $d^H = d^L < L$. For $i > 0$, a comparison of (2),(3) and (4) shows that, if $\frac{p^H(1-p^L)}{p^L(1-p^H)} < \frac{P(i)}{1-P(i)}$, the ordering of net wealth will be $w_H > w_N > w_L$. This implies that $d^L < L < d^H$: when the efficiency of effort, i.e. the difference between p^H and p^L, is small, or when the precision of information is high ($P(i)$ is large), the indemnity in case of the good signal will be larger than the loss. In the opposite case that $\frac{p^H(1-p^L)}{p^L(1-p^H)} > \frac{P(i)}{1-P(i)}$, i.e. if effort is more efficient or the precision of information is lower, there will be partial insurance irrespective of the signal, $d^L < d^H < L$. The ordering of net wealth will then be $w_N > w_H > w_L$. The reason for this result is quite straightforward. In the first case, the state of nature is not as informative of effort as the monitoring signal compared to the second case. Hence, from the viewpoint of the insurer facing a loss and a good signal in the first case, it is more likely that high effort was exerted than when there is a loss and a good signal in the second case. Therefore in the first case the good signal leads to a reward of $d^H > L$ and in the second case the loss leads to a penalty of $d^H < L$.

Especially for a comparison of the different contract schemes and in order to illustrate the effects of increasing precision, it turns out to be helpful to apply a particular utility function. In a different context Kim / Suh (1992) apply $u(w) = 2\sqrt{w}$ in order to illustrate the incentive and risk sharing effects of precision. Similarly to their procedure in Theorem 3 of their paper it can be shown that for $u(w) = 2\sqrt{w}$:
a) $Var(\Psi)$ increases in i (the monitoring signal becomes more informative of effort as precision increases),
b) λ_1 is decreases in i (incentives on effort improve),
c) $Var(\frac{1}{u'(w)})$ decreases in i (risk sharing improves),
d) λ_2 decreases in i (expected utility / efficiency increase in i).

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[6]It is assumed that $w_L > a$, $a > 0$, to ensure the existence of an interior solution.

Proof:
a)

$$Var(\Psi)|_{i>0} = \frac{(p^L)^2}{p^H} + \frac{(1-p^L)^2}{1-p^H}\left(\frac{[1-P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1-P(i)}\right) - 1 \tag{5}$$

$$\frac{\partial Var(\Psi)}{\partial i}|_{i>0} = \frac{\partial Var(\Psi)}{\partial P(i)}\frac{\partial P(i)}{\partial i} = \frac{(2P-1)}{(P-1)^2 P^2}\frac{(1-p^L)^2}{1-p^H}\frac{\partial P(i)}{\partial i} > 0 \tag{6}$$

Without monitoring ($i = 0$), because of $\left(\frac{[1-P(0)]^2}{P(0)} + \frac{[P(0)]^2}{1-P(0)}\right) = 1$,

$$Var(\Psi)|_{i=0} = \frac{(p^L)^2 - p^H}{p^H} + \frac{(1-p^L)^2}{1-p^H} = \frac{[p^H - p^L]^2}{p^H(1-p^H)} < Var(\Psi)|_{i>0} \tag{7}$$

As pointed out in the literature[7], $Var(\Psi)$ can be interpreted as a measure of the informativeness of the state of nature and the monitoring signal taken together on the true level of effort. As noted by Holmström (1979, 79) for the case of continuous distributions, here too Ψ measures how strongly one is inclined to infer from the outcome and the monitoring signal, that actual effort was not the agreed one.
b) c) and d):
The incentive compatibility constraint (IC_e) is equivalent to

$$p^H u(w_N) + (1-p^H)(P(i)u(w_H) + (1-P(i))u(w_L))$$
$$-p^L u(w_N) - (1-p^L)((1-P(i))u(w_H) + P(i)u(w_L)) = v(e^H) - v(e^L). \tag{8}$$

Using that for the chosen utility function it holds that $u(w) = 2\frac{1}{u'(w)}$ and substituting (2), (3) and (4) into the IC_e, one gets

$$2\frac{\lambda_1}{\lambda_2}Var(\Psi) = v(e^H) - v(e^L) = const. \tag{9}$$

As $Var(\Psi)$ increases in i, $\frac{\lambda_1}{\lambda_2}$ must decrease in i in order for the equality to hold (a movement "along" the incentive constraint). The implications for the incentives on effort depend on how λ_1 and λ_2 individually change with i. Next consider

$$Var(\frac{1}{u'(w)}) = E[(\frac{1}{u'(w)} - E(\frac{1}{u'(w)}))^2]. \tag{10}$$

$$E(\frac{1}{u'(w)}) = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}E(\Psi) = \frac{1}{\lambda_2}, \tag{11}$$

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[7]See e.g. Kim / Suh (1992) and Lambert (1985).

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as $E(\Psi) = 0$. Thus one gets that

$$Var(\frac{1}{u'(w)}) = (\frac{\lambda_1}{\lambda_2})^2 Var(\Psi) \,. \tag{12}$$

As $\frac{\lambda_1}{\lambda_2}$ decreases in i and $\frac{\lambda_1}{\lambda_2} Var(\Psi) = const$, it follows that $Var(\frac{1}{u'(w)})$ must decrease in i.

Further, one can use that $w = (\frac{1}{u'(w)})^2$ holds for the chosen utility function and substitute (2), (3) and (4) in order to calculate expected wealth $E(w|e^H)$. But expected wealth is constant for a given level of effort (zero-profit constraint) and, for the high effort level, it is equal to $W - (1 - p^H)L$. Thus, the zero-profit constraint is equivalent to

$$
\begin{aligned}
E(w|e^H) &= E[(\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}\Psi)^2] \\
&= (\frac{1}{\lambda_2})^2 + (\frac{\lambda_1}{\lambda_2})^2 Var(\Psi) \\
&= (\frac{1}{\lambda_2})^2 + Var(\frac{1}{u'(w)}) = W - (1 - p^H)L = const.
\end{aligned} \tag{13}
$$

From this equality it follows that $\frac{1}{\lambda_2}$ must increase in i. Using (11) and the fact that with the chosen function $E[u(w)] = 2E(\frac{1}{u'(w)})$, it follows that $\frac{1}{\lambda_2} = \frac{E[u(w)]}{2}$. Thus, the expected contractual utility increases in i and so does the efficiency of the contract (d). Using this, the above equality is equivalent to $Var(\frac{1}{u'(w)}) = E[w|e^H] - \frac{(E[u(w)])^2}{4}$. Denote by CE the certainty equivalent of the contract, so that $E[u(w)] = u(CE)$. Hence it holds that

$$\left(\frac{1}{\lambda_2}\right)^2 = \frac{(E[u(w)])^2}{4} = \frac{(u(CE))^2}{4} = \frac{(2\sqrt{CE})^2}{4} = CE \,. \tag{14}$$

Now it is obvious, that $Var(\frac{1}{u'(w)}) = E[w|e^H] - CE$ is the risk premium of the contract - denote it by $R(i)$, which measures how efficiently the risk is shared between the insurer and the insured - the smaller it is, the less risky is the contract from the viewpoint of the insured (c).

Even though the spread between the two possible net wealths in the loss state may increase with precision i, the probability of getting the larger net wealth w_H increases in i, which makes the individual better off.

As to b) one can conclude that as $\frac{1}{\lambda_2}$ increases and $\frac{\lambda_1}{\lambda_2}$ decreases in i, it must follow that the shadow price for implementing the higher effort level λ_1 must decrease in i, that is the contract provides better effort incentives.

The marginal effect of precision on expected contractual utility at the solution,

denoted by $h(i)$, is found by

$$\frac{\partial}{\partial i}E[u(w)] = \frac{\partial \mathcal{L}}{\partial i} =$$
$$\underbrace{\frac{\partial P(i)}{\partial i}(1-p^H)((u(w_H)-u(w_L))\lambda_2 \left(\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(1 + \frac{1-p^L}{1-p^H}) - \frac{d^H-d^L}{u(w_H)-u(w_L)}\right)}_{h(i)} \quad (15)$$

where, in accordance to the Envelope theorem, $\frac{\partial \mathcal{L}}{\partial i}$ is the partial derivative of the Lagrangian with respect to i. For $i \in (0,1)$, as $w_H > w_L$, its sign depends on the sign of the term in brackets in the above expression. Considering that

$$\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}\left(1 + \frac{1-p^L}{1-p^H}\right)$$
$$> \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}\left(1 - \frac{1-p^L}{1-p^H}\frac{1-P(i)}{P(i)}\right) = \frac{1}{u'(w_H)} > \frac{d^H-d^L}{u(w_H)-u(w_L)}, \quad (16)$$

where the first inequality follows from a comparison with (2) and the second inequality is due to the concavity of the utility function[8], it follows that $h(i) > 0$ for all $i \in (0,1)$. Thus,

$$\frac{\partial \mathcal{L}}{\partial i} = 0 \quad \text{for} \quad i = 0, \quad \text{because of} \quad w_H = w_L,$$
$$\frac{\partial \mathcal{L}}{\partial i} > 0 \quad \text{for} \quad i \in (0,1),$$
$$\frac{\partial \mathcal{L}}{\partial i} = 0 \quad \text{for} \quad i = 1 \quad \text{because of} \quad \frac{\partial P(i)}{\partial i}|_{i=1} = 0. \quad (17)$$

Expected utility would be highest with maximum precision of information $i = 1$, which result is the expected, when there is a costless additional signal, which is informative of effort[9].

In the particular case of $u(w) = 2\sqrt{w}$, the marginal effect of precision on expected contractual utility can be expressed by means of the Lagrange multipliers and $Var(\Psi)$ (see A2 (i) of the appendix), so that one obtains

$$\frac{\partial \mathcal{L}}{\partial i} = h(i) = \frac{\partial Var(\Psi)}{\partial i}\lambda_1\frac{\lambda_1}{\lambda_2}. \quad (18)$$

As is further shown in A2 (ii) of the appendix, this expression is also equivalent to

$$\frac{\partial \mathcal{L}}{\partial i} = u'\left(\underbrace{E(w|e^H) - R(i)}_{CE}\right) \cdot [-\frac{d\,R(i)}{d\,i}]. \quad (19)$$

Concerning the characteristics of the marginal expected utility, one can ascertain

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[8]For a concave function $u(w)$ with $w_H > w_L$ it holds that $u(w_H) - u(w_L) > u'(w_H)(w_H - w_L)$ and in addition $d_H - d_L = w_H - w_L$.

[9]For example see Holmström (1979, 74-91).

(as is also shown in A2 (iii) of the appendix), that

$$\frac{\partial^2 \mathcal{L}}{\partial i^2}\Big|_{i=0} > 0 \tag{20}$$

and

$$\frac{\partial^2 \mathcal{L}}{\partial i^2}\Big|_{i=1} < 0\,. \tag{21}$$

Thus, expected contractual utility is convex in i around $i = 0$ and there must be a region of i, in which it is concave.[10] Moreover, it is shown in A2 (iii) that around $i = 0$ the marginal value of information is itself concave in i.

$$\frac{\partial^3 \mathcal{L}}{\partial i^3}\Big|_{i=0} < 0\,. \tag{22}$$

$P'''(i) \leq 0$ is a sufficient condition for the marginal value of information to be globally concave.

3.2 Endogenous precision ex ante and privacy costs

In case that individuals incur privacy costs when they reveal information and the terms of the contract prescribe that the level of precision is to be determined ex ante, the optimal contracts are found by

$$\max_{r,d^H,d^L,i} p^H u(w_N) + (1 - p^H)\left(P(i)u(w_H) + (1 - P(i))u(w_L)\right)$$
$$-v(e^H) - (1 - p^H)g(i) \tag{23}$$

s.t.

$$p^H u(w_N) + (1 - p^H)\left[P(i)u(w_H) + (1 - P(i))u(w_L)\right] - v(e^H) - (1 - p^H)g(i) \geq$$
$$p^L u(w_N) + (1 - p^L)\left[(1 - P(i))u(w_H) + P(i)u(w_L)\right] - v(e^L) - (1 - p^L)g(i)$$
$$r - (1 - p^H)\left(P(i)d^H + (1 - P(i))d^L\right) \geq 0$$

Compared to the previous subsection nothing changes with the first order conditions with respect to d^H, d^L and r so that the expressions (2), (3) and (4) remain the same.

However, the marginal effect of precision on expected utility now becomes

$$\frac{\partial}{\partial i}\left[E[u(w)] - (1 - p^H)g(i)\right] = \frac{\partial \mathcal{L}}{\partial i} =$$
$$\underbrace{\frac{\partial P(i)}{\partial i}(1 - p^H)((u(w_H) - u(w_L))\lambda_2\left(\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(1 + \frac{1 - p^L}{1 - p^H}) - \frac{d^H - d^L}{u(w_H) - u(w_L)}\right)}_{h_p(i)}$$
$$\underbrace{-g'(i)((1 - p^H) - \lambda_1(p^H - p^L))}_{k(i)}\,. \tag{24}$$

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[10]Kim / Suh (1992) also derive this conclusion in proposition 2 of their paper.

For $u(w) = 2\sqrt{w}$ and using (18), the above expression is equivalent to

$$\frac{\partial \mathcal{L}}{\partial i} = \underbrace{\frac{\partial Var(\Psi)}{\partial i} \lambda_1 \frac{\lambda_1}{\lambda_2}}_{h_p(i)} \underbrace{-g'(i)((1 - p^H) - \lambda_1(p^H - p^L))}_{k(i)}. \tag{25}$$

Privacy costs affect the marginal effect of precision on expected utility in several different ways.

First consider $k(i)$. In A3 of the appendix it is shown that $(1 - p^H) - \lambda_1(p^H - p^L) > 0$ and thus $k(i)$ is negative for $i > 0$. This shows that restricting individuals to choose between a non-monitoring contract and exogenously fixed precision of monitoring $\bar{i}$ might result in $\frac{\partial \mathcal{L}}{\partial i}|_{\bar{i}} < 0$ for sufficiently large $g'(\bar{i})$, that is it might lead to individuals preferring the non-monitoring contract.

A closer look at $k(i)$ shows that, although it is negative on the whole, this term includes two opposite effects. There is the direct negative effect on expected utility $-(1 - p^H)g'(i)$, which stems from losing privacy in the loss-state, and which is the stronger one. But there is also an indirect positive effect, which is due to improved incentives on effort: $\lambda_1 g'(i)(p^H - p^L)$. Because privacy costs occur only in the state of loss, there is an additional incentive for individuals to avoid it. This positive effect of privacy costs reinforces the effect of a more efficient effort (larger difference between p^H and p^L).

Another way to interpret the positive effect of privacy costs on incentives is to look at the incentive constraint, which is equivalent to

$$p^H u(w_N) + (1 - p^H)(P(i)u(w_H) + (1 - P(i))u(w_L))$$
$$- p^L u(w_N) - (1 - p^L)((1 - P(i))u(w_H) + P(i)u(w_L))$$
$$= v(e^H) - v(e^L) - (p^H - p^L)g(i). \tag{26}$$

Thus, privacy costs effectively decrease the difference between the disutilities of effort and thereby make the implementation of the high effort level e^H compared to the low effort level e^L more attractive. For $u(w) = 2\sqrt{w}$ the incentive constraint is also equivalent to

$$2\frac{\lambda_1}{\lambda_2}Var(\Psi) = v(e^H) - v(e^L) - (p^H - p^L)g(i). \tag{27}$$

When i increases, the right hand side of the above equation decreases. A comparison with the IC_e in the benchmark situation (9) shows that now $\frac{\lambda_1}{\lambda_2}$ has to decrease in i even more. In consequence, the effect of an increase of precision i on risk sharing, i.e. on $Var(\frac{1}{u'(w)})$, will be stronger than in the benchmark situation. From the zero-profit constraint in turn, it follows that with privacy costs $\frac{1}{\lambda_2}$, and thus the expected contractual utility, increases, and λ_1, i.e. the shadow price of incentives, decreases more strongly in i than in the benchmark situation.

To restate this result, denote all variables for the case of privacy costs by the index p. Then one obtains that for any given level of precision it will hold that $\frac{\lambda_1}{\lambda_2} > \frac{\lambda_1^p}{\lambda_2^p}$ by virtue of the incentive constraints and, by combining this with the zero-profit

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constraints, it follows that the risk premium with privacy costs will be smaller than without, $R(i) > R_p(i)$. Hence, it follows directly, that for the same level of monitoring precision, the expected utility from net wealth will be larger if there are privacy costs, $E[u(w)] = 2\frac{1}{\lambda_2} < 2\frac{1}{\lambda_2^p} = E[u(w_p)]$. That is, privacy costs strengthen the effect of monitoring precision on incentives, risk-sharing and contractual utility.

Although there is some positive effect of privacy costs, with the results that λ_1^p and λ_2^p are smaller for any given level of precision as compared to the benchmark situation, it also follows that $h_p(i) < h(i)$ (in (25) and (18)). Thus, all in all, with privacy costs the marginal effect of precision on total expected utility will be smaller for every level of precision i, $\frac{\partial \mathcal{L}_p}{\partial i} < \frac{\partial \mathcal{L}}{\partial i}$. This also implies that $\int_0^{\bar{i}} \frac{\partial \mathcal{L}_p}{\partial i} di|_{with\, g(\cdot)} < \int_0^{\bar{i}} \frac{\partial \mathcal{L}}{\partial i} di|_{ref.sit}$ for all $\bar{i} \in (0,1]$, which is equivalent to $E[U(w_p, e^H, i)] < E[U(w, e^H, i)]$. That is, the existence of privacy costs will decrease welfare as compared to the benchmark situation without privacy costs.

Nevertheless, if an interior solution for the level of precision exists, individuals will prefer some monitoring. As was shown in the benchmark situation, the marginal value of precision, if there are no privacy costs, is concave in i around $i = 0$ and globally concave if $P'''(i) \leq 0$. The marginal privacy cost function is increasing and therefore, in order for an interior solution for i to exist in this scenario, it must hold that

$$\frac{d^2 \mathcal{L}}{d\, i^2}|_{i=0} > 0 \,, \tag{28}$$

which, by using the results of A2 (iii) of the appendix, is equivalent to

$$\frac{\frac{\lambda_1^2}{\lambda_2} \cdot \frac{\partial^2 Var(\Psi)}{\partial P^2} \cdot \left(\frac{\partial P}{\partial i}\right)^2}{1 - p^H - \lambda_1(p^H - p^L)} > g''(0) \,, \tag{29}$$

where all variables in the above expression are evaluated at $i = 0$. This condition also ensures that $i = 0$ will not be optimal. Maximum precision, i.e. $i = 1$, will never be optimal, as $h_p(1) = 0$ in (24) and thus, $\frac{\partial \mathcal{L}}{\partial i}|_{i=1} < 0$. Moreover, if $P'''(i) \leq 0$, the interior solution for i found by the first-order condition will be unique.

The above results are summarized as follows:

Proposition 1: With monitoring conditional on the loss and privacy costs (i) individuals will choose some level of precision $i \in (0,1)$, if $g''(0)$ is not too large. (ii) Privacy costs reinforce the incentives on effort $\lambda_1^p < \lambda_1$ and (iii) improve the efficiency of allocating the risk between the insured and the insurer, $\lambda_2^p < \lambda_2$. (iv) Thereby a higher expected utility from net wealth results, $E[u(w_p)] > E[u(w)]$. (v) However, on the whole the existence of privacy costs reduces the expected total utility, $E[U(w_p, e^H, i)] < E[U(w, e^H, i)]$.

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3.3 Endogenous precision ex post

In this scenario the insurer and the insured do not agree on the level of precision ex ante when the contract is signed. Instead, the individual is allowed to decide on the precision of the information at the time he submits the black box to the insurer. With improved monitoring technologies in the future this could be the natural continuation of the option that insureds currently have, to decide whether or not to reveal information only after it is collected. Although at first sight this scenario might seem beneficial from the viewpoint of the insured, in order for the contract to work, it is necessary to include an incentive constraint with respect to the level of precision i, which imposes a further restriction on the contract. In a first stage the insurer sets the contract d^H, d^L, r. In a second stage the individual chooses the precision of information for a given contract.

Stage 2: In case that a loss occurred, the individual chooses i to maximize $E[u(w)|\text{Loss}] - g(i)$ for given d^H, d^L, r. The first order condition is:

$$\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] - g'(i) = 0 \text{ for } i > 0$$
$$\leq 0 \text{ for } i = 0. \tag{30}$$

Given that $w_H > w_L$, there will be an interior solution for i at stage 2. Only if $w^H \leq w^L$, will the individual choose $i = 0$ at this stage.

In the first stage the insurer sets the contract according to his expectation about the precision the individual will choose. If the insurer expects the insured to prefer a non-monitoring contract $i = 0$, the maximization problem is the standard non-monitoring one and it results in $w_H = w_L$. For $i = 0$ the insurer will never set $w_H < w_L$, so that this case need not be regarded. Hence, for $i \geq 0$, $w_H \geq w_L$ and the optimal contract can be found by adding the above first order condition as an incentive constraint with respect to i (first-order approach[11]) to the optimization problem[12]:

Stage 1:

$$\max_{r, d^H, d^L, i} p^H u(w_N) + (1 - p^H)(P(i)u(w_H) + (1 - P(i))u(w_L))$$
$$-v(e^H) - (1 - p^H)g(i) \tag{31}$$

s.t.

$$p^H u(w_N) + (1 - p^H)(P(i)u(w_H) + (1 - P(i))u(w_L)) - v(e^H) - (1 - p^H)g(i) \geq$$
$$p^L u(w_N) + (1 - p^L)((1 - P(i))u(w_H) + P(i)u(w_L)) - v(e^L) - (1 - p^L)g(i)$$
$$r - (1 - p^H)(P(i)d^H + (1 - P(i))d^L) \geq 0$$
$$\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] - g'(i) = 0$$

---

[11]See e.g. Rogerson (1985) for a justification of this approach in the single-signal case and Jewitt (1988) and Sinclair-Desgagné (1994) for multi-signal cases.

[12]As $w_H < w_L$ will never result, it is not necessary to include an additional constraint requiring that $w_H \geq w_L$.

Let λ_3 be the Lagrange multiplier for the incentive constraint with respect to i (IC_i). In A4 of the appendix it is shown that $\lambda_1, \lambda_2 > 0$ for all i and $\lambda_3 \geq 0$ for $i > 0$. From the first order conditions with respect to d^H, d^L and r the following expressions result[13]:

$$\frac{1}{u'(w_H)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2} \underbrace{(1 - \frac{1-p^L}{1-p^H}\frac{1-P(i)}{P(i)})}_{\Psi_H} + \frac{\lambda_3}{\lambda_2} \underbrace{\frac{\frac{\partial P(i)}{\partial i}}{P(i)}}_{\Omega_H} \tag{32}$$

$$\frac{1}{u'(w_L)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2} \underbrace{(1 - \frac{1-p^L}{1-p^H}\frac{P(i)}{1-P(i)})}_{\Psi_L} + \frac{\lambda_3}{\lambda_2} \underbrace{\frac{-\frac{\partial P(i)}{\partial i}}{1-P(i)}}_{\Omega_L} \tag{33}$$

$$\frac{1}{u'(w_N)} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2} \underbrace{(1 - \frac{p^L}{p^H})}_{\Psi_N} + \frac{\lambda_3}{\lambda_2} \underbrace{0}_{\Omega_N} \tag{34}$$

As in the previous section, Ψ serves to infer the level of effort from the particular outcome and monitoring signal taken together. Ω, on its part, is an indicator of the chosen precision. The larger $\frac{\partial P(i)}{\partial i}$ is (which in fact implies a higher efficiency of the monitoring technology), the stronger are the incentives on precision[14], but it also implies a larger spread between w_H and w_L, which deteriorates risk sharing.
The first order condition with respect to i is

$$\frac{\partial \mathcal{L}}{\partial i} = \underbrace{(1-p^H)\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] - (1-p^H)g'(i)}_{=0}$$

$$+\lambda_1 \underbrace{((1-p^H)\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] - (1-p^H)g'(i)}_{=0}$$

$$+(1-p^L)\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] + (1-p^L)g'(i))$$

$$-\lambda_2(1-p^H)\frac{\partial P(i)}{\partial i}(d^H - d^L)$$

$$+\lambda_3\left(\frac{\partial^2 P(i)}{\partial i^2}[u(w_H) - u(w_L)] - g''(i)\right) \leq 0,\ i \geq 0,\ \frac{\partial \mathcal{L}}{\partial i}i = 0\,. \tag{35}$$

Bearing in mind that $\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] - g'(i) = 0$ must hold in the optimum, the expression $(1-p^L)\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] + (1-p^L)g'(i)$ can be replaced with

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[13]As in the benchmark situation it is assumed that an interior solution with $w_L > a$ exists.
[14]See also Holmström (1979, 79).

$2(1 - p^L)\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)]$, so that the first order condition reduces to

$$\frac{\partial \mathcal{L}}{\partial i} = \frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)](1 - p^H)\lambda_2 \left(2\frac{\lambda_1}{\lambda_2}\frac{1 - p^L}{1 - p^H} - \frac{d^H - d^L}{u(w_H) - u(w_L)} \right)$$

$$+ \lambda_3 \left(\frac{\partial^2 P(i)}{\partial i^2}[u(w_H) - u(w_L)] - g''(i) \right) \le 0, \ i \ge 0, \ \frac{\partial \mathcal{L}}{\partial i}i = 0. \qquad (36)$$

For $i > 0$ and $\lambda_3 > 0$ the second term is negative - these are the additional costs from not being able to set i ex ante, because by choosing d^H, d^L, r the principal has to create incentives not only with respect to effort, but also with respect to the precision of information.[15] Although at the time of revealing the data, the agent also reveals the level of precision, effectively the situation with this contract scheme is as if the principal could not observe the level of precision. The reason for this result is that at the time the contract is signed, the principal has to commit to a certain contract (r, d^H, d^L), which cannot be renegotiated afterwards. However, at this point in time the agent does not commit to a certain level of precision. Therefore, the principal has to anticipate the level of precision and set the contract in a way that takes into account the agent's interest once after he has signed the contract.

In order to get an idea of the effects of precision on the expected utility in this scenario, the same considerations as in the previous sections can be applied for $u(w) = 2\sqrt{w}$. In A5 of the appendix it is shown that for small values of i the risk premium will decrease in i. Therefore a marginal increase of precision, if precision is low, will increase the expected contractual utility. However, for high values of i, the risk premium will increase in i. That is, with a further increase of the level precision, the expected contractual utility will decrease. This result makes the difference compared to the second scenario: when i is agreed upon ex ante, the expected contractual utility $E[u(w)]$ increases in i for all values of i. When the insured chooses i ex post, the marginal effect of precision on the expected contractual utility is similar as in the second scenario only for small values of i. The positive effect of improved effort incentives on expected contractual utility prevails. However, due to the additional information asymmetry (concerning the precision of monitoring), the contract imposes an additional risk on the individual. For large values of i the unfavorable effect of this additional risk outweighs the beneficial incentive effect on effort so that the marginal effect of precision on expected contractual utility becomes negative.

On the whole the effect of this additional information asymmetry on welfare, which results in equilibrium with ex post precision, as compared to the equilibrium welfare with precision ex ante, will be negative. In the following proposition this is shown explicitly for $u(w) = 2\sqrt{w}$.

Proposition 2: Determining ex post the level of i deteriorates risk sharing and reduces the efficiency of the contract.
Proof: see A7 of the appendix.

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[15]The optimal level of precision is determined by (30) whereas the value of λ_3 is obtained from (36).

4 Signal based premiums and indemnities

As was mentioned in the introduction, some insurers offer contracts with monitoring taking place in all states of nature. The collected data are analyzed in the end of a billing period no matter if an accident has occurred or not. According to the driving performance inferred from the records, the premium for the next period is determined. In order to reflect this way of designing the contract in a one-period model, it is assumed that individuals pay the insurance premium for the current period not until the data are transferred to the insurer, i.e. the contract is a binding promise for the insured to pay the premium in the end of the period. Although this assumption concerning the premiums might seem critically unrealistic, a repetition of the same period would yield each time the same results, so that in that case the premiums could be regarded as applying to the next period. This contract scheme is the most straightforward one as it imposes no restrictions on monitoring. In other words, this contract makes unrestricted use of both the outcome and monitoring as signals of the exerted effort and both the premiums and the indemnities can be made dependent on the monitoring signal. The time structure is depicted in Fig.2. The analysis is analogous as for the first contract scheme, so that only the main results will be sketched briefly.

$$
\begin{array}{ll}
-r^H & \text{if NoL}/s^H \\
-r^H + d^H & \text{if L}/s^H \\
-r^L & \text{if NoL}/s^L \\
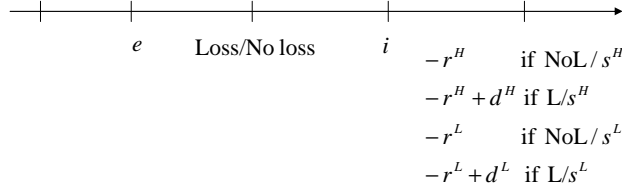-r^L + d^L & \text{if L}/s^L
\end{array}
$$

Figure 2: time structure - basing premiums and indemnities on the signal

To implement the high effort level, the maximization problem with this contract scheme, and with exogenous precision and no privacy costs, is

$$
\max_{r^H, r^L, d^H, d^L} p^H[P(i)u(w_{NH}) + (1 - P(i))u(w_{NL})] + (1 - p^H)[P(i)u(w_{AH})
$$
$$
+ (1 - P(i))u(w_{AL})] - v(e^H) \tag{37}
$$

s.t.

$$
p^H[P(i)u(w_{NH}) + (1 - P(i))u(w_{NL})] + (1 - p^H)[P(i)u(w_{AH}) +
$$
$$
(1 - P(i))u(w_{AL})] - v(e^H) \geq p^L[(1 - P(i))u(w_{NH}) + P(i)u(w_{NL})]
$$
$$
+ (1 - p^H)[(1 - P(i))u(w_{AH}) + P(i)u(w_{AL})] - v(e^L)
$$
$$
P(i)r^H + (1 - P(i))r^H - (1 - p^H)[P(i)d^H + (1 - P(i))d^L] \geq 0,
$$

where $w_{NH} = W - r^H$, $w_{NL} = W - r^L$, $w_{AH} = W - L - r^H + d^H$, $w_{AL} = W - L - r^L + d^L$. After some transformation and for interior solutions, the first order conditions with respect to d^H, d^L, r^H and r^L are equivalent to

$$\frac{1}{u'(w_{AH})} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{1-p^L}{1-p^H}\frac{1-P(i)}{P(i)}}_{\Theta_{AH}}) \tag{38}$$

$$\frac{1}{u'(w_{AL})} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{1-p^L}{1-p^H}\frac{P(i)}{1-P(i)}}_{\Theta_{AL}}) \tag{39}$$

$$\frac{1}{u'(w_{NH})} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{p^L}{p^H}\frac{1-P(i)}{P(i)}}_{\Theta_{NH}}) \tag{40}$$

$$\frac{1}{u'(w_{NL})} = \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(\underbrace{1 - \frac{p^L}{p^H}\frac{P(i)}{1-P(i)}}_{\Theta_{NL}}). \tag{41}$$

A comparison of the above expressions shows that (i) $w_{NH} > w_{NL} > w_{AH} > w_{AL}$, if $\frac{p^H(1-p^L)}{p^L(1-p^H)} > \frac{P^2}{(1-P)^2}$, and (ii) $w_{NH} > w_{AH} > w_{NL} > w_{AL}$, if $\frac{p^H(1-p^L)}{p^L(1-p^H)} < \frac{P^2}{(1-P)^2}$. In the first case, as efficiency of effort is relatively high and the efficiency of the monitoring technology relatively low, the net wealth in the loss state will be smaller than that in the no-loss state irrespectively of the signal. Partial damage insurance is still the more effective way of setting incentives on effort. In the second case, as the efficiency of monitoring is relatively high and the efficiency of effort relatively low, the loss net wealth with the good signal (w_{AH}) is larger than the no-loss net wealth with the bad signal (w_{NL}), i.e. monitoring conveys more information on the chosen effort than the occurrence of the loss does and accordingly it has a stronger influence on the payment. Moreover, the above condition for the efficiency of precision is less restrictive than in the first contract scheme as $\frac{P^2}{(1-P)^2} > \frac{P}{1-P}$, i.e., because monitoring takes place regardless of the occurrence of a loss, the technology need not be as precise as in the first contract scheme in order for the payments to rely relatively more heavily on the monitoring signal as compared to the occurrence of the loss. Note, however, that in contrast to the first contract scheme, with which it was possible to have $d^H > L$, here both d^L and d^H are always below the value of the loss $d^H < L$. One can also ascertain, that for $P(i) \to 1$ (when the efficiency of monitoring is very high), the contract will approach full damage insurance, $w_{AH} \to w_{NH}$.[16]

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[16]In fact it can be shown that conditional monitoring will also approach full damage insurance if the precision of monitoring approaches perfect one. Even though with conditional monitoring $d^H > L$ for high values of $P(i)$, it is still true that $d^H \to L$ for $P(i) \to 1$. One can easily verify by

The marginal effect of precision on expected utility, transformed as a function of net wealth, is

$$\frac{\partial \mathcal{L}}{\partial i} = \frac{\partial P(i)}{\partial i}(1 - p^H)((u(w_{AH}) - u(w_{AL}))\lambda_2 \cdot$$

$$\cdot \left(\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(1 + \frac{1 - p^L}{1 - p^H}) - \frac{w_{AH} - w_{AL}}{u(w_{AH}) - u(w_{AL})} \right)$$

$$+ \frac{\partial P(i)}{\partial i}(1 - p^H)((u(w_{NH}) - u(w_{NL}))\lambda_2 \cdot$$

$$\cdot \left(\frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2}(1 + \frac{1 - p^L}{1 - p^H}) - \frac{w_{NH} - w_{NL}}{u(w_{NH}) - u(w_{NL})} \right) . \tag{42}$$

and by the same arguments as before, it can be shown that $\frac{\partial \mathcal{L}}{\partial i} > 0$ for $i \in (0, 1)$. For $u(w) = 2\sqrt{w}$ the following expressions and equations are obtained, which will be used for the comparison of the different contract schemes:

$$\frac{\partial \mathcal{L}}{\partial i} = \left(\frac{\lambda_1^2}{\lambda_2} \right) \frac{\partial Var(\Theta)}{\partial i}, \tag{43}$$

$$2\frac{\lambda_1}{\lambda_2}Var(\Theta) = v(e^H) - v(e^L), \tag{44}$$

$$Var(\frac{1}{u'(w)}) = (\frac{\lambda_1}{\lambda_2})^2 Var(\Theta), \tag{45}$$

and

$$E(w|e^H) = (\frac{1}{\lambda_2})^2 + Var(\frac{1}{u'(w)}) = W - (1 - p^H)L, \tag{46}$$

where

$$Var(\Theta)|_{i>0} = \left(\frac{(p^L)^2}{p^H} + \frac{(1 - p^L)^2}{1 - p^H} \right) \left(\frac{[1 - P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1 - P(i)} \right) - 1, \tag{47}$$

$$Var(\Theta)|_{i=0} = \frac{[p^H - p^L]^2}{p^H(1 - p^H)} = Var(\Psi)|_{i=0}, \tag{48}$$

and

$$\frac{\partial Var(\Theta)}{\partial i} = \frac{\partial Var(\Theta)}{\partial P(i)}\frac{\partial P(i)}{\partial i} = \frac{(2P - 1)}{(P - 1)^2 P^2} \left(\frac{(p^L)^2}{p^H} + \frac{(1 - p^L)^2}{1 - p^H} \right) \frac{\partial P(i)}{\partial i} > 0. \tag{49}$$

Now consider the case of determining precision ex ante in the presence of privacy costs. With this contract scheme privacy costs arise with certainty in the end of

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considering that $\lambda_1 \to 0$ as $P(i) \to 1$.

the period. As a result, the maximization problem (37) changes only in that $g(i)$ is subtracted from the objective function. The incentive constraint is unchanged as $g(i)$ is subtracted from both sides of it. The first order conditions with respect to r^H, r^L, d^H and d^L are also unchanged. The first order condition with respect to precision becomes for $i > 0$

$$\frac{\partial \mathcal{L}}{\partial i} = \left(\frac{\lambda_1^2}{\lambda_2}\right)\frac{\partial Var(\Theta)}{\partial i} - g'(i) = 0 \,. \tag{50}$$

Compared to the first contract scheme, the negative effect of privacy costs on expected utility as a result of a marginal increase of precision is larger. On the one hand, because privacy is lost in both states of nature, the direct negative effect from losing privacy is larger than in the first contract scheme. On the other hand, there is no positive incentive effect on effort.

5 Comparison of the contract schemes

Note that the alternative contract schemes have a different number of states of nature so that a comparison of the resulting welfare using a general utility function would be difficult, if not impossible. By using the particular function $u(w) = 2\sqrt{w}$, the optimality conditions for each contract scheme can be represented as functions of the Lagrange multipliers and the level of precision, i.e. of the same number of variables. This permits some ordering of the resulting values and drawing some conclusions about the resulting welfare. Let the letter I stand for the first contract scheme with conditional monitoring (where only the *indemnities* are based on the signal), and B stand for the second contract scheme with unrestricted monitoring (where *both* the premiums and the indemnities depend on the signal). From the analysis of the benchmark situation, which was performed for each contract scheme, it is obvious that a comparison of the resulting welfare for a given level of precision i boils down to comparing the values of

$$Var(\Psi) = \frac{(p^L)^2}{p^H} + \frac{(1 - p^L)^2}{1 - p^H}\left(\frac{[1 - P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1 - P(i)}\right) - 1 \tag{51}$$

and

$$Var(\Theta) = \left(\frac{(p^L)^2}{p^H} + \frac{(1 - p^L)^2}{1 - p^H}\right)\left(\frac{[1 - P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1 - P(i)}\right) - 1 \,. \tag{52}$$

The larger the variance with a given level of precision is, the higher is the expected contractual utility and consequently the welfare with the respective contract scheme. For $i = 0$, $Var(\Psi) = Var(\Theta) = \frac{(p^H - p^L)^2}{p^H(1 - p^H)}$.

As was shown in the corresponding sections, both variances increase in precision, but a direct comparison of the above expressions (and bearing in mind that $\frac{[1 - P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1 - P(i)} \geq 1$ increases in i) shows that $Var(\Theta) > Var(\Psi)$ for any level of i. The dif-

ference between those two variances $Var(\Theta) - Var(\Psi) = \frac{(p^L)^2}{p^H}\left(\frac{[1-P(i)]^2}{P(i)} + \frac{[P(i)]^2}{1-P(i)}\right)$ grows with the precision of the monitoring technology and decreases with the efficiency of effort (as effort becomes more efficient, the term $\frac{(p^L)^2}{p^H}$ decreases). This result is not surprising, as it exemplifies the well known principle, that both monitoring and outcome should be used to design the contract, if they are both informative of effort and if they are costless (see Holmström, 1979). When the precision of monitoring increases, the monitoring signal becomes relatively more important for the contract in its being informative of effort, and thus the advantages of the contract scheme B, in which monitoring takes place in all states of nature, are reinforced as compared to the first contract scheme I, in which monitoring is performed only in the loss state. And the other way round, when the efficiency of effort increases, the outcome as a signal of effort gains importance and the advantages of unrestricted monitoring (B) over conditional monitoring (I) are weakened.

As was noted in the previous sections, including privacy costs will affect the optimal level of precision and, in case of conditional monitoring, also the incentives on effort. As was argued above, $Var(\Theta) > Var(\Psi)$, $\forall i \in (0,1]$, i.e., the incentive and risk sharing effects of monitoring precision itself are stronger in the case of B. However, this contract scheme B lacks the additional incentive and risk sharing effects of privacy costs, which are characteristic of the first contract scheme I, and moreover, for the same level of precision, the expected privacy costs in the second contract scheme B are larger than in the first one I.

Denote by i_B^* the optimal level of precision for the second contract scheme B. The resulting total expected utility, net of the disutility of effort, is thereby $E[u(w_B)] - g(i_B^*)$. Denote by $E[u(w_I)]|_{i=i_B^*} - (1-p^H)g(i_B^*)$ the total expected utility which is attained by the first contract scheme I at the same level of precision i_B^*. In $A8$ of the appendix it is shown that

Proposition 3: $E[u(w_I)]|_{i=i_B^*} - (1-p^H)g(i_B^*) > E[u(w_B)] - g(i_B^*)$, which is a sufficient condition for the equilibrium welfare with conditional monitoring (I) to be larger than the equilibrium welfare with unrestricted monitoring (B), if

$$\underbrace{2\sqrt{CE_I|_{i_B^*}} - 2\sqrt{CE_B|_{i_B^*}} + p^H \cdot g(i_B^*)}_{F} > 0 \,, \tag{53}$$

where

$$CE_I|_{i_B^*} = E(w|e^H) - \frac{[\Delta v - (p^H - p^L)g(i_B^*)]^2}{4Var(\Psi)|_{i_B^*}} \,, \tag{54}$$

$$CE_B|_{i_B^*} = E(w|e^H) - \frac{(\Delta v)^2}{4Var(\Theta)|_{i_B^*}} \,, \tag{55}$$

and $\Delta v = v(e^H) - v(e^L)$.

The above condition is more likely to be satisfied, the larger the privacy costs at

the equilibrium level of precision i_B^*, the smaller the probability of loss, the higher the efficiency of effort, the lower the efficiency of precision and the smaller the difference between the disutilities of effort are. A larger value of privacy costs $g(i_B^*)$ implies that the advantage of conditional monitoring, by incurring those costs only in the case of loss $(1 - p^H)g(i_B^*)$, is reinforced (direct effect of privacy costs). Moreover, thereby the incentive and risk-sharing effects of privacy costs on the contract with conditional monitoring I become stronger (indirect effects of privacy costs). A smaller probability of loss essentially works in the same direction. A higher efficiency of effort $(p^H - p^L)$ reinforces the incentive effect of privacy costs with conditional monitoring I, and also weakens the advantage of a larger value for $Var(\Theta)$ as compared to $Var(\Psi)$. A lower efficiency of precision, i.e. a smaller value of $P(i)$ for a given i, also weakens the advantage of $Var(\Theta)$ being larger than $Var(\Psi)$ and indirectly increases the costs of implementing a given level of precision, which again speaks for conditional monitoring I. With a smaller value of Δv, which reduces the costs of implementing the high effort level, also a lower level of precision is sufficient to induce the same incentive and risk sharing effects. This in turn reinforces the relative effect of privacy costs on incentives and risk sharing in the contract scheme with conditional monitoring I.

As was mentioned already, with conditional monitoring I, privacy costs generate an incentive, a risk-sharing and a direct effect on welfare. If the same level of precision generates better incentives with I as compared to B, also risk sharing will be better and hence the expected contractual utility will be larger. By incurring lower expected privacy costs as compared to B, also the total expected utility with I will be larger. But even if incentives and risk sharing are worse(and thus expected contractual utility is smaller) with I, due to the smaller expected privacy costs, total expected utility might still be larger.

6 Concluding remarks

This paper considers the problem of moral hazard in perfectly competitive insurance markets when voluntary monitoring technologies are available and the insureds may choose the comprehensiveness of monitoring data and thereby the level of precision. Also privacy costs incurred to individuals when revealing information are taken into account. Two alternative insurance contract schemes are compared in terms of welfare. The first scheme prescribes monitoring conditional on the loss and that only the insurance indemnities depend on the monitoring signal. The second contract scheme provides unrestricted monitoring and thus completely involves both the outcome and the monitoring signal into the sharing rule. Both the indemnities and the premiums are allowed to depend on the monitoring signal.

If there are no privacy costs, maximum amount of data will be optimal for any contract scheme. For the same level of precision, the second contract scheme attains a higher welfare as it unrestrictedly uses the informativeness of both the outcome and monitoring.

With privacy costs some positive level of monitoring will be optimal for any contract scheme, unless the privacy costs increase too fast in relation to the precision of

monitoring. In the presence of privacy costs, the contract scheme with conditional monitoring has some advantages over unrestricted monitoring. On the one hand, the expected privacy costs are smaller because they are incurred only in the state of loss. On the other hand, privacy costs generate an additional incentive effect on effort and a positive effect on the allocation of risk. When the monitoring costs occur only in the state of loss, individuals have an additional incentive to take care in order to avoid them. The risk borne by the insured is reduced because, for a given level of monitoring precision, when there are privacy costs, a smaller proportion of the risk needs to be transferred to the individual in order to provide him with the same incentives on effort compared to when there are no privacy costs. Because the second contract scheme lacks these positive effects of privacy costs, there will be conditions under which, despite of its better use of information, it will be welfare dominated by the contract scheme with conditional monitoring. In particular, this result is likely to occur if: the efficiency of effort is high, the probability of loss is small, the efficiency of the monitoring technology is low, and the optimal privacy costs resulting with unrestricted monitoring are large.

Even though there is no evidence yet concerning the dimensions of the costs, which are incurred when individuals lose their privacy of driving behavior, there are some reasons to believe that the remaining prerequisites for conditional monitoring to be welfare optimal apply in practice. As noted by Lemaire (1998), the probabilities of loss can be approximated by the claim frequency, which average for Belgium is about 10%. This value is probably small enough to speak in favor of conditional monitoring. There is also some evidence that moral hazard, which is a manifestation of a positive efficiency of effort, is truly an issue in automobile insurance markets. For the French automobile insurance market, Dionne / Michaud / Dahchour (2004) find that, as a result of the stronger incentives created by switching from all-risk to third-party coverage, insureds' probability for filing a claim in a year decreases by 5.9 percentage point. There are also further findings that indicate an existent efficiency of effort in automobile insurance. Dionne / Maurice / Pinquet / Vanasse (2005) also find that driver's risk significantly depends on care. The question of how well the data, measured by the contemporary monitoring systems, are suited to approximate the true risk of the individual, is yet to be examined. Undoubtedly, monitoring technologies will continually improve in the future but, at least currently they are probably far from being perfect. These arguments imply that, in the presence of privacy costs, monitoring conditional on the loss is likely to be welfare maximizing. Finally, even though bonus-malus systems, which are applied in many countries to cope with information asymmetries, cannot cause a loss of privacy, it should be taken into account that, depending on the particular design of the bonus-malus system, for good risks to reach their steady state it can take up to thirty years (see Lemaire (1998)).

A further implication of the results in this paper is that, for a given level of monitoring precision, a contract scheme which imposes conditional monitoring will attain a higher expected utility from welfare if individuals incur privacy costs as opposed to a situation in which individuals do not care about the loss of privacy. However, the overall welfare of individuals will still be lower in the presence of privacy costs.

Because the privacy cost functions in different types of insurance markets might differ and are generally unknown, inferring the efficiency of the respective contracts only on the basis of observable risk sharing might be misleading. Even if a contract offers more coverage and therefore better risk sharing, it need not imply that the precision of monitoring and welfare with this contract is higher. On the one hand, better risk sharing in a market can be equally well explained by a lower precision of monitoring, however accompanied by faster increasing privacy costs. On the other hand, even if the precision of monitoring is the same in both types of insurance markets, in the case of faster increasing privacy costs, risk sharing will indeed be better but the overall welfare of the individual will still be smaller.

Another result in this paper is that letting the individuals choose the precision of the monitoring technology at the time they reveal the data is inefficient. More generally, this result also suggests that, contracts which allow individuals to choose whether or not to reveal information only after they collect and review the data, cannot be expected to persist in the long term.

Appendix

A1: $\lambda_1 > 0$ and $\lambda_2 > 0$.

By combining the first order conditions $\frac{\partial \mathcal{L}}{\partial d^H} = 0, \frac{\partial \mathcal{L}}{\partial d^L} = 0$ and $\frac{\partial \mathcal{L}}{\partial r} = 0$, $r > 0$, one obtains

$$p^H \lambda_2 = p^H u'(W - r) + \lambda_1(p^H - p^L)u'(W - r). \tag{56}$$

The right hand-side of this equality is positive and so must the left hand-side. Hence, $\lambda_2 > 0$. Assume that $\lambda_1 = 0$. From the first order conditions it follows that $w_H = w_L = w_N$, which is not incentive compatible. Therefore $\lambda_1 > 0$.

A2:

(i) Using (2), (3) and (4) one obtains

$$\frac{d^H - d^L}{u(w_H) - u(w_L)} = \frac{w_H - w_L}{u(w_H) - u(w_L)} = \frac{\left(\frac{1}{u'(w_H)}\right)^2 - \left(\frac{1}{u'(w_L)}\right)^2}{2\left(\frac{1}{u'(w_H)} - \frac{1}{u'(w_L)}\right)}$$

$$= \frac{1}{2}\left(\frac{1}{u'(w_H)} + \frac{1}{u'(w_L)}\right)$$

$$= \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2} - \frac{1}{2}\frac{\lambda_1}{\lambda_2}\frac{1 - p^L}{1 - p^H}\frac{[1 - P(i)]^2 + [P(i)]^2}{P(i)[1 - P(i)]} \tag{57}$$

and

$$u(w_H) - u(w_L) = 2\frac{\lambda_1}{\lambda_2}\frac{1 - p^L}{1 - p^H}\left(\frac{P(i)}{1 - P(i)} - \frac{1 - P(i)}{P(i)}\right) \tag{58}$$

which can be substituted into $h(i)$. After some transformation the marginal effect of precision is equivalent to

$$\frac{\partial \mathcal{L}}{\partial i} = \frac{\partial P(i)}{\partial i}\lambda_1\frac{2P(i) - 1}{P(i)[1 - P(i)]}\frac{\lambda_1}{\lambda_2}\frac{(1 - p^L)^2}{1 - p^H}\frac{1}{P(i)[1 - P(i)]} \tag{59}$$

Now using (6), one obtains for the marginal effect of precision on expected utility

$$\frac{\partial \mathcal{L}}{\partial i} = h(i) = \frac{\partial Var(\Psi)}{\partial i}\lambda_1\frac{\lambda_1}{\lambda_2} \tag{60}$$

(ii) Having that $R(i) = \left(\frac{\lambda_1}{\lambda_1}\right)^2 Var(\Psi)$, it follows that

$$\frac{d\,R(i)}{d\,i} = 2\frac{\lambda_1}{\lambda_2}\frac{d}{d\,i}\left(\frac{\lambda_1}{\lambda_2}\right)Var(\Psi) + \left(\frac{\lambda_1}{\lambda_2}\right)^2\frac{\partial Var(\Psi)}{\partial i} \tag{61}$$

Taking the total differential of IC_e (9), one obtains

$$2\frac{d}{di}\left(\frac{\lambda_1}{\lambda_2}\right)Var(\Psi) + 2\frac{\lambda_1}{\lambda_2}\frac{\partial Var(\Psi)}{\partial i} = 0 \tag{62}$$

Thus,

$$\frac{d}{di}\left(\frac{\lambda_1}{\lambda_2}\right)Var(\Psi) = -\frac{\lambda_1}{\lambda_2}\frac{\partial Var(\Psi)}{\partial i} \tag{63}$$

This can be substituted into $\frac{d\,R(i)}{d\,i}$, which becomes

$$\frac{d\,R(i)}{d\,i} = 2\frac{\lambda_1}{\lambda_2}\left(-\frac{\lambda_1}{\lambda_2}\frac{\partial Var(\Psi)}{\partial i}\right) + \left(\frac{\lambda_1}{\lambda_2}\right)^2\frac{\partial Var(\Psi)}{\partial i} = -\left(\frac{\lambda_1}{\lambda_2}\right)^2\frac{\partial Var(\Psi)}{\partial i} \tag{64}$$

Now for the marginal value of information one obtains that

$$\frac{\partial\mathcal{L}}{\partial i} = \frac{\partial Var(\Psi)}{\partial i}\lambda_1\frac{\lambda_1}{\lambda_2} = \frac{\partial Var(\Psi)}{\partial i}\left(\frac{\lambda_1}{\lambda_2}\right)^2\lambda_2 = -\frac{d\,R(i)}{d\,i}\lambda_2 \tag{65}$$

As was shown in the main text, $\left(\frac{1}{\lambda_2}\right)^2 = CE$. Hence, $\lambda_2 = \frac{1}{\sqrt{CE}}$. But then, considering that

$$u'(CE) = \left(2\sqrt{CE}\right)' = \frac{1}{\sqrt{CE}} = \lambda_2\,, \tag{66}$$

it follows, that

$$\frac{\partial\mathcal{L}}{\partial i} = u'(CE)\left(-\frac{d\,R(i)}{d\,i}\right)\,. \tag{67}$$

(iii) Consider again the former expression for the marginal value of information:

$$\frac{\partial\mathcal{L}}{\partial i} = h(i) = \frac{\partial Var(\Psi)}{\partial i}\lambda_1\frac{\lambda_1}{\lambda_2}\,. \tag{68}$$

From it one obtains that

$$\frac{\partial^2\mathcal{L}}{\partial i^2} = \frac{\lambda_1^2}{\lambda_2}\cdot\left(\frac{\partial Var(\Psi)}{\partial P}\cdot\frac{\partial^2 P}{\partial i^2} + \frac{\partial^2 Var(\Psi)}{\partial P^2}\left(\frac{\partial P}{\partial i}\right)^2\right)\,. \tag{69}$$

Here it was used, that

$$\frac{\partial Var(\Psi)}{\partial i} = \frac{\partial Var(\Psi)}{\partial P}\cdot\frac{\partial P(i)}{\partial i}\,, \tag{70}$$

and hence, for the second derivative of $Var(\Psi)$, also that

$$\frac{\partial^2 Var(\Psi)}{\partial i^2} = \frac{\partial Var(\Psi)}{\partial P} \cdot \frac{\partial^2 P}{\partial i^2} + \frac{\partial^2 Var(\Psi)}{\partial P^2} \left(\frac{\partial P}{\partial i}\right)^2 . \tag{71}$$

One can verify, that

$$\frac{\partial^2 Var(\Psi)}{\partial P^2} = \frac{2(1 - 3P + 3P^2)}{(1 - P)^3 P^3} \frac{(1 - p^L)^2}{1 - p^H} , \tag{72}$$

which is positive for all values of $P(i) \in [\frac{1}{2}, 1)$.

Now consider the sign of $\frac{\partial^2 \mathcal{L}}{\partial i^2}$ in (69) for $i = 0$. From (6), as $P(0) = \frac{1}{2}$, one can ascertain that $\frac{\partial Var(\Psi)}{\partial P}|_{i=0} = 0$. Thus,

$$\frac{\partial^2 \mathcal{L}}{\partial i^2}|_{i=0} = \frac{\lambda_1^2}{\lambda_2} \cdot \frac{\partial^2 Var(\Psi)}{\partial P^2} \left(\frac{\partial P}{\partial i}\right)^2 > 0 . \tag{73}$$

For $i = 1$ one obtains, due to $\frac{\partial P}{\partial i}|_{i=1} = 0$,

$$\frac{\partial^2 \mathcal{L}}{\partial i^2}|_{i=1} = \frac{\lambda_1^2}{\lambda_2} \cdot \frac{\partial Var(\Psi)}{\partial P} \cdot \frac{\partial^2 P}{\partial i^2} < 0 . \tag{74}$$

For the third derivative of the value of information at $i = 0$ one obtains

$$\frac{\partial^3 \mathcal{L}}{\partial i^3}|_{i=0} = \frac{\lambda_1^2}{\lambda_2} \left(\frac{\partial^3 Var(\Psi)}{\partial P^3} \cdot \left(\frac{\partial P}{\partial i}\right)^3 + \frac{\partial^2 Var(\Psi)}{\partial P^2} 2 \frac{\partial P}{\partial i} \frac{\partial^2 P}{\partial i^2}\right) . \tag{75}$$

For

$$\frac{\partial^3 Var(\Psi)}{\partial P^3} = \frac{6(-1 + 4P - 6P^2 + 4P^3)}{(1 - P)^4 P^4} \frac{(1 - p^L)^2}{1 - p^H} \tag{76}$$

it can be verified, that it is negative $\forall P(\frac{1}{2}, 1]$ and is equal to zero for $P(0) = \frac{1}{2}$. Hence, it follows that

$$\frac{\partial^3 \mathcal{L}}{\partial i^3}|_{i=0} < 0 . \tag{77}$$

Thus, the marginal value of information is concave around $i = 0$ and there must also be a region of $i \in (0, 1]$ of local concavity for it. Moreover, for any value of i,

$$\frac{\partial^3 \mathcal{L}}{\partial i^3} = \frac{\lambda_1^2}{\lambda_2} \cdot \left(\frac{\partial^2 Var(\Psi)}{\partial P^2} \frac{\partial P}{\partial i} \frac{\partial^2 P}{\partial i^2} + \frac{\partial Var(\Psi)}{\partial P} \frac{\partial^3 P}{\partial i^3} + \frac{\partial^3 Var(\Psi)}{\partial P^3} \cdot \left(\frac{\partial P}{\partial i}\right)^3 + \frac{\partial^2 Var(\Psi)}{\partial P^2} 2 \frac{\partial P}{\partial i} \frac{\partial^2 P}{\partial i^2}\right) . \tag{78}$$

$\frac{\partial^3 P}{\partial i^3} \leq 0$ is a sufficient condition for $\frac{\partial^3 \mathcal{L}}{\partial i^3} < 0 \ \forall i$, i.e. for the marginal value of information to be globally concave. The value of information is convex in i around $i = 0$, and there is a region of i, for which it is concave.

A3: $k(i) = -g'(i)((1 - p^H) - \lambda_1(p^H - p^L)) < 0$.
Proof: Solving (4) and (3) for λ_2 and setting them equal, one obtains for λ_1

$$\lambda_1 = \frac{[u'(w_L) - u'(w_N)]p^H(1 - p^H)(1 - P)}{u'(w_N)(p^H - p^L)(1 - p^H)(1 - P) - p^H u'(w_L)[(1 - p^H)(1 - P) - (1 - p^L)P]} \tag{79}$$

The numerator is positive due to $u'(w_L) > u'(w_N)$. Because $\lambda_1 > 0$, also the expression in the denominator - denote it by Γ - must be positive.
Assume that $(1 - p^H) - \lambda_1(p^H - p^L) < 0$. After some transformation this is equivalent to

$$\frac{u'(w_L)p^H(1 - p^H)(1 - 2P)(1 - p^L) - u'(w_N)(p^H - p^L)(1 - p^H)(1 - P)}{\Gamma} > 0 \tag{80}$$

However, $1 - 2P < 0$ because of $P > \frac{1}{2}$ for $i > 0$. The first term in the above nominator is thus negative, the second is also negative, which is a contradiction to the above assumption. Thus $(1 - p^H) - \lambda_1(p^H - p^L) > 0$ and $k(i) < 0$.

A4: $\lambda_1, \lambda_2 > 0$ for all i, $\lambda_3 = 0$ for $i = 0$ and $\lambda_3 \geq 0$ for $i > 0$.
Proof: As in the benchmark situation, by combining $\frac{\partial \mathcal{L}}{\partial d^H} = 0, \frac{\partial \mathcal{L}}{\partial d^L} = 0$ and $\frac{\partial \mathcal{L}}{\partial r} = 0$, one obtains that $\lambda_2 > 0$. For $\lambda_3 = 0$, in order for the contract to be incentive compatible, $\lambda_1 > 0$ must hold. For $\lambda_3 > 0$ assume that $\lambda_1 = 0$. From the first order condition with respect to i it follows that $\frac{\partial \mathcal{L}}{\partial i} < 0$, $i = 0$. However, this solution is not incentive compatible and hence $\lambda_1 > 0$ must hold also in this case.
Assume that in the optimum $i > 0$ and $\lambda_3 = 0$. In this case the results for w_H, w_L, w_N, λ_1 and λ_2 in the third scenario (i cannot be determined ex ante) must correspond exactly to the results of the second scenario (i is set ex ante). Looking at the first order condition with respect to i (36) shows that it is satisfied for $\lambda_3 = 0$ only if

$$2\frac{\lambda_1}{\lambda_2}\frac{1 - p^L}{1 - p^H} \leq \frac{d^H - d^L}{u(w_H) - u(w_L)} . \tag{81}$$

That is, $\lambda_3 = 0$, only if the optimal values of the variables as found by solving the problem in the second scenario (precision ex ante) satisfy the above equation. If the opposite holds, then with $\lambda_3 = 0$ and $i > 0$, $\frac{\partial \mathcal{L}}{\partial i} > 0$ would result and hence, in that case λ_3 must be positive. In the following it will be shown that $\lambda_3 > 0$ can be the case for small values of $P(i)$.
On the one hand, from the first order condition (2) it follows that for small values of P

$$\frac{1}{\lambda_2} > \frac{1}{u'(w_H)} . \tag{82}$$

will hold. On the other hand, from (79) one obtains that for small values of $P(i) \rightarrow$

$\frac{1}{2}$+

$$\lim_{P \to \frac{1}{2}_+} \lambda_1 = \frac{[u'(w_L) - u'(w_N)]p^H(1-p^H)\frac{1}{2}}{\frac{1}{2}u'(w_N)(p^H - p^L)(1-p^H) - p^H u'(w_L)[(1-p^H)\frac{1}{2} - (1-p^L)\frac{1}{2}]}$$

$$= \frac{[u'(w_L) - u'(w_N)]p^H(1-p^H)}{(p^H - p^L)[(1-p^H)u'(w_N) + p^H u'(w_L)]} . \tag{83}$$

It can be verified that λ_1 will be larger than 1, if

$$[u'(w_L) - u'(w_N)][p^H(1-p^H) - (p^H)^2 + p^H p^L] \geq (p^H - p^L)u'(w_N) \tag{84}$$

holds. This will be the case, if risk aversion and herewith the difference $[u'(w_L) - u'(w_N)]$ is large enough or the efficiency of effort $(p^H - p^L)$ is sufficiently low. Hence, for small values of $P(i)$, $\lambda_1 > 1$ is a sufficient condition for

$$2\frac{\lambda_1}{\lambda_2}\frac{1-p^L}{1-p^H} > 2\lambda_1\frac{1-p^L}{1-p^H}\frac{1}{u'(w_H)} > \frac{d^H - d^L}{u(w_H) - u(w_L)}, \tag{85}$$

where the first inequality follows directly from (82), and the second inequality follows from $\lambda_1 > 1$, and $\frac{1}{u'(w_H)} > \frac{d^H - d^L}{u(w_H) - u(w_L)}$ (due to the concavity of the utility function). In this case $\lambda_3 > 0$.

A5: Choosing precision ex post.
For $u(w) = 2\sqrt{w}$, it can be shown that now the incentive constraint with respect to effort IC_e is equivalent to

$$2\left[\frac{\lambda_1}{\lambda_2}Var(\Psi) + \frac{\lambda_3}{\lambda_2}E(\Psi \cdot \Omega)\right] = v(e^H) - v(e^L) - g(i)(p^H - p^L), \tag{86}$$

where $Var(\Psi)$ is calculated as before (with $E(\Psi) = 0$ and $\frac{\partial Var(\Psi)}{\partial i} > 0$) and

$$\frac{\partial P(i)}{\partial i}\frac{(1-p^L)[2P(i) - 1]}{[1 - P(i)]P(i)} = E(\Psi \cdot \Omega). \tag{87}$$

One can ascertain, that $E(\Omega) = 0$ and thus $E(\Psi \cdot \Omega) = Cov(\Psi, \Omega)$. For the effect of precision on it consider

$$\frac{\partial E(\Psi \cdot \Omega)}{\partial i} = (1 - p^L)\left(\frac{\partial P(i)}{\partial i}\right)^2 [\frac{1}{P(i)^2} + \frac{1}{(1 - P(i))^2}]$$

$$+ \frac{\partial^2 P(i)}{\partial i^2}(1 - p^L)\frac{2P(i) - 1}{(1 - P(i))P(i)} . \tag{88}$$

The first term of the above expression is positive. The second term is negative due to $\frac{\partial^2 P(i)}{\partial i^2} < 0$. However, if $\left|\frac{\partial^2 P(i)}{\partial i^2}\right|$ is sufficiently small, then $\frac{\partial E(\Psi \cdot \Omega)}{\partial i} > 0$. If the probability of observing s^H does not increase too slowly in the amount of data i,

then the covariance between the measure of the exerted effort and the measure of the chosen precision will increase with the amount of data.

When i increases, the right hand-side of (86) decreases and so must the left hand side (movement "along" the IC_e). λ_3 is the shadow price of setting incentives for the individual to reveal a certain level of precision i. When the level of precision i grows, this multiplier will also become larger, as it is more costly to implement a higher level of precision (see A6 of the appendix). The above equation (86) implies that it is not possible for both $\frac{\lambda_1}{\lambda_2}$ and $\frac{\lambda_3}{\lambda_2}$ to increase as the level of precision becomes larger. Using the previous result that the shadow price of incentives on effort λ_1 decreases in i, the following can be inferred: if a larger level of precision causes $\frac{1}{\lambda_2}$ to increase, then $\frac{\lambda_3}{\lambda_2}$ will also increase and $\frac{\lambda_1}{\lambda_2}$ must decrease; if a larger level of precision reduces $\frac{1}{\lambda_2}$, then $\frac{\lambda_1}{\lambda_2}$ will also decline and $\frac{\lambda_3}{\lambda_2}$ might either decrease or increase. In any case $\frac{\lambda_1}{\lambda_2}$ will decrease when the level of precision is increased.

Comparing (86), (27) and (9) for the same value of i, where $Var(\Psi)$ has the same value in all three scenarios, it is obvious (as was noted in the previous section) that $\frac{\lambda_1}{\lambda_2}$ has to be smaller in the second scenario compared to the benchmark situation. As was mentioned before, this is because privacy costs improve incentives on effort. Further, it follows that $\frac{\lambda_1}{\lambda_2}$ is even smaller in this third scenario, in which precision is specified ex post.

With $u(w) = 2\sqrt{w}$ the incentive constraint with respect to i IC_i is equivalent to

$$\frac{\partial P(i)}{\partial i}[u(w_H) - u(w_L)] = g'(i) \iff$$
$$2\left[\frac{\lambda_1}{\lambda_2}\frac{\partial P(i)}{\partial i}\frac{(1-p^L)[2P(i)-1]}{[1-P(i)]P(i)} + \frac{\lambda_3}{\lambda_2}\left[\frac{\partial P(i)}{\partial i}\right]^2\frac{1-p^H}{P(i)[1-P(i)]}\right]$$
$$= (1-p^H)g'(i) \tag{89}$$

It can be shown that

$$\left[\frac{\partial P(i)}{\partial i}\right]^2\frac{(1-p^H)}{P(i)[1-P(i)]} = Var(\Omega). \tag{90}$$

The partial derivative of $Var(\Omega)$ with respect to i is equal to

$$\frac{\partial Var(\Omega)}{\partial i} = \frac{(1-p^H)(P'(i))^3(2P(i)-1)}{(1-P(i))^2 P(i^2)} + \frac{2(1-p^H)P'(i)P''(i)}{(1-P(i))P(i)}. \tag{91}$$

The first term is positive, the second one is negative. But if $\left|\frac{\partial^2 P(i)}{\partial i^2}\right|$ is sufficiently small (which implies that the monitoring technology is efficient), then $\frac{\partial Var(\Omega)}{\partial i} > 0$, i.e. the ability to infer from Ω the actual chosen level of precision, grows with the level of precision. Hence IC_i is equivalent to

$$2\left[\frac{\lambda_1}{\lambda_2}E(\Psi \cdot \Omega) + \frac{\lambda_3}{\lambda_2}Var(\Omega)\right] = (1-p^H)g'(i). \tag{92}$$

Having $g''(i) > 0$, the right hand side of the above equation increases in i and so

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must the left hand side (movement along the IC_i).

As in the previous scenarios, these expressions for IC_e (86) and IC_i (92) can be used in order to calculate the risk premium

$$Var\left[\frac{1}{u'(w)}|e^H\right] = E\left[\left[\frac{1}{u'(w)} - E\left(\frac{1}{u'(w)}\right)\right]^2\right].$$

As before $E(1/u'(w)) = \frac{1}{\lambda_2}$ and hence for the risk premium one obtains,

$$
\begin{aligned}
Var\left[\frac{1}{u'(w)}|e^H\right] &= E\left[\left[\frac{\lambda_1}{\lambda_2}\Psi + \frac{\lambda_3}{\lambda_2}\Omega\right]^2\right] \\
&= \left(\frac{\lambda_1}{\lambda_2}\right)^2 Var(\Psi) + 2\frac{\lambda_1}{\lambda_2}\frac{\lambda_3}{\lambda_2}E(\Psi\cdot\Omega) + \left(\frac{\lambda_3}{\lambda_2}\right)^2 Var(\Omega) \\
&= \underbrace{\frac{\lambda_1}{\lambda_2}\left(\frac{\lambda_1}{\lambda_2}Var(\Psi) + \frac{\lambda_3}{\lambda_2}E(\Psi\cdot\Omega)\right)}_{IC_e} + \underbrace{\frac{\lambda_3}{\lambda_2}\left(\frac{\lambda_1}{\lambda_2}E(\Psi\cdot\Omega) + \frac{\lambda_3}{\lambda_2}Var(\Omega)\right)}_{IC_i}.
\end{aligned}
\tag{93}
$$

From (86) it follows, that the term in the first brackets declines in i and so does $\frac{\lambda_1}{\lambda_2}$. From (92) it follows that the term in the second brackets increases in i and $\frac{\lambda_3}{\lambda_2}$ might either increase or decrease. For small i, λ_3 and the term in the second brackets (IC_i) will be small and the effect of the first term will prevail, i.e. for small i an increase of the level of precision will decrease the risk premium $Var\left[\frac{1}{u'(w)}|e^H\right]$, i.e. it will improve risk sharing. The reverse can be argued for large values of i. Then λ_3 will grow and as $g'(i)$ grows large enough, the effect of a marginal increase of precision on $Var\left[\frac{1}{u'(w)}|e^H\right]$ will be positive, i.e. risk sharing will become less efficient. The repercussions of these considerations on expected contractual utility $E[u(w)] = 2\frac{1}{\lambda_2}$ are reversed and can be derived as before from the zero-profit constraint. It can be verified that

$$E(w|e^H) = \left(\frac{1}{\lambda_2}\right)^2 + Var\left[\frac{1}{u'(w)}|e^H\right] = W - (1-p^H)L \tag{94}$$

applies also in this case. It follows that, for small values of i, as $Var\left[\frac{1}{u'(w)}|e^H\right]$ decreases in i, a marginal increase of precision will increase expected contractual utility. For high values of i, or when $Var\left[\frac{1}{u'(w)}|e^H\right]$ increases in i, the expected contractual utility will fall with a further increase of i.

A6: λ_3 increases when precision i increases

Proof: Assume that i^* and λ_3 are the values of the variables in the optimum such that all 4 equations (30) and (36), (92) and (86) are satisfied. Specifically it holds

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that

$$\frac{\partial P(i)}{\partial i}\Big|_{i^*}[u(w_H) - u(w_L)] - g'(i^*) = 0. \tag{95}$$

Now assume an exogenous reduction of $g''(i)$ for all i which leads to

$$\frac{\partial P(i)}{\partial i}\Big|_{i^*}[u(w_H) - u(w_L)] - g'(i^*) > 0. \tag{96}$$

For $u(w) = 2\sqrt{w}$ one obtains from the expressions in (32) and (33) that

$$u(w_H) - u(w_L) = 2\frac{\lambda_1}{\lambda_2}\frac{1-p^L}{1-p^H}\left(\frac{P(i)}{1-P(i)} - \frac{1-P(i)}{P(i)}\right)$$

$$+ 2\frac{\lambda_3}{\lambda_2}\frac{\frac{\partial P(i)}{\partial i}}{P(i)(1-P(i))}. \tag{97}$$

Observing the above expressions, it follows that, in order to restore the equality in (96), and holding everything else constant, either i has to increase, or λ_3 to decrease.

A7: Proof of Proposition 2
Let the index F stand for fixed, in the sense that i is fixed ex ante when the contract is signed, and N the index for not fixed, in the sense that at the time the contract is signed, no agreement concerning the level of precision is made (precision is chosen ex post). For a *given level of precision i*, in which case the right hand sides of (27) and (86) are equal, by combining those two equalities it follows that

$$\frac{\lambda_1^F}{\lambda_2^F}Var(\Psi) = \frac{\lambda_1^N}{\lambda_2^N}Var(\Psi) + \frac{\lambda_3^N}{\lambda_2^N}E(\Psi \cdot \Omega), \tag{98}$$

from which

$$\frac{\lambda_1^N}{\lambda_2^N} = \frac{\lambda_1^F}{\lambda_2^F} - \frac{\lambda_3^N}{\lambda_2^N}\frac{E(\Psi \cdot \Omega)}{Var(\Psi)}. \tag{99}$$

When precision is determined ex ante, the risk premium is

$$Var\left(\frac{1}{u'(w^F)}\right) = \left(\frac{\lambda_1^F}{\lambda_2^F}\right)^2 Var(\Psi) \tag{100}$$

(see (12), which expression holds both for the benchmark situation and for the second scenario).
When precision is determined ex post (93) $Var\left(\frac{1}{u'(w)}\right)$ rewritten again is

$$Var\left(\frac{1}{u'(w^N)}\right) = \left(\frac{\lambda_1^N}{\lambda_2^N}\right)^2 Var(\Psi) + 2\frac{\lambda_1^N}{\lambda_2^N}\frac{\lambda_3^N}{\lambda_2^N}E(\Psi \cdot \Omega) + \left(\frac{\lambda_3^N}{\lambda_2^N}\right)^2 Var(\Omega). \tag{101}$$

Substituting for $\frac{\lambda_1^N}{\lambda_2^N}$ the expression from (99), the above equation reduces to

$$Var\left(\frac{1}{u'(w^N)}\right) = \left(\frac{\lambda_1^F}{\lambda_2^F}\right)^2 Var(\Psi) + \left(\frac{\lambda_3^N}{\lambda_2^N}\right)^2 Var(\Omega). \tag{102}$$

Comparing (100) and (102) shows that

$$Var\left(\frac{1}{u'(w^N)}\right) > Var\left(\frac{1}{u'(w^F)}\right), \tag{103}$$

i.e. for any level of precision the risk premium with precision ex post is larger than the risk premium with precision ex ante. It follows directly that $E[u(w^N)] < E[u(w^F)]$. That is, for any arbitrary level of precision the expected contractual utility is smaller when precision is chosen ex post. Therefore, for any arbitrary level of precision, also the total expected utility is smaller when precision is chosen ex post than when it is chosen ex ante, $E[u(w^N)] - (1-p^H)g(i) < E[u(w^F)] - (1-p^H)g(i)$. Although the optimal level of precision will generally be different in the two scenarios, the above relation holds for any $i \in (0,1]$. In other words, the total expected utility in the third scenario (precision ex post) will be below the total expected utility in the second scenario (precision ex ante) throughout the whole support of i, from which it follows that welfare at the optimum level of precision for the third scenario will be smaller than the welfare at the optimal level of precision for the second scenario.

A8: Comparison of I and B in the presence of privacy costs:
(27), (12) and (13) for I and (44) to (46) for B can be transformed to obtain:

$$\frac{\lambda_1^B}{\lambda_2^B} = \frac{\Delta v}{2Var(\Theta)} \tag{104}$$

$$Var\left(\frac{1}{u'(w_B)}\right) = \frac{(\Delta v)^2}{4Var(\Theta)} \tag{105}$$

$$\left(\frac{1}{\lambda_2^B}\right)^2 = E[w|e^H] - \frac{(\Delta v)^2}{4Var(\Theta)} \tag{106}$$

and

$$\frac{\lambda_1^I}{\lambda_2^I} = \frac{\Delta v - (p^H - p^L)g(i)}{2Var(\Psi)} \tag{107}$$

$$Var\left(\frac{1}{u'(w_I)}\right) = \frac{(\Delta v - (p^H - p^L)g(i))^2}{4Var(\Psi)} \tag{108}$$

$$\left(\frac{1}{\lambda_2^I}\right)^2 = E[w|e^H] - \frac{(\Delta v - (p^H - p^L)g(i))^2}{4Var(\Psi)} \tag{109}$$

From the above expressions it follows that for a given level of precision i, $\frac{\lambda_1^I}{\lambda_2^I} < \frac{\lambda_1^B}{\lambda_2^B}$, if

$$\frac{\Delta v - (p^H - p^L)g(i)}{\Delta v} < \frac{Var(\Psi)}{Var(\Theta)} \tag{110}$$

In that case, risk-sharing is better, expected contractual utility and total expected utility are larger with I than with B and incentives on effort with I are better than with B ($\lambda_1^I < \lambda_1^B$).

But even if the reverse is true, $\frac{\lambda_1^I}{\lambda_2^I} > \frac{\lambda_1^B}{\lambda_2^B}$, it is still possible that risk-sharing with I might be better than with B, $Var\left(\frac{1}{u'(w_I)}\right) < Var\left(\frac{1}{u'(w_B)}\right)$, which also implies larger contractual utility with I ($\lambda_2^I < \lambda_2^B$):

$$\frac{(\Delta v - (p^H - p^L)g(i))^2}{(\Delta v)^2} < \frac{Var(\Psi)}{Var(\Theta)} \,. \tag{111}$$

This in turn implies better incentives with I, $\lambda_1^I < \lambda_1^B$. Note that $\frac{\Delta v - (p^H - p^L)g(i)}{\Delta v}$ is less than 1, so that the above condition is less restrictive than the first one.

Finally, even if risk sharing with I is worse than with B, i.e. $E[u(w_I^*)] < E[u(w_B^*)]$, it is still possible, that total expected utility is larger with I due to the smaller privacy costs, which are incurred with conditional monitoring. Note, that $E[u(w_I)] - (1 - p^H)g(i) > E[u(w_B)] - g(i)$ for $i = i_B^*$, which is the optimal level of precision for the third contract scheme B, is a sufficient condition for the *equilibrium* level of total expected utility with I to dominate the equilibrium level of total expected utility with B. This in turn, using the fact that $E[u(w)] = 2\frac{1}{\lambda_2} = 2\sqrt{CE}$ and the above expressions, is equivalent to

$$\underbrace{2\sqrt{CE_I} - 2\sqrt{CE_B} + p^H \cdot g(i)}_{F} > 0 \,, \tag{112}$$

where

$$CE_I = E(w|e^H) - \frac{[\Delta v - (p^H - p^L)g(i)]^2}{4Var(\Psi)} \tag{113}$$

and

$$CE_B = E(w|e^H) - \frac{(\Delta v)^2}{4Var(\Theta)} \tag{114}$$

Denote the left hand-side of the above inequality by F. It can be verified, that

$\frac{\partial F}{\partial g(i)} > 0$, $\frac{\partial F}{\partial p^H} > 0$, $\frac{\partial F}{\partial p^L} < 0$ and $\frac{\partial F}{\partial \Delta v} < 0$. For the sign of the last derivative,

$$\frac{\partial F}{\partial \Delta v} = -\frac{(\Delta v - (p^H - p^L)g(i))}{2Var(\Psi)}\frac{1}{\sqrt{CE_I}} + \frac{\Delta v}{2Var(\Theta)}\frac{1}{\sqrt{CE_B}} \tag{115}$$

it was taken into account, that the contractual utility with I being smaller than the contractual utility with B for a given level of precision, $E[u(w_I)] < E[u(w_B)]$, which is equivalent to $\frac{1}{\lambda_2^I} < \frac{1}{\lambda_2^B}$, is only possible, if $\frac{\lambda_1^I}{\lambda_2^I} > \frac{\lambda_1^B}{\lambda_2^B}$. From the above expressions for these terms, it follows that $\frac{(\Delta v - (p^H - p^L)g(i))}{2Var(\Psi)} > \frac{\Delta v}{2Var(\Theta)}$. Further $E[u(w_I)] < E[u(w_B)]$ is equivalent to $CE_I < CE_B$ and thus $\frac{1}{\sqrt{CE_I}} > \frac{1}{\sqrt{CE_B}}$, from which the sign of the derivative is unambiguously negative.
For the sign of $\frac{\partial F}{\partial P(i_B^*)}$, one can verify that

$$\frac{\partial F}{\partial P(i_B^*)} = \frac{\partial Var(\Psi)}{\partial P}\frac{(\lambda_1^I)^2}{\lambda_2^I} - \frac{\partial Var(\Theta)}{\partial P}\frac{(\lambda_1^B)^2}{\lambda_2^B} \tag{116}$$

If, at $i = i_B^*$, incentives and risk-sharing are better with I than with B ($\lambda_1^I < \lambda_1^B$, $\lambda_2^I < \lambda_2^B$) and $\frac{\lambda_1^I}{\lambda_2^I} < \frac{\lambda_1^B}{\lambda_2^B}$, it was shown above that total expected utility with I will be larger than with B, then $F > 0$. Due to $\frac{\partial Var(\Psi)}{\partial i} < \frac{\partial Var(\Theta)}{\partial i}$, $\frac{\partial F}{\partial P(i_B^*)} < 0$. In that case, an improvement of the monitoring technology, i.e. larger $P(i)$ for any value of i, will reduce the advantage of I as compared to B. Note that I being better than B is more likely if $P(i_B^*)$ is small, and generally, if the efficiency of the monitoring technology is low.

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