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Abstract

In this paper, we investigate the relationship between market dynamics, dynamic resource management and environmental policy. In contrast to static market entry games, this paper draws attention to the effects of market dynamics on resource dynamics et vice versa, because (1) we show that feedback processes are necessary for obtaining a better understanding of what drives the *dynamics* between the evolution of common-pool resources and the number of harvesters and more importantly, (2) this analysis provides an environment discussing sustainability in an appropriate inasmuch dynamic way. The paper makes following major points: (1) Interpreting the monopoly-scenario as a non-cooperative solution and the firm coexistence solution as a cooperative solution, it is shown that the coexistence solution of this model implies a degenerate saddle-node equilibrium. (2) An increasing number of harvesters does not necessarily imply a lower stock of the common-pool resource in the long run. (3) The paper introduces a way establishing an output-sharing solution by implementing an output tax, which turns out to be a pure effort tax in the long run. (4) Strong resource sustainability is not possible, given cost reducing technological progress is relevant and policy interventions ceased. With respect to environmental policy, we can conclude that a tax scheme is not a substitute to a partnership solution dealing with the common-pool problem, but is treated as an instrument establishing such a solution in the sense of a policy mix approach.

Keywords: Sustainability, resource management, environmental policy, common resources, population dynamics

JEL Classification Number: Q28, Q57, C61

1 Motivation

In the 1990s, with ecological economics, a new discipline of research has been founded as the consequent answer to serious ecological problems which cannot be tackled by traditional environmental and resource economics (Costanza et al. (1991)). In contrast to this well-introduced mainstream disciplines, ecological economics as a transdisciplinary academic discipline combines ecological as well as economic issues in a more pluralistic and eclectic approach than traditional resource and environmental economics. Ecological economics can be interpreted as economics for sustainability, or more precisely it promotes the sustainable development of ecosystems and societies.

Albeit the watch-wording *sustainable development* has been the focal point of many recent works, both in economics and ecology, surprisingly or not, there is a considerable hot debate on the conceptual as well as on the operational level of what sustainability actually contains and implies (Baumgärtner and Quaas (2010)). Ecological economics propagates the concept of strong sustainability, which is, in contrast to the neoclassical view, proposing that natural capital can be entirely replaced by man-made capital (Illge and Schwarze (2009)). At the end of the day, *sustainability* remains a more or less obscure item. Being aware of an imprecise definition of sustainability, in this paper, we refer to a very broad definition of what the goal of sustainability is: to maintain a (diverse) biological system over time and space. We can operationalize this goal by pointing out that strong sustainability emphasizes resource conservation over time (Howarth (2007)).

One point on the ecological as well as on the environmental economics' research agenda is the resource and ecosystem management issue. It seems that traditional resource economics is based on a merely inflexible construct of what can be called mechanistic resource stock sustainability (Van den Bergh (2007)), whereas ecological economics relies on a more satisfactory approach which explicitly acknowledges the interaction between resource dynamics and market development. This implies an important step towards finding a satisfactory answer to the problem about the risk of overexploitation of common-pool resources.

The massive destruction of common-pool resources caused by an excessive extraction from the resource stock is generally known as the *tragedy of the commons* (Hardin (1968)). One general result stemming from Cournot-competition-style harvesting games is that myopic agents neither take into account that choosing their individual harvesting level affects the future biomass evolution (Sandal and Steinshamn (2004)), nor these agents take into account that an increase in individual harvesting efforts leads to a decrease in individual profits (Atzenhoffer (2010)). As shown by Harms and Sylvia (2001), commercial fishermen often base their harvesting decisions on short time horizons. As a result, the exploitation effort regularly exceeds Pareto-optimality (Atzenhoffer (2010)) and further the exploitation level of the common-pool resource increases steadily with the number of harvesters (Sandal and Steinshamn (2004), Atzenhoffer (2010)). However, the empirical evidence concerning this issue is mixed. In a survey dealing with the over-exploitation of the *prunus africana* in Cameroon, Stewart (2003) reports that an increasing number of harvesters eroded the traditional resource protection ethic by the *kwifon*, as the former harvesting monopolist lost his right for exclusively harvesting the bark of the *prunus africana*. Ito et al. (1995) and Lopez (1998) however show that even if the number of harvesters is limited, the exploitation level of the common-pool resource often exceeds the Nash equilibrium. The latter insight is also fleshed out by laboratory experiments (Walker et al. (1990) and Walker and Gardner (1992)). All these studies have in common that the discussion of the effects of market entry on the incentive of harvesting the common-pool resource is in the front. However, in this paper we draw attention to the effects of market dynamics on resource dynamics et vice versa, because (1) we believe that feedback processes are necessary for obtaining a better understanding of what drives the *dynamics* between the evolution of common-pool resources and the number of harvesters and more importantly, (2) this analysis provides an environment discussing sustainability in an appropriate inasmuch dynamic way.

Despite of the awareness that there is obviously no escape from the tragedy, state-imposed regulations or decommissioning schemes often fail establishing a sustainable exposure with the common-pool resources, as shown empirically by Ostrom (1990), Ostrom et al. (1994) and Berkes et al. (2000). However, the empirical evidence to this issue is mixed. As mentioned by Jensen (2002), lump-sum taxation on fishermen´s capital could be an appropriate way reducing this tendency. Evaluating the theoretical literature, Ruseski (1998), Copeland (1994) and Bulte and Damania (2005) discuss the effectiveness of effort subsidies and effort taxes for renewable resource management by using strategic trade models. Assuming myopic agents, Bulte and Damania (2005) show that an imposed effort tax on harvesting output, which is not sufficiently high, results into a suboptimal harvesting level. Of course, environmental policy actions relying on taxes require an accurate knowledge about the common-pool resource, and further, the effectiveness seriously depends on the acceptance of the resource users.

Alternatively, harvesting-sharing is suggested as a promising environmental policy instrument substitute for taxes or quotas (e.g. Yamamoto (1995), Schott et al. (2007)). Albeit harvesting-sharing seems to be attractive to cope with the common-property problem even without communication between the harvesters (Schott et al. (2007)), the implementation, as well as the perpetuation of such a partnership is difficult, e.g. because of harvester´s heterogeneity. We have to assert that the relevant literature to this topic is rather silent (Heintzelman et al. (2008)). Insofar, a dynamic resource management approach is required establishing a long-lasting sustainability concept.

Another point which justifies the implementation of a dynamic resource management system is that, even if we had found a perfect environmental policy strategy in the short run, the design of policy instruments often neglects the fact that environmental-related technological progress affects common-pool resource degradation and does not necessarily lead to a more efficient use of the resource as it is generally argued for the case of non-common-pool resources (Bretschger and Smulders (2007)). Regev et al. (1998) investigate, whether technological progress endangers resource sustainability. In contrast to the prevailing economic wisdom, they find that in a competitive economy, the appearance of technological progress is detrimental to resource preservation. Empirically, this is particularly true for the worldwide fisheries which are suffering from technological improvements which often results in increasing harvesting efforts (Hilborn et al. (1995)).

From the discussion above, it seems to be necessary to amalgamate market dynamics, resource dynamics, environmental technology progress and environmental policy in a comprehensive co-evolutionary model. One novelty of this paper is that this co-evolutionary model can be further used to discuss environmental policy strategies in the context of a dynamic resource management frame. This framework perfectly fits to the discipline of ecological economics, and its importance and amenities compared to neoclassical approaches has been clearly emphasized by Kenneth E. Boulding (Boulding (1978), Boulding (1981)).

Further, the paper makes the following major points: (1) Interpreting the monopoly-scenario as a non-cooperative solution and the firm coexistence solution as a cooperative solution, we show that the coexistence solution of this model implies a degenerate saddle-node equilibrium. (2) An increasing number of harvesters does not necessarily imply a lower stock of the common-pool resource in the long run. (3) The paper introduces a way establishing an output-sharing solution by implementing an output tax, which turns out to be a pure effort tax in the long run. (4) Strong resource sustainability in the sense of resource conservation is not possible, given cost-reducing technological progress is relevant and policy interventions ceased.

The outline of the paper is as follows: In the next section the co-evolutionary model is introduced. Section 2.1 highlights the main elements of the model. Section 2.2 discusses the dynamic behaviour of the model and provides a stability analysis of the obtained steady-states. This is followed by section 2.3 which highlights the link between dynamic resource management and environmental policy. In section 2.4 a numerical exercise is conducted. Section 3 summarizes the main results and presents the conclusion.

2 Model

2.1 Basic setup

In this section, we present the basic elements of the model, which consists of three basic elements: Firstly, a dynamic rule which describes the dynamic behaviour of the natural resource N[1], secondly, the market share evolution $s_h \in [0, 1]$ of the h-th firm and thirdly, the dynamic cost structure of the h-th firm, C_h.

2.1.1 Evolution of the natural resource N

We assume that production exclusively depends on a scarce and regenerative but exhaustible natural resource N which is growing with an exogenously given rate ξ[2]. To keep the model tractably simple, we refer on the so called *Schaefer equation*, which is gathered from the *Gordon-Schaefer* model (Gordon (1954)). This equation reads as

$$\dot{N} = \xi N \left[1 - \frac{N}{M}\right] - E(N). \tag{1}$$

$E(N)$ represents the aggregate harvesting function depending on the aggregate stock of N. We assume that aggregate harvesting is indirectly linked to N which can be expressed as:

$$E(N) = \sum_h s_h f_h(N), \tag{2}$$

with $f_h(N) = e_h N$ as a simple production function with constant returns to scale in the firm-specific harvesting effort e_h and N.

[1]We skip the variable-specific time-indices in the following paragraphs.

[2]See, for instance, Dasgupta and Heal (1979).

2.1.2 Evolution of the market share s_h

To keep the model simple without loss of generality, we restrict ourselves on the $h = \{i, j\}$ firm case. The firms are assumed to be totally symmetric with the exception of their individually given harvesting cost level C_h, and, as shown later, with their harvesting effort e_h. It is further supposed that the cost level C_h is directly linked to the market share evolution s_h. Borrowed from evolutionary game theory, we employ a so-called replicator dynamics approach to model the market share evolution of the h-th firm. To guarantee an analytical solution of the model, we have decided to refer to a simple selection mechanism based on the firm-specific profits per unit $\pi_h \equiv \frac{\Pi_h}{N+1}$[3]: If π_h is below the average market profit per unit, $\bar{\pi}$, this leads to a market share decrease of firm h, whereas a positive value of $\pi_h - \bar{\pi}$ leads to a market share gain of the h-th firm. If $\pi_h - \bar{\pi} = 0$, the market share evolution s_h remains unchanged. Accordingly, we can model the market share evolution $\dot{s}_i$ for firm i as[4]:

$$\dot{s}_i = s_i(\pi_i - \bar{\pi}), \tag{3}$$

with $c_i \equiv \frac{C_i}{1+N}$, $\pi_i = e_i \frac{N}{1+N}[p - c_i]$, $\bar{\pi} = \sum_h s_h \pi_h$ with $\bar{c} = \sum_h c_h s_h$ and $\sum_h s_h = 1$. p stands for the market price and is treated as a global parameter. For firm i, equation (3) can be expanded and rewritten as

$$\begin{aligned} \dot{s}_i &= s_i(1 - s_i)(\pi_i - \pi_j) \\ &= s_i(1 - s_i)\left[p\frac{N}{N+1}(e_i - e_j) - \frac{N}{N+1}(e_i c_i - e_j c_j)\right] \\ &= s_i(1 - s_i)\Theta, \ \ i \neq j, \end{aligned} \tag{4}$$

with $\Theta \equiv \left[p\frac{N}{N+1}(e_i - e_j) - \frac{N}{N+1}(e_i c_i - e_j c_j)\right]$, $\frac{\partial \dot{s}_i}{\partial c_i} < 0$ and $\frac{\partial \dot{s}_i}{\partial c_j} > 0$.

[3] We define the profits per unit analogously to the per unit cost function as suggested by Bradley (2001).

[4] Because of the fact that firms are assumed to be symmetric with the exception of their cost and harvesting effort, the same arguments as stated in the text above can be used to model the market share evolution of firm j.

2.1.3 Evolution of the harvesting costs C_h

Firms are treated to be heterogeneous with respect to their harvesting cost and harvesting effort structure. The total cost structure C_h of firm h consists of a fix block C_{fix}, which is unaffected by cost reducing technological progress. In contrast to $C_{fix,h}$, the variable cost block $C_h - C_{fix,h}$ can be reduced by the rate of technological progress $\theta \in (0, 1)$. From this point of view, we can specify the cost function for firm h as

$$\dot{C}_h = \theta(C_{fix,h} - C_h). \tag{5}$$

The solution to this differential equation is straightforwardly given by:

$$C_h = (C(0)_h - C_{fix,h})exp[-\theta t] + C_{fix,h}, \tag{6}$$

with $C(0)_h > C_{fix,h}$ as a necessary condition for a falling cost structure over time. Using equation (6), it can be shown that for $t \mapsto \infty$ the limit of C_h is $C_{fix,h}$. If $\theta = 0$, $C_h = C(0)_h$ which implies a constant cost structure over time.

2.1.4 Harvesting effort vs. harvesting costs

As mentioned in the introduction, an increased harvesting effort transforms into an increased harvesting cost level. Following (Noailly et al. (2003)), it is assumed that without loss of generality,

$$e_i C_i > e_j C_j \tag{7}$$

holds for all t. Together with $C_{fix,i} \geq C_{fix,j}$, we have introduced a trade-off scenario: firm i is, on the one hand, harvesting more than firm j, but is on the other hand confronted with higher extraction costs C_i. Hence, in this way, we introduce heterogeneity in our model.

2.2 Dynamic analysis of the co-evolutionary system

2.2.1 The co-evolutionary system

The model consists of five differential equations, namely, two expressions of equation (1) for firm i and j, equation (4) and two expressions of equation (5) for firm i and j. Avoiding the loss of analytical tractability, we can reduce the dimension of the system from five to three equations[5].

This is due because we know that $\sum_h s_h = 1$ and further we can solve equation (5) for firm j. So, at the end of the day, we arrive at a system of three differential equations which is necessary to describe the dynamic of firm i in the way of market share and profit evolution[6]. The system reads as

$$\begin{cases} \dot{N} &= \xi N\left[1 - \frac{N}{K}\right] - \left[\sum_h s_h f_h(N)\right] \\ \dot{s}_i &= s_i(1 - s_i)(\pi_i - \pi_j) \\ \dot{C}_i &= \theta(C_{fix,i} - C_i). \end{cases} \tag{8}$$

2.2.2 Identification of the steady states

In this section we identify the steady states of system (8).

Definition: Steady state vector Γ. *A* $(n \times 1)$ *steady state vector* Γ *of an* n*-dimensional system is defined as the collectivity of intersections of the* n *nullclides of system (8).* For system (8) we can identify $n = 3$ nullclides. Thus, $\Gamma = [\dot{N}, \dot{s}_i, \dot{C}_i]' = 0$. We find for $\dot{N} = 0$

$$s_i \equiv g(N) = \frac{\xi\left(1 - \frac{N}{M}\right) - e_j}{e_i - e_j} \tag{9}$$

and the trivial condition $N = 0$. Next, we obtain $\dot{s}_i = 0$ for $s_i = 0$, $s_i = 1$ and

$$N \equiv h(C_i) = \left(\frac{e_i C_i - e_j C_j}{p(e_i - e_j)}\right) - 1. \tag{10}$$

[5]Of course, we can reduce the dimension from five to two equations if we solve the differential equation (5) for firm i. We are going to exploit this fact later on when discussing the global stability of the obtained steady states of the co-evolutionary system.

[6]Analogously, we can use the same harvesting strategy to discuss the dynamic of firm j.

Finally, $\dot{C}_i = 0$ results for $C_i = C_{fix,i}$[7].

Solving system (8), we can identify ten steady state conditions which are summarized in table (1) in the appendix.

2.2.3 Existence conditions

1. A necessary condition for the existence of equilibrium A_1, A_2, A_3 and A_4 is that $N > 0$, which implies that $\xi > e_i$. To avoid resource eradication, we impose the restriction that $\xi > e_i > e_j$. Further, $C_i > \underline{C}_i$.

2. A necessary condition for the existence of equilibrium B_1 and B_2 is that $N > 0$, which implies that long-run resource existence requires $\xi > e_j$. Further, $C_i < \underline{C}_i$.

3. Necessary conditions for the existence of equilibrium D_1 and D_2 are:

   (a) $N > 0$, which is ensured by the condition

   $$C_i > \frac{1}{e_i}\left[p(e_i - e_j) + e_j C_j\right] \equiv \Psi. \tag{11}$$

   (b) C_i must be situated between the cost range defined as

   $$(\underline{C}_i; \overline{C}_i), \tag{12}$$

   with

   $$\underline{C}_i \equiv \frac{1}{e_i}\left\{[p(e_i - e_j)]\left[\left(1 - \frac{e_i}{\xi}\right)M + 1\right] + e_j C_j\right\} \tag{13}$$

   and

   $$\overline{C}_i \equiv \frac{1}{e_i}\left\{[p(e_i - e_j)]\left[\left(1 - \frac{e_j}{\xi}\right)M + 1\right] + e_j C_j\right\}. \tag{14}$$

   $\underline{C}_i < \overline{C}_i$ is ensured by the assumption that $e_i > e_j$.

4. Given $N = 0$, equilibria H_1 and H_2 are always realized. Because of sustainability, which requires $N > 0$, we can exclude this two types of equilibria from our analysis because they are not of further interest.

[7] Of course, if $\theta = 0$, we meet the condition $\dot{C}_i = 0$.

2.2.4 Stability analysis

Proposition I: Equilibrium A_1, A_2, A_3 ***and*** A_4 ***are globally asymptotically stable.*** *Equilibrium* A_1, A_2, A_3 *and* A_4 *are globally stable given,* $\xi > e_i$ *and* $C_i < \underline{C}_i$. $\square$

Proof: Refer to appendix 4.4.2

Firstly, let us focus on equilibrium A_1 and A_2. These equilibria are met if $\xi > e_i$ and $C_i < \underline{C}_i$. Note that for $\xi > e_i$ it necessarily follows that $\underline{N} < N < \overline{N}$, which leads to the conclusion that equilibrium B_1 and B_2 cannot be reached. Further, if $C_i < \underline{C}_i$, consequently $C_i \notin (\underline{C}_i; \overline{C}_i)$ and this implies that equilibria D_1 and D_2 can be ruled out. Additionally, $C_{fix,i} > C_{fix,j} \neq 0$ which eliminates equilibrium A_3 and A_4 for $\theta \in [0, 1)$. Therefore, the remaining and stable equilibria A_1 and A_2 can be met, which depends on the fact, whether technological progress is zero (which implies that equilibrium A_1 is realized) or if $\theta \in (0, 1)$, which means that the flow of system (8) converges to equilibrium A_2. Secondly, let us discuss the global stability of equilibrium A_3 and A_4. In this case, either the entire cost structure tends to zero (equilibrium A_3) or both firms are confronted with the same fix cost block for $t \mapsto \infty$, given technology progress is not equal to zero. Accordingly, firm i rules out firm j because of the fact that $e_i > e_j$ for all t. Therefore, equilibria B_1 and B_2 cannot be reached, just like the coexistence equilibria D_1 and D_2. For this case, the dynamic of the system (8) converges to equilbrium A_3 or A_4.

Proposition II: Equilibrium B_1 ***and*** B_2 ***are globally asymptotically stable.*** *Equilibrium* B_1 *and* B_2 *are globally asymptotically stable, given* $\xi > e_i > e_j$ *and* $C_i > \overline{C}_i$. $\square$

Proof: Refer to appendix 4.2.3

For this constellation, the dynamic of system (8) drives firm i out of the market. In the case of $C_i > \overline{C}_i$, together with $\overline{N} > N > \underline{N}$, it follows that the remaining fixed points A_1, A_2, A_3 and A_4 cannot be met. Asymptotically, equilibrium B_1 will be realized if technological progress is excluded and $C(0)_i > C(0)_j \neq 0$. If $\theta \in (0, 1)$ and further $C_{fix,i} > C_{fix,j} \neq 0$ equilibrium B_2 will be realized in the long run.

Proposition III: Equilibrium D_1 ***and*** D_2 ***are globally asymptotically saddle-***

node stable. *Equilibrium D_1 and D_2 are globally asymptotically saddle-node stable given, if and only if, $C_i \in (\underline{C}_i; \overline{C}_i)$ and $C_i > \frac{1}{e_i}\left[p(e_i - e_j) + e_j C_j\right]$ holds.* $\square$

Proof: Refer to appendix 4.2.4

Proposition III indicates that if $C_i \in (\underline{C}_i; \overline{C}_i)$ and $C_i > \frac{1}{e_i}\left[p(e_i - e_j) + e_j C_j\right]$ holds, only equilibrium D_1 or D_2 can be met which depends on the fact whether technology progress is excluded from the model (equilibrium D_1) or not (equilibrium D_2). Only for these two types of equilibria, a coexistence solution is possible which implies that $\underline{N} < N < \overline{N}$ and $s_i \in (0, 1)$ holds in equilibrium. Consequently, the profit-streams of both firms h tend to be asymptotically equal in the long run, which indicates that the equality

$$\left[(N+1) - \left(\frac{e_i\Omega_i - e_j\Omega_j}{p(e_i - e_j)}\right)\right] = \left[\frac{(e_i C_{fix,i} - e_j C_{fix,j})}{p(e_i - e_j)}\right] \tag{15}$$

with $\Omega_h \equiv (C_h(0) - C_{fix,h})exp[-\theta t]$ for $\{i, j\} \in h$ holds[8]. If $\theta = 0$, it follows from expression (15) that $N + 1 = \frac{e_i C_i(0) - e_j C_j(0)}{p(e_i - e_j)}$. Further, if $t \mapsto \infty$, $N + 1 \mapsto \frac{(e_i C_{fix,i} - e_j C_{fix,j})}{p(e_i - e_j)}$.

The latter two derived conditions can be traced back to equation (10) which coincides with $\dot{s}_i = 0$. The implication is that for meeting the steady state D_1, the fix cost structure is irrelevant but the initial cost structure is, whereas for the steady state D_2 this is quite the opposite[9]. Figure 1 provides a graphical representation of equilibrium D_1 and D_2 as the intersection of the three nullclides $\dot{N} = \dot{s}_i = \dot{C}_i = 0$.

2.3 Dynamic resource management and environmental policy

Our analysis conducted in section 2.2.2 and section 2.2.3 has identified three scenarios to which the dynamics to system converges in the long run: (1) firm i, linked with the high harvesting effort remains in the market. (2) Firm j, connected with the low harvesting effort survives, and (3) both firms coexist in the market, which leads

[8]The right hand's side of expression (15) is strictly positive, given $(e_i C_{fix,i} > e_j C_{fix,j})$ which is ensured by the assumption stated in equation (7) and $e_i > e_j$. The left hand side of equation (15) is strictly non- negative, given $(N + 1) \geq \left(\frac{e_i\Omega_i - e_j\Omega_j}{p(e_i - e_j)}\right)$.

[9]In fact, the irrelevance of fix costs qualitatively does not change the results, unless the initial cost structure is zero which is not imposed for the steady state D_2.

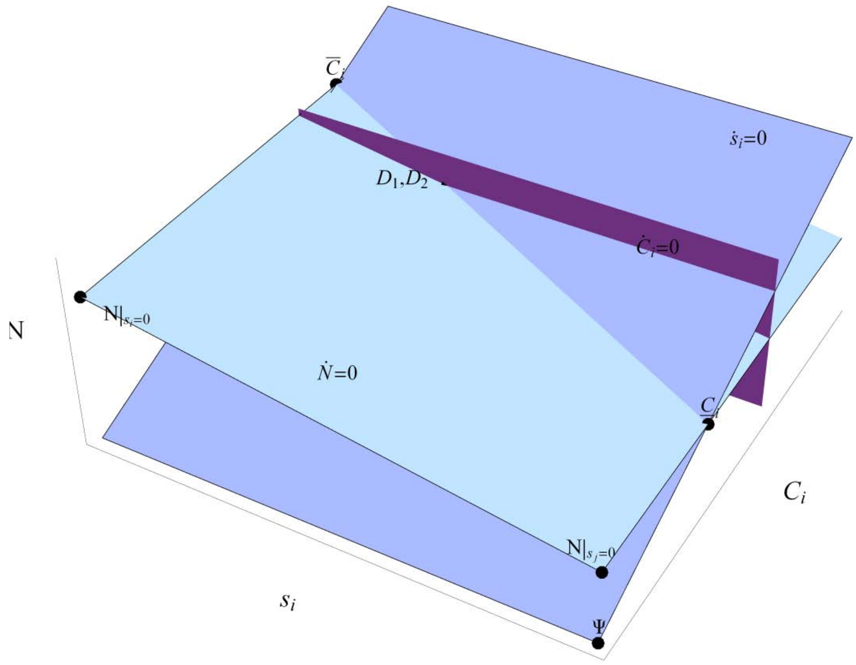

Figure 1: Graphical representation of equilibrium D_1 and D_2 of system (8)

to a sharing of the common-pool resource. In section 2.2.3, we have pointed to the fact that the cost structure heavily determines the long run outcome of the model. From a policy maker's point of view, this leads to the question whether there exists an environmental instrument affecting the outcome of the model regarding resource and market share evolution. For instance, output-sharing often fosters the commons problem by increased free-riding and therefore leads to insufficient effort levels, induced by unstable Nash equilibria. Heintzelman et al. (2008), however, show within a two-stage game that free-riding in partnerships could be beneficial because the *partner solution* naturally offsets the tendency of over-extraction by the tendency of free-riding (Schott et al. (2007)).

Albeit the partnership solution offers advantages compared to other resource management issues, such as the imposition of tax instruments or quotas, a practical problem coming along with this solution is the duration and stability of the incured partnership (Schott et al. (2007)). Because of the fact that the *partner solution* is indeed a one-shot, non-repeated game there is no guarantee that this solution holds forever. Hence, it should be clear that we have to ask for a dynamic resource management system. In section 2.2.3 together with section 2.2.4 (proposition III), we have shown that the coexistence-solution is, technically spoken, a saddle-node bifucation point which is in fact a degenerate equilibrium because small changes of the bifurcation parameter Θ lead to a sudden qualitative change of the entire dynamic system. Translated into economic terms, the coexistence solution is only realized given $C_i \in (\underline{C}_i; \overline{C}_i)$. Therefore, the policy maker is confronted with the problem how to meet $C_i \in (\underline{C}_i; \overline{C}_i)$. One way is to implement an *indirect effort* tax by taxing the profit streams. This leads to two problems: firstly, the implementation of an effective tax scheme requires the knowledge of the effort parameter e_h, and secondly, an issue which is only rarely considered in the relevant literature, technological progress leads to cost reduction over time, affecting the profits and consequently the tax rate. Related to our model, technological progress reduces firm-specific heterogeneity from two to one dimensions because equation (7) changes into $e_i > e_j$ due to the fact that $C_{fix,j} = C_{fix,i}$ or $C_{fix,j} = C_{fix,i} = 0$ holds in the long run, given $\theta \in (0, 1)$. But why is there an incentive for policy actions at all? The answer to this question gives the following proposition.

Proposition IV: Strong resource sustainability in the sense of resource conservation is not possible, given cost reducing technological progress is relevant and policy interventions ceased. □

Proof of Proposition IV

The following proof is based on a contradiction.

1. Case: $C_{fix,j} = C_{fix,i}$

    (a) An existence condition for equilibrium B_2 is that $C_i > \underline{C}_i$ for all t. Obviously, this is not the case if $C_{fix,j} = C_{fix,i}$, because from equation (6) it follows for $t \mapsto \infty$ that $C_{fix}e_1 > C_{fix}e_2$ must hold in the long run. Realizing B_2, we must show that $C_{fix} > \overline{C}_{fix}$. Inserting C_{fix} in equation (12), we come to the conclusion that $e_2 > e_1$ which obviously contradicts $C_{fix}e_1 > C_{fix}e_2$. Therefore equilibrium B_2 cannot be met.

(b) From 1a), we know that $C_{fix} \leq \overline{C}_{fix}$. This implies that equilibrium D_2 theoretically can be met. To exclude this fix point, we have to show that $C_{fix} < \underline{C}_{fix}$. Evaluating this last expression yields that $e_1 > e_2$ which holds per assumption. Therefore, the only fixed point which can be realized is A_3.

2. Case: $C_{fix,j} = C_{fix,i} = 0$

   (a) Again, we concentrate on one of the existence condition for equilibrium B_2: $C_i > \overline{C}_i$ for all t. Inserting $C_{fix} = 0$ in equation (12), we find $e_2 > e_1$ which is in conflict to $e_1 > e_2$. Therefore, equilibrium B_2 cannot be met.

   (b) Further, we can exclude equilibrium D_2 as an potential equilibrium because of two reasons: firstly, for $C_{fix} = 0$, N would become negative ($N = -1$) and further $C_{fix} \notin (\underline{C}_{fix}, \overline{C}_{fix})$. Hence, only equilibrium A_4 can be realized.

From the proof above we can deduce that if on the one hand the heterogeneous cost structure across firms disappears, but on the other hand heterogeneity with respect to harvesting effort remains, firm i always remains in the market. Given this, equilibrium A_3 or A_4 are realized for sure. Clearly, this solution is suboptimal with respect to resource heterogeneity because there exists other equilibria resulting in higher levels of the resource stock in the long run due to the fact that $M\left(1 - \frac{e_j}{\xi}\right) > h(C_i) > M\left(1 - \frac{e_i}{\xi}\right)$, given $e_i > e_j$. ■

As we have seen from the proof of proposition IV, firm i associated with the high harvesting effort remains in the market. If the policy aim is to establish the output-sharing solution across heterogeneous firms or if resource conservation is the relevant point, introducing a tax scheme with the cut of tax rate

$$\bar{\tau}_{\pi_i} = 1 - \left[\frac{\left(p - \frac{C_j}{1+N}\right) e_j}{\left(p - \frac{C_i}{1+N}\right) e_i}\right] \in [0,1] \tag{16}$$

is one dynamic instrument realizing this goals, even in the long run.

Proposition V: Only if $\bar{\tau}_{\pi_i} \geq \tau$, the tax scheme is effective. *More specifically, if $\bar{\tau}_{\pi_i} = \tau$, the coexistence solution is realized, and given $\bar{\tau}_{\pi_i} > \tau$, resource conservation is guaranteed in the long run.* □

Proof of Proposition V

Given $\bar{\tau}_{\pi_i} > \tau$, it follows that $\pi_j > \pi_i(1-\tau)$ which implies $\Theta < 0$. Further, $\bar{\tau}_{\pi_i} = \tau$, we have $\pi_j = \pi_i(1-\tau)$ which coincides with $\Theta = 0$. If $\pi_j < \pi_i(1-\tau)$, this is equivalent to $\Theta > 0$. Therefore, only $\bar{\tau}_{\pi_i} \geq \tau$ guarantees an effective tax scheme. ■

Provided technological progress is a relevant issue, we are able to derive the following proposition VI.

Proposition VI: If technological progress is relevant, the profit tax turns out to be a pure effort tax in the long run. □

Proof of proposition VI

Given $\theta > 0$, $C_{fix,j} = C_{fix,i}$ or $C_{fix,j} = C_{fix,i} = 0$ holds in the long run, and thus the effective tax rate changes to $\bar{\tau}_{\pi_i} = 1 - \left[\frac{e_j}{e_i}\right] \in [0, 1]$. ■

In this section, we have shown that output-sharing can be realized only under some conditions, given technological progress is relevant. It is worth to note that the derived effective tax rate (16) is not an one-shot tax rate, but even holds in the long run, with the emphasis that it changes its interpretation from a profit to a pure effort tax, due to the fact that $\bar{\tau}_{\pi_i} \to 1 - \left[\frac{e_j}{e_i}\right]$ for $t \to \infty$. Thus, the tax scheme is temporal effective and can be used to implement the output-sharing solution for all t, even for the case of heterogeneous firms.

2.4 Numerical illustration and discussion

In this section, we conduct a simulation exercise of our model. The main target of this numerical exercise is first to illustrate the stability of the different equilibria with and without technological progress. Secondly, we proof the remaining propositions numerically.

For the simulation exercise, we arbitrarily choose the parameter values as follows: We set $C(0)_i = 10$, $C(0)_j = 9$, $C_{fix,i} = 7$, $C_{fix,j} = 6$, $e_i = 0.5$, $e_i = 0.4$, $\xi = 0.6$ $M = 2$, $N(0) = 1$, $p = 9$ $s(0)_i = 0.75$ and $s(0)_j = 0.25$.

Obviously, $e_1 C_1 > e_2 C_2$ holds. For the cost range $(\underline{C}_i; \overline{C}_i)$ we compute $(9.6; 10.2)$ and thus we will realize the coexistence equilibrium D_1, with the corresponding steady state values $s_i^{D_1} = 0.33$, $N^{D_1} = 0.56$ and $C_i^{D_1} = 10$ for firm i and $s_j^{D_1} = 0.67$ and $C_j^{D_1} = 9$ for firm j. The flow of system (17) is graphically represented with figure 2. Additionally, a representative trajectory is represented as a dashed line, which starts from $s(0)_i = 0.75$ and $N(0) = 1$ together with $C(0)_i = 10$. The plotted vector field supports the saddle-node birfucation point analysis conducted in proof 4.2.4.[10]

In appendix 4.3, you will find plotted time series of the the firm-specific market shares s_h, the profits per unit π_h, the average profit across the market $\overline{\pi}$ as well as the evolution of N for $T = 500$ simulated periods. From proposition III together with equation (15), we know that firm specific profit streams must coincide in the steady state. Looking at panel (a) of figure 5, we observe that after around 400 periods the steady state is realized.

The profit streams coincide which can be seen from the bottom graph of panel (a) of figure 5. This is conform to proposition III. But what is the inherent dynamic of this model? Let us have a look at the upper graph of panel (a) of figure 5.

Right at the beginning of the market share evolution process, firm i is confronted with a cost disadvantage compared to firm j which cannot be balanced entirely by the effort advantage given by $e_i > e_j$. Hence, $\pi_i < \pi_j$ which leads to a considerably market share loss of firm i for the first 150 periods. At the same time, firm j can dramatically improve the market share ratio compared to firm i which is due to the selection process defined in equation (4). At the same time, N is growing as the market share of firm j rises. At around 400 periods the market share ratio becomes constant because of the fact that the profit share is constant. This implies that N must be constant, too. Interestingly, the average profit per unit nearly collapses to zero, as firm-specific profits do. This is due to the fact that firm i starts with a relatively high market share $s(0)_i = 0.75$, and thus N is considerably decreasing as $\frac{N}{N+1}$ does. Consequently, firm-specific profits per unit are decreasing as the

[10]With respect to figures 1, 2 and 3, $\underline{s}_i$ is directly associated with $\underline{C}_i$ whereas $\overline{s}_i$ is linked to $\overline{C}_i$.

average profit per unit $\overline{\pi}$. After 25 periods the profits per unit pass through their individual reversal points because the market share of j is increasing and further $\frac{N}{N+1}$ is increasing in N which leads to a recovery of the profit streams until period $t = 400$, where profit streams coincide at a steady state value of $\pi_h = \overline{\pi} = 0.46$. At the end of the day, we observe a coexistence of both firms engaged in harvesting, even in the long run. Again, this can be interpreted as a output-sharing solution without communication (Schott et al. (2007)).

The before derived results are only valid if technological progress is excluded. Let us assume from now that technological progress is relevant. Without loss of generality, we set $\theta = 0.50$. The cost range $(\underline{C}_i; \overline{C}_i)$ is now a function of firm-specific fix costs $C_{fix,h}$. This follows from equation (6). For given values $C_{fix,i} = 7$, $C_{fix,j} = 6$, the cost range can be computed as $(7.2; 7.8)$ and therefore the existence condition for equilibrium A_2 is met. This implies that firm j which earns profits considerably below the average cost level leaves the market[11]. N, under this scenario, is a falling function in N until the steady state level with $N^{A_2} = 0.33$, $\overline{\pi} = \pi_j = 0.46$ and $\pi_i = 0.44$ is reached. Compared to the coexistence solution, N^{A_2} is clearly below N^{D_1}, because $e_i > e_j$. Panel (b) in figure 5 replicates this scenario. Figure 3 shows the dynamic of the reduced system (8) and the asymptotically globally stability of the fixed point A_2. It is worth to note that realizing the coexistence solution requires an effective cut off tax rate of $\bar{\tau}_{\pi_i} = 4.28\%$, given $\theta = 0.50$[12].

Now assume that $C_{fix,i}$ is set to $C_{fix,i} = 8$ which is obviously above the defined cost range of $(7.2; 7.8)$. From this point of view, only the asymptotically globally stable fixed point B_2 can be met which can be seen in figure 4. In this case, firm j earns profits slightly above π_i for all t. Because of the inherent dynamic of the reduced system (8), firm i is shacked out of the market indicating that the resource conservation of the common-pool resource can be meet at best.

Because of the resulting monopoly of firm j, the stock of N will be the highest in the steady state compared to N^{A_2} and N^{D_1} with $N^{B_2} = 0.67$. As a consequence of that, profits per unit N will be the highest in the steady state for both firms, with

[11] This result is conform to proposition IV.

[12] This replicates the implications derived from proposition V and VI

$\overline{\pi} = \pi_j = 0.85$ and $\pi_i = 0.83$, which can be seen in panel (c) of figure 5.

The last point which remains open is to illustrate proposition IV numerically. Reducing firm-specific heterogeneity which implies setting $C_{fix,h} = 6, \forall :$, only equilibrium A_4 can be realized[13]. Secondly, we set $C_{fix,h} = 0$. Now fix point A_3 is obtained which is qualitatively comparable to A_2 and A_4[14]. Obviously, proposition IV holds numerically.

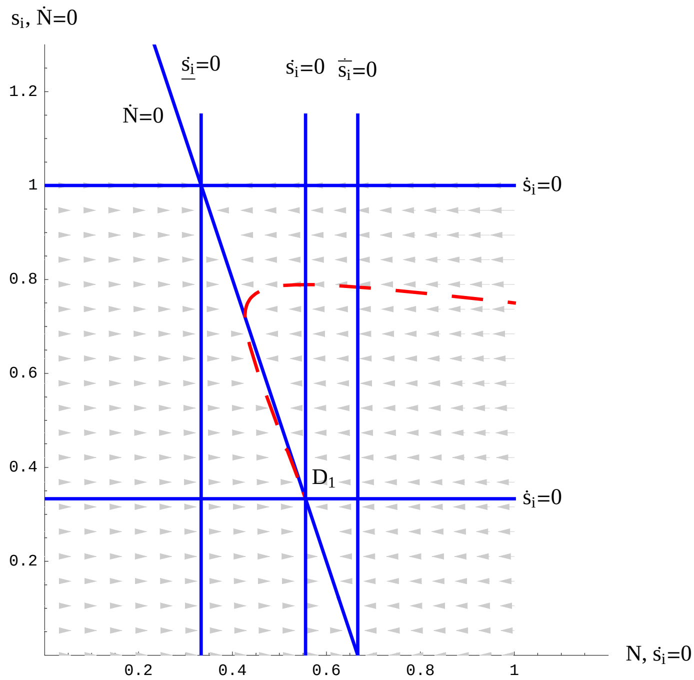

Figure 2: Trajectory and corresponding steady state D_1 of the reduced system (8)

3 Conclusion

In this paper, we have investigated the relationship between market dynamics, dynamic resource management and environmental policy in a comprehensive co-evolutionary framework. This framework perfectly fits to the discipline of ecological economics (Boulding (1978), Boulding (1981)).

[13]Realizing A_2, the profits per unit N reads as: $\overline{\pi} = \pi_i = 0.56$ and $\pi_j = 0.44$.

[14]The profits per unit N reads as: $\overline{\pi} = \pi_i = 1.12$ and $\pi_j = 0.89$.

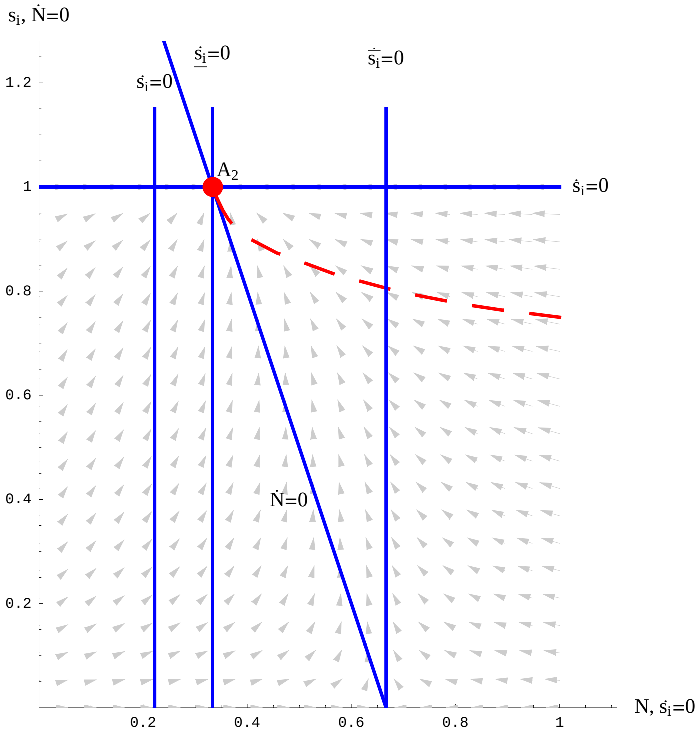

Figure 3: Trajectory and corresponding steady state A_2 of the reduced system (8)

In contrast to static market entry games, in this paper we draw attention to the effects of market dynamics on resource dynamics et vice versa, because (1) we have shown that feedback processes are necessary for obtaining a better understanding of what drives the *dynamics* between the evolution of common-pool resources and the number of harvesters and more importantly, (2) this analysis has provided an environment discussing sustainability in an appropriate inasmuch dynamic way.

By explicitly controlling for feed-back processes between market share-, resource- and cost evolution, our analysis conducted in section 2.2.2 and section 2.2.3 has identified three scenarios to which the dynamics of this co-evolutionary system converge in the long run: (1) firm i, linked with the high harvesting effort remains in the market. (2) Firm j, connected with the low harvesting effort survives, and (3) both firms coexist in the market which leads to a sharing of the common-pool resource. We have proofed that the derived equilibria associated with (1) and (2) are globally, asymptotically stable, whereas the equilibria (3) are globally saddle-node stable.

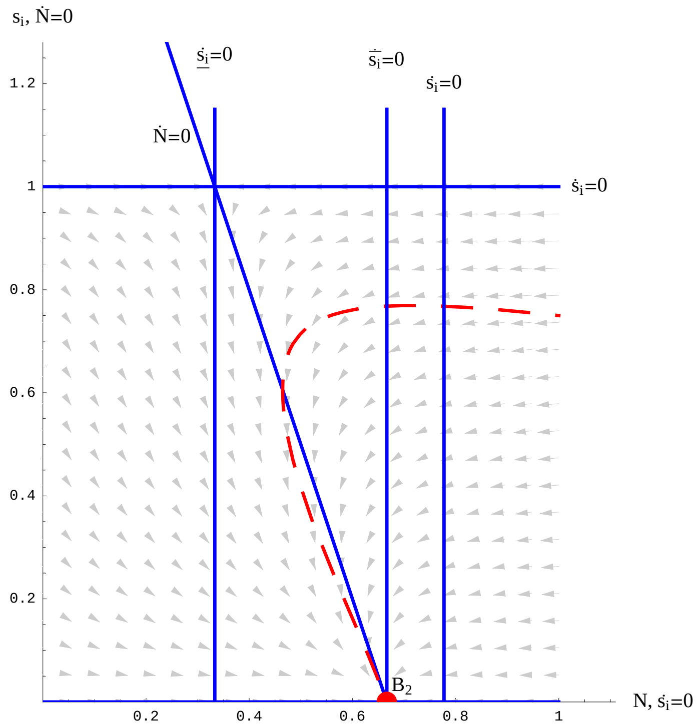

Figure 4: Trajectory and corresponding steady state A_2 of the reduced system (8)

Moreover, we have shown that the cooperative solution (2) does not necessarily lead to a lower stock of the common-pool resource compared to the non-cooperative solutions. Allowing harvesting cost-reducing technological progress, we further conclude that strong resource sustainability is not possible, given policy interventions ceased. This implies a dynamically sustainable resource management plan together with a concerted policy scheme. On the basis of a profit-tax, which turns out to be a pure effort tax in the long run, we introduce a time efficient tax scheme which guarantees strong sustainability in the long run even across heterogeneous firms. Therefore, we can conclude that a tax scheme is not seen as a substitute to a partnership solution dealing with the common-pool problem, but should be rather treated as an instrument establishing such a solution in the sense of a policy mix.

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4 Appendix

4.1 Steady states

Steady State	N^*	C_i^*	s_i^*	θ	cost structure	market structure
B_1	$M\left(1-\frac{e_j}{\xi}\right)$	$C_i(0)>0$	0	0	$C_i(0)>C_j(0)\neq 0$	monopoly for firm j
A_1	$M\left(1-\frac{e_i}{\xi}\right)$	$C_i(0)>0$	1	0	$C_i(0)>C_j(0)\neq 0$	monopoly for firm i
D_1	$h(C_i)$	$C_i(0)>0$	$g(N)$	0	$C_i(0)>C_j(0)\neq 0$	duopoly
H_1	0	$C_i(0)>0$	$s_i(0)$	0	$C_i(0)>C_j(0)\neq 0$	depends on $s(0)_i$
B_2	$M\left(1-\frac{e_j}{\xi}\right)$	$C_{fix,i}>0$	0	$\theta\in(0,1)$	$C_{fix,i}>C_{fix,j}\neq 0$	monopoly for firm j
A_2	$M\left(1-\frac{e_i}{\xi}\right)$	$C_{fix,i}>0$	1	$\theta\in(0,1)$	$C_{fix,i}>C_{fix,j}\neq 0$	monopoly for firm i
D_2	$h(C_i)$	$C_{fix,i}>0$	$g(N)$	$\theta\in(0,1)$	$C_{fix,i}>C_{fix,j}\neq 0$	duopoly
H_2	0	$C_{fix,i}$	$s_i(0)$	$\theta\in(0,1)$	$C_{fix,i}\geq 0$ for $\{i,j\}\in h$	depends on $s(0)_i$
A_3	$M\left(1-\frac{e_i}{\xi}\right)$	0	1	$\theta\in(0,1)$	$C_{fix,h}=0$ for $\{i,j\}\in h$	monopoly for firm i
A_4	$M\left(1-\frac{e_i}{\xi}\right)$	$C_{fix,i}>0$	1	$\theta\in(0,1)$	$C_{fix,i}=C_{fix,j}\neq 0$	monopoly for firm i

Table 1: Summary of the derived steady state conditions for system (8)

4.2 Proof of the global stability of the obtained steady states of system (8)

4.2.1 Remarks

In this section, we proof the local as well as the global stability of the identified fix-points of system (8). First, we proof the local stability of the obtained steady states of the non-linear system (8). It is known that for a hyperbolic fix-point (the real part of the Eigenvalues are nonzero), the flow of the linearized fix-point is homeomorphic to the non-linear flow (sufficiently close to the fix-point). Before we proceed, we transform system (8) which consists of three differential equations to a system of two differential equations. We can follow this way because we know that equation (5) is a first order autonomous differential equation with the solution given by equation (6). Inserting expression (6) into equation (8) leads to

$$\begin{cases} \dot{N} &= \xi N \left[1 - \frac{N}{K}\right] - \left[\sum_h s_h f_h(N)\right] \\ \dot{s}_i &= s_i(1-s_i)(\pi_i - \pi_j). \end{cases} \tag{17}$$

The Jacobian matrix of system (17) reads as:

$$Jac \equiv \begin{bmatrix} \frac{\partial \dot{N}}{\partial N} & \frac{\partial \dot{N}}{\partial s_i} \\ \frac{\partial \dot{s}_i}{\partial N} & \frac{\partial \dot{s}_i}{\partial s_i} \end{bmatrix} =$$

$$= \begin{bmatrix} -\xi(\frac{N}{M}) & -N(e_i - e_j) \\ s_i(1-s_i)\frac{\Theta}{(1+N)N} & (1-2s_i)\Theta \end{bmatrix}, \tag{18}$$

with $\Theta \equiv \frac{N}{N+1}\left[p(e_i - e_j) - e_i c_i + e_j c_j\right]$.

4.2.2 Proof of proposition I

We start with the proof of the local stability of equilibrium A_1, A_2, A_3 and A_4 by evaluating the Jacobian matrix at A_1, A_2, A_3 and A_4 each. Table 2 shows the Eigenvalues and their respective signs for each fix-point.

The sign of the second Eigenvalue χ_2 for the steady states A_1, A_2, A_3 and A_4 is clearly determined to be negative due to the fact that $N > 0$. However, the sign of the first Eigenvalue for A_1 and A_2 is not determined ex ante: It could be positive, zero or negative.

If the sign is positive, we obtain a saddle path for A_1 and A_2. If χ_1 and χ_2 are negative, A_1 and A_2 define a stable node. Given the Eigenvalues are zero, we obtain a later line equilibrium because the Jacobian matrix turns out to be singular.

From proposition I, it follows that $s_i = 1$ which leads to the conclusion that A_1 and A_2 are strictly negative because $\Theta > 0$ which stems directly from $\pi_i > \pi_j$. The first Eigenvalue for the fix-points A_3 and A_4 is clearly negative due to the fact that $e_i > e_j$. Therefore, the fix-points A_1, A_2, A_3 and A_4 are locally stable.

Steady State	Eigenvalue χ_1	Eigenvalue χ_2	Fix-point
A_1	$-\Theta^{A_1} \frac{N^{A_1}}{1+N^{A_1}} < 0$	$-\frac{\xi N^{A_1}}{K} < 0$	stable node
A_2	$-\Theta^{A_2} \frac{N^{A_2}}{1+N^{A_2}} < 0$	$-\frac{\xi N^{A_2}}{K} < 0$	stable node
A_3	$\frac{p(e_j - e_i)}{1+N^{A_3}} < 0$	$-\frac{\xi N^{A_3}}{K} < 0$	stable node
A_4	$\frac{p(e_j - e_i)}{(1+N^{A_4})} + \frac{C_{fix}(e_i - e_j)}{(1+N^{A_4})^2} < 0$	$-\frac{\xi N^{A_4}}{K} < 0$	stable node

Table 2: Summary of the derived steady state conditions for system (8)

The next point is to show that the equilibrium points A_1, A_2, A_3 and A_4 are asymptotically globally stable. This can be done by ruling out closed orbits. Referring to the literature, two important results provide sufficient conditions that rule out the possibility of periodic solutions: the Benedixon's and Dulac's criterion, whereas it turns out that the Benedixon´s criterion can be traced back to the Dulac's criterion as a special case.[15]

Theorem 1: Dulac's criterion. Assume that $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ is a continuously differentiable vector field, on a simply connected open subset of $\mathcal{R} \times \mathcal{R}$. If there exists a continuously differentiable, real-valued function $k(\boldsymbol{x})$ with $\{N, s_i\} \in \boldsymbol{x}$ such that

[15] The implication is, if neither of these two mentioned criteria are satisfied, there may be periodic solutions or not.

$\nabla \cdot (k\dot{\boldsymbol{x}})$ has one sign throughout $\mathcal{R}$, there are no periodic orbits of the autonomous system in $\mathcal{R}$. □

Proof: Assume that there is a closed orbit $\mathcal{C}$ in the simple connected region $\mathcal{R}$. Let $\mathcal{D}$ denote the interior of $\mathcal{C}$. When $\mathcal{C}$ is transversed counterclockwise, Green's Theorem in the plane gives the following identity:

$$\int\int_{\mathcal{D}} \nabla \cdot (k\dot{\boldsymbol{x}})\, d\mathcal{D} = \oint_{\mathcal{C}} (k\dot{\boldsymbol{x}} \cdot \boldsymbol{n})\, dl, \tag{19}$$

where $\boldsymbol{n}$ is the outward normal and dl is the element of *arc* length along $\mathcal{C}$. The integral on the left hand's side of equation (19) must be nonzero, because $\nabla\ (k\dot{x})$ has one sign in $\mathcal{R}$. The line integral on the right hand side of equation (19) equals to zero because $\mathcal{C}$ is a trajectory. Thus, the tangent vector of $\mathcal{C}$, $\dot{\boldsymbol{x}}$, is orthogonal to $\boldsymbol{n}$ which leads to $\dot{\boldsymbol{x}}\boldsymbol{n} = 0$. This contradiction implies that no such $\mathcal{C}$ can exist. ■

The problem which comes along with applying the Dulac's and Benedixon's criterion is that it is sometimes pretty hard determining an appropriate weighting function $k(\boldsymbol{x})$, because there is no algorithm providing such a function. With respect to system (17), we choose $k(\boldsymbol{x}) = 1$[16] which, as we will see, is a good choice helping us to provide a sufficient condition for the exclusion of periodic orbits.

Applying Dulac's criterion, we can write:

$$\nabla \cdot (k\dot{\boldsymbol{x}}) = \frac{\partial}{\partial s_i}(k\dot{s}_i) + \frac{\partial}{\partial N}\left(k\dot{N}\right). \tag{20}$$

Using the steady state condition $s_i = 1$ associated with the steady states A_1, A_2, A_3 and A_4, we can evaluate expression (20) further as:

$$\nabla \cdot (k\dot{\boldsymbol{x}}) = -\frac{\xi N}{K} + (1 - 2s_i)\Theta = -\frac{\xi N}{K} - \Theta < 0 \tag{21}$$

Expression (21) turns out to be strictly negative, given $N > 0$ by assumption and $\Theta > 0$ because of the fact that $\pi_i > \pi_j$. Hence, $\nabla(k\dot{\boldsymbol{x}})$ does not change sign in $\mathcal{D}$. Since the region $s_i = 1$ and $N > 0$ is simply connected and $k(\cdot)$ and $\boldsymbol{f}(\cdot)$ are smooth, Dulac's criterion implies that there are no closed orbits in the positive quadrant. Therefore, A_1, A_2, A_3 and A_4 are asymptotically globally stable. ■

[16] For this choice, the Dulac's and Benedixon's criterion coincide.

4.2.3 Proof of proposition II

The steady states B_1 and B_2 are locally stable which can be seen from table 3. Please note that Θ turns out to be negative because of $\pi_j > \pi_i$.

Steady State	Eigenvalue χ_1	Eigenvalue χ_2	Fix-point
B$_1$	$\Theta^{B_1}\frac{N^{B_1}+1}{N^{B_1}} < 0$	$-\frac{\xi N^{B_1}}{K} < 0$	stable node
B$_2$	$\Theta^{B_2}\frac{N^{B_2}+1}{N^{B_2}} < 0$	$-\frac{\xi N^{B_2}}{K} < 0$	stable node

Table 3: Summary of the derived steady state conditions for system (8)

Applying Dulac's criterion once again, we find with the steady state condition $s_i = 0$:

$$\nabla \cdot (k\dot{\boldsymbol{x}}) = -\frac{\xi N}{K} + \Theta < 0 \tag{22}$$

which is clearly negative, since $\Theta < 0$. Hence, the fix-points B_1 and B_2 are asymptotically globally stable since the region $N > 0$ and $s_j = 1$ is simply connected and $k(\cdot)$ and $\boldsymbol{f}(\cdot)$ fulfill the smoothness conditions implied by Dulac's criterion. ■

4.2.4 Proof of proposition III

Proposition III tells that $C_i \in (\underline{C}_i; \overline{C}_i)$, which implies $\pi_i = \pi_j$. Further, the condition $C_i > \frac{1}{e_i}\left[p(e_i - e_j) + e_j C_j\right]$ must hold. If $\pi_i = \pi_j$, it follows immediately that $\Theta = 0$. For $\Theta = 0$, the Jacobian becomes singular. Thus, at least one Eigenvalue must be zero which can be seen from table 4.

Steady State	Eigenvalue χ_1	Eigenvalue χ_2	Fix-point
D$_1$	0	$-\frac{\xi N^{D_1}}{K} < 0$	saddle-node
D$_2$	0	$-\frac{\xi N^{D_2}}{K} < 0$	saddle-node

Table 4: Summary of the derived steady state conditions for system (8)

For this constellation, system (17) exhibits an entire line of fix-points on $g(N)$, whereas the Eigenvector associated with the Eigenvalue zero is the direction vector of the system. We know that one existence condition for D_1 and D_2 postulates that $\Theta = 0$. Obviously, any marginally, infinitesimally change of Θ in the positive or negative direction drives the flow of system (17) completely out of these equilibria towards A_1 or A_2 [17] or to B_1 or B_2[18]. In the case of $\Theta = 0$, linearization of system (17) is not sufficient to deduce any information about the dynamic behaviour of the non-linear system (17), because $\Theta = 0$ serves as a cut-off variable which separates saddle path equilibria and nodes[19]. Hence, Θ is defined as the so-called fold bifurcation point, because one stable node and one saddle collide and both disappear right at $\Theta = 0$. A necessary condition for fold bifurcation is that $det(Jac) = 0$ or $\chi_1 = 0$, which obviously holds for system (17). Because of the fact that $tr(Jac) \neq 0$, we call the fold bifurcation equilibrium a degenerate equilibrium[20].

If we could further exclude the possibility of closed orbits, obviously no limit circle bifurcation can occur globally. To show this, we use the Poincaré map, together with Dulac's criterion. As shown in section 4.2.2 and section 4.2.3, the steady states A_1 to A_4 and B_1 and B_2 are globally asymptotically stable. With respect to a Poincaré mapping, this implies that every trajectory from a given starting point converges to the steady state und thus no closed orbits may occur. Referring to equation (20) and considering the fact that $\Theta = 0$, we obtain:

$$\nabla \cdot (k\dot{\boldsymbol{x}}) = -\frac{\xi N}{K} < 0 \tag{23}$$

which is clearly negative due to the fact that $N > 0$. Since the region $N > 0$ and $s_i \in (0, 1)$ is simply connected and $k(\cdot)$ and $\boldsymbol{f(\cdot)}$ again fulfill the smoothness conditions implied by Dulac's criterion, the existence of limit circles can be ruled out. Hence, no limit circle bifurcation can occur globally and thus the steady states D_1 and D_2 are saddle-node-stable. ■

[17] This implies that $\Theta > 0$ and $\theta \in (0, 1)$ or $\theta = 0$.

[18] This requires that $\Theta < 0$ and $\theta \in (0, 1)$ or $\theta = 0$.

[19] Suppose for a moment that $\Theta > 0$. Then A_1 and A_2 can be described as stable nodes, but B_1 and B_2 are saddle path equilibria, et vice versa for $\Theta < 0$.

[20] The non-degenerate conditions are $tr(Jac) = 0$ or $\chi_2 \neq 0$.

4.3 Time series for s_h, π_h, $\bar{\pi}$ and N

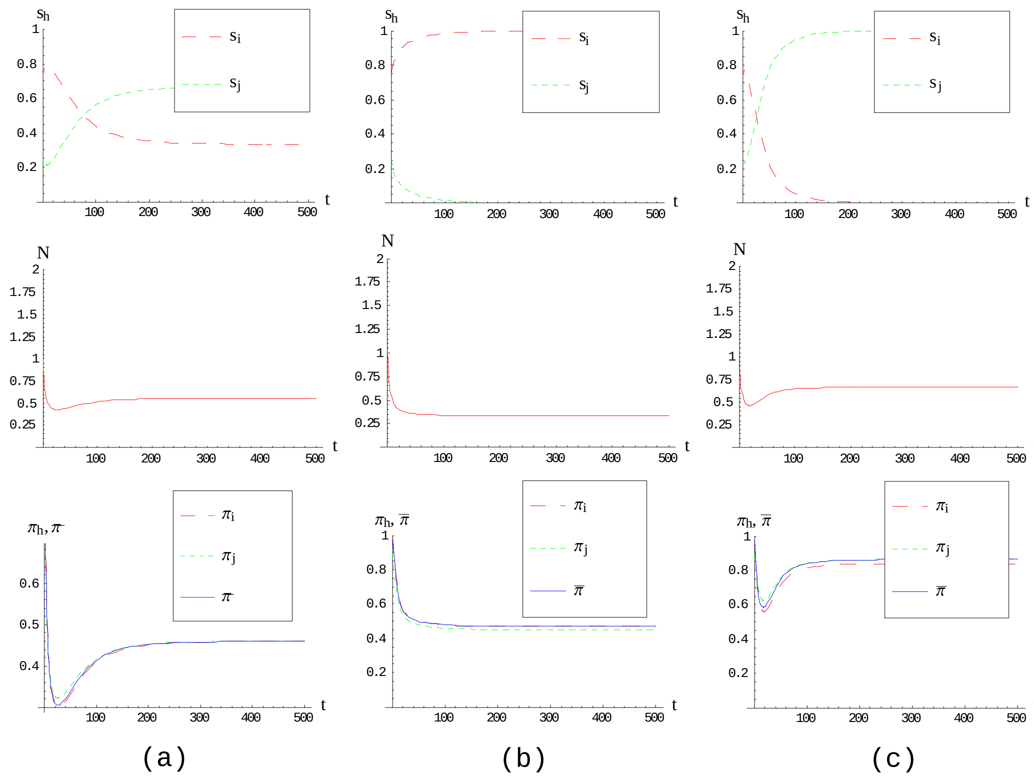

Figure 5: Computed time series based on the simulation exercise conducted in section 2.4