A Supply Side Story for a Threshold Model:
Endogenous Growth of the Free and Open Source Community

By

Francesco Rullani and Lorenzo Zirulia

Danish Research Unit for Industrial Dynamics

www.druid.dk

A Supply Side Story for a Threshold Model: Endogenous Growth of the Free and Open Source Community

Francesco Rullani
Department of Economics and Business
LUISS Guido Carli
E-mail: frullani@luiss.it

Lorenzo Zirulia
Department of Economics
University of Bologna
KITeS, Bocconi University
RCEA
E-mail: Lorenzo.zirulia@unibo.it

Abstract:
The study of social institutions producing and disseminating knowledge has mainly concentrated on two main concepts: Science and Technology. This paper examines a recent institutional form that seems not to resemble either of the other two; that is, knowledge-intensive communities, where individuals freely exchange knowledge through information and communication technology. Using free and open source software as an example, we develop a model where this phenomenon is confronted with Technology with respect to its ability to attract researchers.

1. Introduction

One of the main challenges the development of what has been called a "knowledge society" imposes to economic theory is the assessment of the changes that occurred in the institutions enabling knowledge production and diffusion. Moving from the production of physical goods to the production of knowledge, in fact, implies a reshaping of the structures upon which the economy has been constructed.

The economic discourse on institutions connected to knowledge production has its modern origins in the work of Dasgupta and David's (1987, 1994), who recognize two main institutional models, "Science" and "Technology", whose real manifestations are the academic world for the former and markets for technology (Arora et al., 2001) for the latter. Science is based on disclosure, rewards from priority and peer recognition and, today, on the public funding of knowledge production. Technology is based on secrecy and/or intellectual property rights and is profit-motivated. However, in the "shaded areas" of this dual system, we observed the emergence of a series of examples of an open model of knowledge production—where most agents develop and distribute knowledge without direct external funding or rents assured by the Intellectual Property Rights (IPR). Collective invention (Allen, 1983, Nuvolari, 2004), or communities of user innovators (Jeppesen and Frederiksen, 2006; von Hippel, 1988), are just two examples of the specific forms this model can take, and represent a challenge to the explanatory power of the Science/Technology dual system.

Nowadays, a particularly important and pervasive role is played by what David and Foray (2003) call knowledge-intensive communities. These communities are characterized by a significant number of members who produce and reproduce knowledge in a 'public' (often virtual) space, in which new information and communication technologies are intensively used to codify and transmit knowledge. The economic and social relevance of these communities is such that firms are in the need of understanding their principles to be able to relate to, and benefit, from them (Bonaccorsi et al., 2006).

One of the most prominent examples of knowledge-intensive communities, both in terms of economic and social impact, is the free and open source software community. In this community, a large number of individuals spread all over the world (Gonzalez-Barahona et al., 2008) cooperate online to create software, and release it freely and openly through the Internet. Anyone can enter the production process and report bugs, propose patches, cooperate with other developers on existing software, or launch new projects; while—thanks to the license scheme adopted by the community (mostly the General Public License, GPL) —no one can appropriate the software jointly developed. Firms have created business models to be able to

leverage the capabilities of the community and create a positive coexistence with it (Dahlander and Magnusson, 2005; Dahlander and Wallin, 2006; Dahlander, 2007; Bonaccorsi et al., 2006; Fosfuri et al., 2008)

Our paper develops a formal analysis that aims at capturing the essence of free and open source community, and of knowledge-intensive communities more in general, as *institutions*. In particular, we develop a model where "Community" confronts Technology with respect to the ability of attracting researchers. In the specific case of software, our interest is to understand the conditions under which voluntary, open source software development (Community) can co-exist in productive balance with proprietary software development (Technology), together with the determinants of the relative size of the two institutions and of the stability of the equilibrium configurations. In the modeling exercise, we represent the broad set of motivations that affects the functioning of communities and the individual choices between the two institutions, with a particular attention on the social dimension of communities. The abstraction of the model, which typifies both Community and Technology taking "a radical view" on its constituents, allows the identification of the determinants of different dynamics of these institutions, and allows economic actors, such as managers and practitioners, to better understand how the free and open source community works in an environment where it needs to compete for resources (i.e., developers) with other institutions traditionally related to the business sector. Moreover, it explicitly takes into account the role of each researcher's externalities towards those who work in her same institution and in the other institution, a mechanism usually left in the background in the literature.

Our first result show that Technology and Community may coexist. From an empirical point of view, this is consistent to what is observed in sectors like software, where similar, competing products are offered under proprietary and open regimes. What is notable in our discussion, and to the best of our knowledge new in the literature on open source, is that this result has been obtained with developers identical in every the respect via endogenous mechanisms within each institutional regime.

A second result is that multiple equilibria are pervasive. In particular, we identify a threshold that divides the realm of communities doomed to remain small from the set of communities that are able to grow *endogenously* fast and large. This threshold has been widely recognized in the literature about communities (e.g., Bonaccorsi and Rossi, 2003); what is new in our argument is that the threshold is not based on demand factors, but on the structure on developers' motivation, i.e., on supply side factors.

The paper is organized as follows. Section 2 elaborates the appreciative theory upon which the

argument is based. Section 3 describes the model, which takes as its starting point the analysis made by Carraro and Siniscalco (2003) comparing Technology and Science. Section 4 derives the main results, while Section 5 discusses their properties in light of the discussion in Section 2. Finally, Section 6 concludes.

2. The free and open source community as a knowledge-related institution

This section develops an appreciative theory the institutional status of knowledge-intensive communities, which will be formalized in Section 3. In particular, we will focus on the individual and social motivations prevailing in this institution, in order to characterize the main determinants of the payoff of the individuals acting in Community. In Section 3, we will confront them with motivations prevailing in Technology, whose characteristics are instead well known since the description of the dual system represented by Technology and Science in Dasgupta and David (1987, 1994).

As a reference point, we use the existing literature on free and open source. It was, in fact, in this literature that the question was asked relative to whether, and to what extent, the community model seemed to just resemble the academic world, i.e. Science in the Dasgupta and David formulation, so that, indeed, a new theory of communities as institution, and an explicit analysis of the comparison between Science and Technology, would be unnecessary.

In that respect, what we claim is that Science and communities are similar *ab origine*, in that they have the same *nature*: signaling, reputation, own-use, and social interaction are, in both cases, among the main factors determining the payoff function. However, the relative importance of those factors differs, since Science can rely on State intervention assuring funds and reputation-based incentives, while endogenous mechanisms, notably social motivations, are most important in Community.

2.1 Individual motivations in Community: just a "fancy Science"?

Bezroukov (1999a; 1999b) was among the first authors identifying a possible homomorphism between Community and Science in terms of the produced outcome, the involved incentives, the typology of teamwork and institution of collaboration, and the way in which the activity is financially supported. In particular, Bezroukov stresses the similar role of financial organizations, such as research institutes, universities, or private research labs, in providing the individuals with the funds to undertake their activities in the directions they desire; and the similarity

3

between the rules upon which Science is based and the practices typical of the free and open source community, which are also based on a public debate where priority over solutions and peer review are the crucial mechanisms used to regulate and direct individuals' activities (Dasgupta and David, 1987; Lee and Cole, 2003). Kelty (2001) stresses the same similarities. On the one hand he states that "[…T]he funding that supports many projects (in most cases indirectly) comes from those well-known scientific institutions" (Kelty, 2001, online). On the other hand, he also argues that the structure of incentives and the organization of the collaborative effort of developers and scientists are very close to one another, both based on rules connecting the openness of the results to the individual pursuit of recognition and reputation (see also Lerner and Tirole, 2002). Mustonen (2003) shares the same point of view: "The essential property of the copyleft licensing scheme [i.e. GPL] is that it creates a particular incentive structure… [that] has properties that are equivalent to the incentive structures of scientific communities" (Mustonen, 2003, p. 104). Following a similar path, Bonaccorsi and Rossi (2003) recall the origins of free and open source inside the university labs to claim that "Emerging as it does from the university and research environment, the movement adopts the motivations of scientific research" (Bonaccorsi and Rossi, 2003, p. 1245). Dalle and David (2003) also share a similar point of view, stressing the parallelism between the free and open source institutional setting and the rules of "open science," where "the norm of openness is incentive compatible with a collegiate reputational reward system based upon accepted claims to priority" (Dalle and David, 2003, pp. 3, 4). A similar point is made by Raymond (1998), who suggests that the correspondence between the two phenomena is just the outcome of the fact that the scientific and the free and open source enterprises had simply given the same answer to the same problem of collective knowledge production.

In addition to reputation-based incentives that relate peer-judgement to possible psychological and also financial rewards (Lerner and Tirole, 2002), own-use has also been underlined as an important motivation both in Science and in free and open source communities. This relates to the literature, inspired by von Hippel (1988), that has highlighted the role of users as a source of innovation in a wide range of fields (e.g. sports equipment, as in Franke and Shah, 2003). In the software case, an individual who has the knowledge and the tools to develop software can easily customized the software she uses and even produce the one she needs (von Hippel, 2001). As Bessen (2006) showed, in fact, software is a complex good that can be personalized much more effectively by skilled users than by manufacturers. Once produced, the software is very inexpensive to exchange through the Internet, so that even a very small reward can push developers to exchange the codes they have written (von Hippel and von Krogh, 2003). Own-

use has a relevant role also in the scientific environment, at least relative to software development. In the research fields where software is a fundamental instrument, as it happens in econometrics, for example, scientists often decide to develop the tools they need, and sometimes they decide to distribute their work widely and freely (Gambardella and Hall, 2006). Thus, the free and open source community and science also "overlap" with respect to the own-use incentive.

So, signaling one's talent, reputation and own-use, are the main individual incentives in action both in free and open source community and Science. However, similarities between Science and free and open source do not imply that the two systems simply coincide. Indeed, the point made by our paper is exactly that Science and free and open source do differ in some fundamental aspects, and those aspects matter in the ability of free and open source to attract researchers vis-à-vis Technology.

The first difference we stress refers to elements characterizing the modern functioning of academia are absent, or at least less relevant, in free and open source: the crucial role played by the State and the professionalization of the scientific career (Dalle and David, 2007). Even in a period of reduced bugdets, the public sector intervention in paying researcher wages and allocating funds is prominent. This direct involvement is much less relevant in free and open source. As long as professionalization is concerned, career advancements and access to funds in Science are strictly related to structured "reputational games" to which scientists must participate. As Dalle and David suggest "…one should observe that that the parallel [between Science and free and open source] is by no means exact: formal professional accreditation and institutional affiliation are salient de facto requirements for active participation in modern academic and public sector research communities, yet the computer programming and other software development tasks—whether in the commercial or the free and open-source spheres— remain activities that have resisted becoming 'professionalized.'" (Dalle and David, 2007, p. 393n4). Until now, free and open source has attracted many firms, and its economic dimension has considerably grown (Ghosh, Haaland, and Hall, 2008; Henkel, 2006). However, activity is still generally characterized by a large number of volunteers performing a large amount of the coding and of the related activities, and it is still based on a very informal and unspecified set of rules changing from one open source project to another (O'Neil, 2009).[1]

The State intervention and the professionalization of the scientific world have made priority rule

---

[1] Indeed, "In an historical perspective, the current structure of the [free and open source] community in these dimensions still resembles the initial stages of the development of open science, the era of "the West's 'amateur' and 'gentlemen scientists' of the late eighteenth and nineteenth centuries" (David et al., 2001, online).

and signaling much more prominent in Science that it is in open source. As a consequence, Science and free and open source do not differ in terms of the components of the utility function they assure to their researchers; they differ in terms of the weights they assign to these components. Indeed, in free and open source, surveys and empirical studies, such as the FOSS-EU survey (Ghosh et al., 2002), the Boston Consulting Group survey (Lakhani et al., 2002) and many others (Bonaccorsi and Rossi, 2006; David and Shapiro, 2008; Elliott and Scacchi, 2003), confirm that own-use related incentives are among the most important motivations, but find that reputation and signaling play a role (e.g., Roberts et al 2006) that is not crucial (Lakhani and Wolf, 2005).

A similar difference can be found with respect to the social dimension of the two institutions. The next paragraph is dedicated to this issue.

2.2. Social motivations in Community

In the context of the free and open source community, the social dimension has been analyzed with respect to such theories as gift economy (Raymond, 1998), communities of practice, or epistemic communities (Cohendet et al., 2001; Edwards, 2001; Lin, 2003a, 2003b, 2004a, 2004b; Mateos-Garcia and Steinmueller, 2008), and general reciprocity (Luo, 2002). In particular, the community of practice perspective (Wenger, 1998) can be particularly useful to describe in detail the passages of the social processes at work in the free and open source community.

Applying this perspective to the free and open source community means recognizing the central role of the process of "negotiation" (Lin, 2004a) that developers are involved in when creating software. Developing a common project together makes people continuously "renegotiate" the meanings connected with their own actions, giving sense to the common action and to the social context where it takes place. This negotiation of meanings leads to a continuous reshaping of the participants' vision of the world to adapt their *identities* to the social circumstances they are embedded in (Wenger, 1998). But changing individuals' identities means configuring in a new way the principles guiding their actions and their priorities, i.e., precisely those principles represented by their payoff functions. In other words, the interaction between the community members' leads to change in their identities that ultimately results in a modification of the importance in the elements of their payoff functions, that now take into account the priorities and rules shared by the whole community (Muller, 2006).

An empirical account of this process can be found in Bagozzi and Dholakia (2006) and Shah

(2006). Shah (2006) describes the evolution in developers' motivations as follows: "... a need for software-related improvements drives initial participation. The majority of participants leave the community once their needs are met, however, a small subset remains involved. For this set of developers, motives evolve over time and participation becomes a hobby." (Shah, 2006, p. 1000). Among possible explanations for this process, the author also identifies the hypothesis that the "interaction with the community leads to a shift in the individual's identity and self-perception" (Shah, 2006, p. 1011). This is the perspective taken by Bagozzi and Dholakia (2006), who write: "Initial participation by novice users is driven by specific task-oriented goals .... But over time, as the user comes to form deeper relationships with other [free and open source community] members, the community metamorphosizes into a friendship group and a social entity with which one identifies." (Bagozzi and Dholakia, 2006, p. 1111).

If the free and open source community is conceived as a community of practice, following Wenger's intuition (1998) the social mechanisms described above should act along the nexus of the communitarian ties, impacting on the structure of the payoff function and the relative importance of its constituting elements. This payoff structure can be adequately represented including and giving importance to three factors taken directly from Wenger's (1998) idea of community of practice.

The first aspect is related to the *communitarian activity*. What makes a group of people becoming a community is the construction of a social environment where identities are defined through a process that is interwoven with the activity of the community (e.g., in the free and open source case, producing software). All the processes take place *in* that environment and *thanks to* that environment. Thus, the common activity has a central role in the payoff function and depends on the effort of all the members of the community.

The second aspect is *personal involvement*: if a member's identity is strongly tied to the common activity —i.e. the project undertaken by the community—the effect of that activity on her payoff function is greater. For example, in the free and open source case, the GNU/Linux development has a greater effect on the payoff of a person who "believes" in the GNU/Linux project, greater than the payoff ensured by the simple usage of GNU/Linux (the most famous open source operating system). This translates in the model of Wenger's (1998) idea of *engagement*, where individuals are involved into a process of "renegotiation" of their visions of the world and reciprocal influence between them and the social environment of the community. The more a member invests in -and counts on- the common activity undertaken in the community to define her identity, the higher is the psychological payoff she gets from that activity. Thus, personal involvement is intended as endogenous to the development of the

community: it develops and becomes stronger (in terms of affecting members' behaviors) as the "volume" of the interaction grows. So, when a community grows, it not only becomes "quantitatively" stronger (e.g. it produces more software), but also "qualitatively," determining a higher average rate of the personal involvement of its core members. Thus, personal involvement can be considered a function of the community size.

The third aspect refers to *cooperation costs*: a group of people who collaborate is subject to free riding episodes. The group must then create some rules and enforcing mechanisms to sustain the cooperation and avoid free riding. The costs originate from activities such as monitoring others' behavior, spreading the information about it, discovering the break of a rule, and punishing the free rider. These costs are increasing with the number of the community members, so that they pose a limit to growth opportunities of the community.

Taking seriously the social side of the community means recognizing its role *also* in the scientific institution. Like the community, in fact, Science can also be considered a structure internally organized around a system of meanings debated and shaped by scientists. The scientific community—which is in fact a community—is the real protagonist of the scientific institution, allowing internalization of the rules, learning, production and allocation of reputation, and providing the space in which the produced knowledge is composed into a meaningful system. However, as discussed before, Science can rely on State intervention assuring funds and reputation-based incentives, so that the role of social motivation is less prominent. A community, instead, has a bottom-up growth, and must be able to generate its own reproduction process without external help. That is why social motivations, less relevant for the survival of Science, appear to have a central role in the free and open source community.

3. The model

Our paper builds on the work presented in Carraro and Siniscalco (2003), who compare Science and Technology in their "capability" to attract researchers. We substitute Science with the community, and perform the comparison with Technology along the same lines.

Our model can be summarized in the following terms. A population of *N* researchers is active in a given field of research. Researchers are identical, both in terms of preferences and productivity. Researchers exert effort to produce knowledge, and they can do this in two institutional settings: Technology and Community. It is assumed that researchers, before choosing how much effort to exert, choose which institution they intend to belong, based on an

expected payoff comparison. Participation to one institution is exclusive[2].

Technology and Community differ in their pay-off structure[3]. In the field of Technology, new knowledge is embodied in patents, so that it can be sold: economic return constitutes the main source motivation. In Community, benefits of new knowledge are associated to the own-use, signaling, and reputation motives (simplified in the model by a single parameter), while the social dimension is linked to the degree of personal involvement and to the communitarian activity. In terms of costs, in addition to effort costs, participation to Community involves cooperation costs. Technology and Community differ also in terms of spillovers from other researchers. In Technology, the knowledge produced by a researcher choosing this institution has a negative impact on the probability of others' success in knowledge creation, due to competition and limits imposed by property rights; in community instead the effect is positive because openness and cooperation. For the same reasons, when spillovers occur across institutions, it is assumed that externalities from Community to Technology are positive, while externalities from Technology to Community are negative.

Formally, the payoff from participating to Technology and Community are respectively:

$$\Pi_i^T = \Pr^T(x_i^T, X_{-i}^T, X^C)R^T - c^T(x_i^T) \tag{1}$$

$$\Pi_i^C = \Pr^C(x_i^C, X_{-i}^C, X^T)k^C - c^C(x_i^C) + e(n)Y(x_i^C, X_{-i}^C, X^T) - C(n) \tag{2}$$

In equation (1) and (2), *n* denote the number of researchers in Technology (and thus being *(N-n)* the number of researchers in Community). Defined $I = T, C$ as the index for institution, x_i^I is the individual effort of researcher *i* if active in institution *I*. X_{-i}^I and X^I represent the sum of efforts for all the researchers in *i*'s institution (excluding *i*) and in the other institution, respectively, $\Pr^I(\circ)$ is the probability of innovation (successful production of knowledge). It is assumed that $\dfrac{\partial \Pr_i^I}{\partial x_i^I} > 0$ (the higher the effort of the researcher, the higher the probability of innovation); $\dfrac{\partial \Pr_i^I}{\partial X_{-i}^T}(\dfrac{\partial \Pr_i^I}{\partial X_i^T}) < 0$ (externalities from Technology are always negative);

---

[2] As an alternative interpretation, researchers belong to both institutions, and the relevant decision is the allocation of resources (e.g. time and effort) to each organization.

[3] Of course, in the reality these differences are much more blurred. A researcher working for firm but embedded in the scientific debate with her colleagues from other firms can reach the same social motivation as an open source developer. Vice versa, the latter can find a job in an open source based company receiving a monetary incentive similar to that of the former. However, we seek to grasp the inner difference between the two institutions, and thus we magnify the difference in the payoff functions they offer to their researchers.

$\frac{\partial \Pr_i^I}{\partial X_{-i}^C} (\frac{\partial \Pr_i^I}{\partial X_i^C}) > 0$ (externalities from Community are always positive).[4] $c_i^I(\circ)$ represents the

cost of effort, with $\frac{\partial c_i^I}{\partial x_i^I} > 0$ and $\frac{\partial^2 c_i^I}{\left(\partial x_i^I\right)^2} < 0$

In equation (1), R^T represents the economic return from innovation in Technology in case of success. It may be represented by profits from entrepreneurial activity; or alternatively, $\Pr^T(x_i^T, X_{-i}^T, X^C)R^T$ may represent the expected wage for employed software developers. Similarly, in equation (2), k^C is the "prize" obtained by the success in innovating. k^C captures the different motivational dimensions already identified in the literature on the open source movement, from the increased reputation among peers and in the job market (Lerner and Tirole, 2002), to the possibility to use the produced knowledge (von Hippel, 2001). The other terms in (2) instead unfold the social dimension as we have defined it in the previous section. $e(n)$ is the

personal involvement in Community (with $\frac{\partial e(n)}{\partial n} < 0$); $Y(x_i^C, X_{-i}^C, X^T)$ is the value of the

communitarian activity, with $\frac{\partial Y}{\partial x_i^C} > 0$, $\frac{\partial Y}{\partial X^T} < 0$ and $\frac{\partial Y}{\partial X_{-i}^C} < 0$[5]; $C(n)$ are the cooperation costs

expressed as a function of n, with $\frac{\partial C(n)}{\partial n} < 0$. $\frac{\partial^2 C(n)}{\partial n^2} > 0$ and $C(N) = 0$.

Researchers' interaction is represented by a two-stage non-cooperative game where in the first stage each researcher decides whether entering Technology or Community, and in the second stage, after observing n, each agent decides simultaneously her effort level. The game is solved backward, computing the optimal effort of each researcher given N and n. Then, the analysis moves to the first stage. Following Carraro and Siniscalco (2003) we look at pure strategy Nash equilibria in which n^* researchers choose Technology in equilibrium, and consequently $N-n^*$

---

[4] We shall assume, and use these assumptions in the proof of Proposition 1, that the probability of innovation is strictly concave in x_i^I and that $\frac{\partial \Pr_i^I}{\partial x_i \partial X_{-i}^T} (\frac{\partial \Pr_i^I}{\partial x_i \partial X_i^T}) < 0$ and $\frac{\partial \Pr_i^I}{\partial x_i \partial X_{-i}^C} (\frac{\partial \Pr_i^I}{\partial x_i \partial X_i^C}) > 0$, i.e. individual efforts are strategic complements with total efforts in Community and strategic substitutes with total efforts in Technology.

[5] We shall also assume that Y is strictly concave in x_i^I and that $\frac{\partial Y_i^I}{\partial x_i \partial X_{-i}^T} < 0$ and $\frac{\partial Y_i^I}{\partial x_i \partial X_{-i}^C} > 0$, i.e. individual efforts are strategic complements with total efforts in Community and strategic substitutes with total efforts in Technology.

choose Community. Furthermore, we restrict to symmetric equilibria in terms of efforts within each institution. Consequently, we define $\Pi^T(n)$ and $\Pi^C(n)$, the reduced-form payoff in the first stage for a researcher choosing Technology and Community, as a function of the number of researchers in Technology. If *N* is large enough, the determination of an interior equilibrium *n** is well approximated by the condition:

$$\Pi^T(n*) = \Pi^C(n*) \tag{3}$$

so that *n* is treated as a continuous variable. In Section 4 we will also look at the stability of equilibria. An equilibrium *n** is (locally) stable if:

$$\frac{d\Pi^T(n*)}{dn} - \frac{d\Pi^C(n*)}{dn} < 0 \tag{4}$$

which implies that there is a neighborhood of *n** such that for any *n* in such a neighborhood the myopic (with respect to the choice of institution) best response dynamic adjustment process converges to *n**. Informally, an allocation of researchers between Technology and Community is stable if (sufficiently small) exogenous shocks in institutions size do not move the equilibrium away (permanently) from the initial configuration.

3.1 Technology, Science and Community: our model and Carraro and Siniscalco (2003)

As we previously mentioned, our framework is based on the model developed by Carraro and Siniscalco (2003), who describes the choice of researchers between Science and Technology. In that paper, the authors assume the following payoff function for participation to Science:

$$\Pi_i^S = F(n) + \Pr^S(N, x_i^S, X_{-i}^S, X^T)k^S - c^S(x_i^S) \tag{5}$$

Some of differences we put forth between Community and Science in Section 2 are captured by differences in the elements constituting equation (2) and (5). The role of the State in Science lead to the presence of a fixed salary *F(n)* (increasing in *n*), which moves upward the payoff from Science. Firms involved in free and open source can also provide their employees with fixed salary to work on their projects, but as discussed in the previous sections, this will be true only for a minor fraction of the millions of free and open source developers and contributors. Equation (2), instead, includes the positive value attached to personal involvement and communitarian activity that is present in Community.

Apart from the structural properties of the pay-off function, the discussion in Section 2 reflects also in the values reasonably taken by similar parameters in equations (2) and (5). It is the case

11

of k^S and k^C, which measure the private value of innovation in Science and Community, and thus the own-use, signaling and reputational incentives. We showed that while own-use is likely to have similar values in both environments, signaling and reputational incentives do not appear as fundamental in free and open source (in relative terms). We can thus assume that $k^S > k^C$. Science and Community may also differ in terms of externalities towards Technology. In Science the produced knowledge can be easily adapted and translated into an IPR regime by someone other than the innovator[6]. In the free and open source community, however, the GPL protects the produced knowledge, thus limiting this possibility (Gambardella and Hall, 2006). This does not cancel out the benefit that Technology has from the community production of software (ideas can be reused because GPL is not a patent), but GPL does limit the effects of communitarian externalities on the Technology payoff function. This implies that we could assume that the marginal effect of total efforts in Science on the probability of innovation in Technology is higher than the marginal effect of total efforts in community on the same probability.

3.2 Our contribution to the formal literature on free and open source

This paper contributes to a fast-growing literature developing formal models on diverse aspects of free and open source communities. A first stream of literature has looked at the conditions for developers (or user-developers) to contribute to free and open source communities, thus emphasizing supply-side and motivation issues. Bitzer and Schroder (2005), for instance, consider open source software as a public good, and develop a game-theoretic model of contribution by self-interested individuals, while Gambardella and Hall (2006) and Johnson (2006) considered the competition of the free and open source community and the IPR-based system in attracting developers. Our work is closely related to this literature. More specifically its contribution to it is twofold: on the one hand, it explicitly analyses the role of social motivations in explaining the relative attractiveness of the community model, and on the other hand it does that taking into account how spillovers link not only the members of the same institutions, but also those part of the competing one.

A second stream of literature has looked at competition between proprietary and open source software more from the consumers' point of view. Among others, Casadesus-Masanell and

---

[6] The Bay-Dhole act and the recent increase in the importance of patents in the scientific world enhanced precisely this process easier. However, the investigation of the impact of the adoption of Technology-based practices by a Science environment is outside the scope of this paper.

Ghemawat (2006) developed a dynamic, duopoly model between a profit-oriented firm and an open-source community; Economides and Katsamakas (2006) consider the two-sided competition between proprietary and open source platforms, with a particular attention to the incentives for complimentary good production; Lanzi (2009) jointly considers product differentiation, lock-in and network externalities, and consumers' experience in software use and implementation; Dalle and Jullien (2003) and Bonaccorsi and Rossi (2003) take a technology diffusion perspective, studying the conditions under which open source software can overcome an existing and dominant proprietary software. Our model can also be seen a contribution to this literature, since it can explain the co-existence of the models of software production, and then their competition, on the basis of the structure of developers' motivations.

As a third contribution of our model is its capability to show that the two institutions can coexistence even when developers are identical. Another model tries to bridge the demand and the supply sides of literature (Mustonen, 2003), but it assumes that developers are heterogeneous in their productivity, and that more productive developers choose the open source model.

4. Results

To solve the model, we first determine the equilibrium efforts in the second stage of game for given allocation of researchers in Technology and Community. Then we proceed backward by analyzing the first stage decision and determining equilibria and their stability properties.

4.1 Equilibrium efforts in the second stage

In the second stage of the game, each researcher, both in Technology and Community chooses the effort that maximizes her payoff given *n* and the effort choices of the other researchers. The first order conditions for payoff maximization in Technology and Community are given by:

$$\frac{\partial \Pi_i^T}{\partial x_i^T} = \frac{\partial \mathrm{Pr}^T(x_i^T, X_{-i}^T, X^C)}{\partial x_i^T} R^T - \frac{\partial c^T(x_i^T)}{\partial x_i^T} = 0 \tag{6}$$

$$\frac{\partial \Pi_i^C}{\partial x_i^C} = \frac{\partial \mathrm{Pr}^C(x_i^C, X_{-i}^C, X^T)}{\partial x_i^T} k^C - \frac{\partial c^C(x_i^C)}{\partial x_i^C} + e(n)\frac{\partial Y(x_i^C, X_{-i}^C, X^T)}{\partial x_i^C} = 0 \tag{7}$$

Since we are interested in symmetric Nash equilibria, equilibrium efforts in Technology and Community (as a function of *n*), denoted by $\hat{x}^T(n)$ and $\hat{x}^C(n)$, are implicitly defined by:

13

$$\frac{\partial \Pr^T(\hat{x}^T(n),(n-1)\hat{x}^T(n),(N-n)\hat{x}^C(n))}{\partial x_i^T}R^T - \frac{\partial c^T(\hat{x}^T(n))}{\partial x_i^T} = 0 \tag{8}$$

$$\frac{\partial \Pr^C(\hat{x}^C(n),(N-n-1)\hat{x}^C(n),n\hat{x}^T(n))}{\partial x_i^T}k^C - \frac{\partial c^C(\hat{x}^C(n))}{\partial x_i^C} + e(n)\frac{\partial Y(\hat{x}^C(n),\hat{x}^C(n),\hat{x}^T(n))}{\partial x_i^C} = 0 \tag{9}$$

Proposition 1, proved in the Appendix, characterizes the effect of *n* on the effort exerted by each researcher in Technology and Community.

Proposition 1 *An* increase *in group size reduces individual effort in Technology and increases it in Community. i.e.* $\dfrac{\partial \hat{x}^T(n)}{\partial n} < 0$ *and* $\dfrac{\partial \hat{x}^C(n)}{\partial n} < 0$.

The intuition of this result is straightforward. In Technology, an increase in group size increases competition within the group and reduces spillovers from Community, both effects being detrimental to the productivity of individual effort. In Community, an increase in size leads to more efforts because of the complementarity among researchers' investments and because of the lower negative externalities from Technology.

When we look at total efforts in each institution, i.e. $\hat{X}^T(n) = n\hat{x}^T(n)$ and $\hat{X}^C(n) = (N-n)\hat{x}^C(n)$, it is immediate to see that total efforts in Community is decreasing in *n*, i.e. increasing in its size. For Technology, instead, the effect is ambiguous. Following Carraro and Siniscalco (2003) we solve this ambiguity by assuming that the total effort is always increasing in group size also in Technology, i.e. $\dfrac{d\hat{X}^T(n)}{dn} > 0$.

Plugging equilibrium efforts in the payoff functions, we have the reduced-form payoff used for comparison in the first stage:

$$\Pi_i^T(n) = \Pr^T(\hat{x}^T(n),(n-1)\hat{x}^T(n),(N-n)\hat{x}^C(n))R^T - c^T(\hat{x}^T) \tag{10}$$

$$\Pi_i^C(n) = \Pr^C(\hat{x}^C(n),(N-n)\hat{x}^C,n\bar{x}^T)k^C - c^C(\hat{x}^C) + e(n)Y(\hat{x}^C(n),(N-n)\hat{x}^C,n\bar{x}^T) - C(n) \tag{11}$$

4.2 Equilibrium in the *first stage*

In order to identify the equilibria and their stability properties it is useful to derive the first derivatives of $\Pi_i^T(n)$ and $\Pi_i^C(n)$. By use of the envelope theorem, we obtain:

14

$$\frac{d\Pi_i^T(n)}{dn} = \left\{ \frac{\partial \Pr^T}{\partial X_{-i}^T} \frac{dX_{-i}^T}{dn} + \frac{\partial \Pr^T}{\partial X^C} \frac{dX_{-i}^C}{dn} \right\} R^T \tag{12}$$

$$\frac{d\Pi_i^C(n)}{dn} = \left\{ \frac{\partial \Pr^C}{\partial X_{-i}^C} \frac{dX_{-i}^C}{dn} + \frac{\partial \Pr^C}{\partial X^T} \frac{dX^T}{dn} \right\} k^C + \frac{\partial e}{\partial n} \left[\frac{\partial Y}{\partial X_{-i}^C} \frac{dX_{-i}^C}{dn} + \frac{\partial Y}{\partial X^T} \frac{dX^T}{dn} \right] - \frac{dC}{dn} \tag{13}$$

In order to simplify the proofs, but without affecting the qualitative discussion that follows, we shall assume that $\frac{d^2\Pi_i^T(n)}{dn^2} > 0$ (which is satisfied if the effort cost function is sufficiently convex) and $\frac{d^2\Pi_i^C(n)}{dn^2} < 0$ (which is satisfied whenever the coordination costs are sufficiently convex.). These assumptions guarantee the existence of at most three equilibria, reducing in this way the number of cases to be considered.

Next Proposition is proved in Appendix.

Proposition 2 *Payoff from* Technology *are always decreasing in the number of researchers in the group, i.e.* $\frac{d\Pi_i^T(n)}{dn}$ *is always positive. Instead, payoffs from Community are always increasing, always decreasing or first increasing and then decreasing in group size, i.e.* $\frac{d\Pi_i^C(n)}{dn}$ *can be i) always negative ii)* always *positive or ii) first positive and then negative.*

The intuition of the results in Proposition 2 closely mimics Proposition 1. Payoffs in Technology are decreasing in the size of this group because, first of all, more researchers in Technology implies more competition in the "patent races" and, second, it implies less researchers active in Community, and then lower positive spillovers. In Community, size of the group has a positive effect on researchers' payoff for three reasons: i) larger positive spillover within the group; ii) a positive impact on the communitarian activity; iii) a lower negative externalities from Technology. However, large communities incur in large coordination costs. This negative effect of group size can easily prevail for large groups.

We are now ready to state our main proposition on equilibria existence and stability.[7]

Proposition 3 *The equilibria of the game are characterized as follows:*

(Scenario I*) If* $\Pi_i^T(0) > \Pi_i^C(0)$ *and* $\Pi_i^T(N) > \Pi_i^C(N)$ *, but* $\Pi_i^T(n) < \Pi_i^C(n)$ *for some values of n, then there are* two *stable equilibria* $n_1^* \in (0, N)$ *(coexistence of Technology and Community) and* $n^* = N$ *(all* researchers *in Technology), and one unstable equilibria* $n_2^* \in (0, N)$ *(coexistence of Technology and* Community*), with* $n_1^* < n_2^*$ *.*

(Scenario II*) If* $\Pi_i^T(0) < \Pi_i^C(0)$ *and* $\Pi_i^T(N) > \Pi_i^C(N)$ *, then there are three equilibria: two stable equilibria,* $n^* = 0$ *(all* researchers *in Community), and* $n^* = N$ *(all researchers in Technology),, and one unstable equilibria* $n^* \in (0, N)$ $n_2^* \in (0, N)$ *(coexistence of* Technology *and Community).*

(Scenario III*) If* $\Pi_i^T(0) > \Pi_i^C(0)$ *and* $\Pi_i^T(N) < \Pi_i^C(N)$ *, then the equilibrium value* $n^* \in (0, N)$ *(coexistence of Technology and Community) is* unique *and stable.*

A graphical representation of equilibria determined in Proposition 3 is shown in Figure 1

INSERT FIGURE 1 ABOUT HERE

5. Discussion

5.1 The social *dimension of Community*

In this section we comment upon the different scenarios described in Proposition 3, in particular relating them to the social dimension of Community.

Before entering into the discussion, we first argue how variations in the level of personal involvement, the value of communitarian activity and the coordination costs impact on the payoff function in Community

An increased importance of personal involvement *e(n)* and of the value of communitarian activity has the effect of moving the payoff from the Community upwards, making the Community more attractive for any *n*. This effect is likely to be more significant for large size of Community, increasing the (positive) sensitiveness of the payoff of researcher belonging to Community to her group size, i.e. making $\dfrac{d\Pi_i^C(n)}{dn}$ more negative.

As for coordination costs, their increase has the primary effect of reducing the Community payoff for all *n*. However, we could expect that any increase in coordination costs would have a greater impact for small *n* (large community). If this is the case we could expect $\Pi_i^C(0)$ to move

16

down and $\dfrac{d\Pi_i^C(n)}{dn}$ to be increasing for small values of n (i.e., for large communities).

In Scenario I, two stable equilibria exist, one in which all researchers choose Technology and one in which Community is "large" (while Technology is "small"); on the contrary, the equilibrium with a "small" Community is unstable. In this scenario, $\Pi_i^T(0) > \Pi_i^C(0)$ and $\Pi_i^T(N) > \Pi_i^C(N)$: $\Pi^C(n)$ must have an inverted-U shape. This is consistent with a situation where coordination costs, communitarian activity and personal involvement are all significant, i.e., all factors we identified as peculiar of knowledge intensive communities are present. High coordination costs would lead to $\Pi_i^T(0) > \Pi_i^C(0)$ and to $\dfrac{d\Pi_i^C(n)}{dn}$ being increasing for low n; important communitarian activity and personal involvement effects, making Community payoff highly (and positively) dependent on group size, would make $\Pi_i^C(n)$ strongly decreasing for high values of n, inducing an inverted-U relationship in the Community payoff and $\Pi_i^T(N) > \Pi_i^C(N)$.

As a first remark, we notice that in this Scenario there exists one stable equilibrium in which Technology and Community coexist, with groups size depending on parameters values. From an empirical point of view, this equilibrium is clearly consistent to what is observed in sectors like software, where similar, competing products are offered under proprietary and open regimes. As anticipated, a notable result is that this has been obtained with ex-ante symmetric researchers and it is the outcome of endogenous mechanisms within each institutional regime.

While a large community is stable, the equilibrium where the community is small is an unstable equilibrium. As we suggested in the previous section, the model admits a dynamic interpretation, where individuals choose sequentially, and play a best response strategy to the current allocation of researchers between institutions. In this case, the unstable equilibrium constitutes a threshold that divides the realm of small communities from the set of communities that are able to grow fast and large. In each one of those spaces, the dynamics of the model shows a sort of bandwagon effects. If a community, for whatever reason, is able to grow enough and overcomes the threshold, then it grows endogenously until the large equilibrium, which in a sense expresses the full potential of a community. This appears the case of the free and open source community, as widely recognized in the literature (e.g. Bonaccorsi and Rossi, 2003, and Bitzer and Schröder, 2005).[8] What is new in our "Critical Mass" argument for free and open

---

[8] Notice that this approach takes into account the quantitative aspect of the free and open source community growth, but not its qualitative side. When communities grow, their social space becomes more complex, and their forms of participation and governance structures are put under pressure. The case of Debian is a clear example of the radical

source development is that it is not based on demand factors, (such as, for instance, in Bonaccorsi and Rossi, 2003), but instead it is based on the structure on developers' motivation. It is the shape of the social forces we described and rooted in Wenger's (1998) community of practice that determine the behavior we observe in the model.

Consider now Scenario II. It occurs when $\Pi_i^T(0) < \Pi_i^C(0)$ and $\Pi_i^T(N) > \Pi_i^C(N)$. In this case, the stable equilibria correspond to the corner solutions, while an unstable interior equilibrium separates the two "basins of attraction". This scenario corresponds to a situation where personal involvement and communitarian activity are important, but coordination costs are small.[9] In a sense, this scenario is a special case of Scenario I: small coordination costs lead to $\Pi_i^T(0) < \Pi_i^C(0)$, instead of $\Pi_i^T(0) > \Pi_i^C(0)$, posing no limit to Community growth. What this scenario shows with clarity is that strong communitarian activity may create a large community, but this is not necessarily so: to be fully expressed, the self-reinforcing growth process needs a critical mass at the beginning.

Finally, consider Scenario III, in which the unique and stable equilibrium is the coexistence Technology-Community. This case requires $\Pi_i^T(0) > \Pi_i^C(0)$ and $\Pi_i^T(N) < \Pi_i^C(N)$, and consequently the "absolute" value of $\dfrac{d\Pi_i^C(n)}{dn}$ is "small" (compared to $\dfrac{d\Pi_i^T(n)}{dn}$). Therefore, this case is consistent with a situation where the value of communitarian activity, the degree of personal involvement and coordination costs are small. Low values of communitarian activity and personal involvement tend to induce low values of $\Pi_i^C(0)$[10]; low values of communitarian activity, personal involvement and coordination costs tend to make $\Pi_i^C(0)$ relatively insensitive to n, i.e. $\dfrac{d\Pi_i^C(n)}{dn}$ "small".

In light of our previous discussion in Section 2, low coordination costs, low value of communitarian activity and low degree of personal involvement make communities resembling closely scientific community, where these elements are certainly present but not as crucial for the existence of the institutions. Indeed, this scenario is isomorphic to one of those identified in

---

transformation needed to make a growing project able to bear the challenges determined by its own growth (Mateos-Garcia and Steinmueller, 2008; O'Mahony and Ferraro, 2007; Sadowski et al., 2008).

[9] The symmetric case, where coordination costs are large, but the communitarian activity and the degree of personal involvement are low, would lead toward Scenario III, or a situations where all researchers choose Technology as unique equilibrium, in case they are very large.

[10] Since low coordination costs tend to increase $\Pi_i^C(0)$, we are assuming that this effect is dominated by the other.

Carraro and Siniscalco (2003). However, differences between Science and Community are not limited to the importance of the social dimension. In Section 3.1 we argued that the professionalization of modern Science leads to two other differences in payoffs. First of all, researchers in Science are paid by the State a fixed wage, which moves upwards their payoff. Second, we could expect that the individual return from innovation (k) is lower in Community than in Science. This implies that this type of equilibrium should be less likely to be observed when Technology faces Community than when it faces Science; or, if it is observed in both cases, the size of Community will be smaller than the size of Science. So, even when arguing that social motivations in the functioning of communities are not a necessary condition for their existence, it must be acknowledged that they clearly have a positive impact on their establishment and growth. Moreover, social motivations are crucial to generate the threshold level between small unstable communities and large stable communities, which is observed in the free and open source case.

5.1 Path dependency and the growth of communities

In the dynamic interpretation, the basin of attraction of the two stable equilibria, in terms of initial condition for n, is determined by the unstable equilibrium, whose values depend on the parameters of the model. The path dependency revealed by the importance of initial condition for n in determining, as first, which equilibrium is selected, and then the size of Community, points at the fundamental role that the initial ability of attracting researchers has for the establishment and growth of this institutional mode.

Considering the free and open source case, we can observe that communities become economically relevant when they fill an unfilled market, creating one *ex novo* or providing the conditions to fill an established one (Bonaccorsi and Rossi, 2003). The definition of "market," of course, must be interpreted in a wide sense, so that not only the product is important, but also the model of production—in the free and open source case allowing users to be part of the process—.The simple existence of a community attracts all the individuals interested in that market (Green, 1999). Thus, the more the community responds to unfilled gaps, the more attractive it becomes to interested individuals.

Moreover, communities, as other institutions, cover a particular space of social interaction. They provide members with a specific interaction environment, ruled by implicit laws and grounded in peculiar identities, i.e. structures of meanings, principles and values. One of the debates around

---

If this is not case, we could expect to prevail the situation where all researchers choose Community.

which the free and open source community is structured concerns the concepts of free and open source software (Dahlander, 2007). This debate shapes the environment in which developers act, defining rules (from rules against free-riding to recognition by peers), meanings (what "openness" means), values (whether software should be always free), and visions of the world (whether all the produced knowledge should be free). Such interaction contributes then to the building of the "identity" of the community. Non-members interested in this debate and sensitive to such an identity are then attracted to the community, and may become eventually members.

Another mechanism can be activated also by trust building, which in small communities can lead towards a common language, established rules and an improved efficiency at the organizational level. This implies that over time, for given n, coordination costs may decrease for a sort of "free-riding exclusion effect": when member i starts to engage in the communitarian activity and to believe in the common enterprise, she begins to *perceive* the community as a trustworthy environment. Thus, the simple fact that j also belongs to the community is taken by i as a signal of j's trustworthiness. j's potential free-riding behavior is perceived by i as an almost-irrelevant exception, and i reduces her monitoring and punishing activities, decreasing the cooperation costs. This maps the results obtained by Bagozzi and Dholakia (2006), who, as already noted, find that "the community metamorphosizes into a friendship group and a social entity with which one identifies" (Bagozzi and Dholakia, 2006: 1111). If this is coupled with community rules that regulate the gradual acceptance of new members into the community of the kind described by Lave and Wenger (1991) as legitimate peripheral participation, the free-riding exclusion effect can increase payoff participation and make the community more attractive for potential members (and able to handle them without increasing again coordination costs), fueling community growth (Bonaccorsi and Rossi, 2003).

Filling an unfilled market, identity building, and trust building, can all attract new potential members, and trigger the self-reinforcing growth described in the model as a movement from a community below the threshold to one above the threshold, able to grow endogenously.

5.2 Intellectual property rights: strength of patent protection and GPL

The possibility to patent software code, and in general the capability of IPR to enhance software production, is still a challenging debate. Although the level of protection granted by the IPR (in particular patent protection) is not explicitly included in the model, it is easy to consider its effect on the payoff functions. As a first order effect, an increase in the strength of intellectual property

rights in Technology implies an increase in the economic return R, which brings about an increase in the payoff from Technology. Second, stronger IPR limit the scope of the innovative activity (constraining the "field" in which research could be exploited without violating them), in both Technology and Community cases. Formally, this is captured by a more negative effect on the probability of innovation for any value of the total effort in Technology. While in Technology this effect is most likely to be dominated by the increase in R, for Community the negative effect is the only effect. This unambiguously leads to equilibria in which a community grows much less than before, if it is created at all. This result is consistent with the concerns about extending the right of software producers to patent their code in Europe. As Linus Torvalds and Alan Cox, two opinion leaders in the open source world, put it: "Software patents are also the utmost threat to the development of Linux and other free software products, as we are forced to see every day while we work with the Linux development" (Torvalds and Cox, 2003).

A related discussion concerns the role of GPL licences in the development of communities. In the model, the direct effect of GPL is seen in the reduction of positive externalities from Community to Technology. This has the consequence of moving down payoff from Technology, and leading towards equilibrium in which Community are of larger size and/or enlarging the basin of attractions of these equilibria. So, the effect of the GPL is fundamental in enhancing the community sustainability, creating the condition of the community growth (Gambardella and Hall, 2006), although it is not part of the engine sustaining it. Connecting to our previous point on IPR, the role of GPL is particular important in creating a space in which the free and open source community can develop *inside* the property rights structure. Furthermore, GPL can help to create the critical mass at the initial stage of community development, by attracting individuals that share the ideological component that at the basis of GPL.

5.3 Heterogeneous agents

In our model, we excluded structural idiosyncrasy among agents, which was instead adopted by other authors in the same field (Mustonen, 2003). We can relate directly to this point of view sketching here few possible consequences of researchers' heterogeneity on our game.

Let us consider the problem in a dynamic interpretation of the model, and focus for illustration on Scenario I. Initially, the community is set up by people with a high interest in the activity that the community is going to undertake (for instance captured by a high value of k) and in the "vision" it embodies (for instance captured by a high value of personal involvement e even when if $N-n$ is small), an interest high enough to make them bear the costs connected with the small

size of the community. The community, then, can be created and developed, even if linking only a few individuals. Once again, it is crucial whether the early choice of community by these researchers would lead to overcome the threshold previously discussed. If that is the case, the community starts to develop a structured identity, and to develop reciprocal trust and legitimate peripheral participation processes (Lave and Wenger, 1991). The artifacts produced start to fill a new market, and as its identity becomes more precise, trustworthy and well known, other agents could find it desirable to join the communitarian project. In terms of our model, the communitarian terms of the payoff function, namely $e(n)$ and $Y(.)$, will trigger community growth, leading towards the equilibrium characterized by a stable large community. In this description, it is easy to recognize the actual evolution free and open source community (Bonaccorsi and Rossi, 2003; Bitzer and Schröder, 2005). What our argument suggests is that, thanks to the logic of threshold models (Granovetter, 1978) and for given characteristics of the "average" researcher, higher heterogeneity would favor the constitution a community.

6. Conclusions

In this paper we developed a model where knowledge-intensive communities (a notable example being free and open source communities) are confronted with Technology in their ability to attract researchers. On one side, our results suggest that, even if researchers are ex ante symmetric, Community and Technology can coexist. On the other side, the social nature of Communities, as capture by the degree of personal involvement, the value of communitarian activity and coordination costs, induces multiple equilibria where communities *may* growth endogenously when their size overcomes a certain threshold.

For managers, our results may provide some interesting insights. In particular, firms interested in the development of communities should be particularly concerned with their initial stages. Firms should try to activate, for instance by sponsoring the participation of their employees, the dynamic mechanisms we identified, based on social motivation and interaction of developers, which can trigger the growth of communities in a self-reinforcing way. In communities, the initial stage is what matters the most; beyond the threshold the endogenous mechanisms induced by the dynamics of the developers' motivations can directly provide enough support for sustainable growth.

References

Arora, A., A. Fosfuri, A. Gambardella. 2001. Markets for Technology: The Economics of Innovation and Corporate Strategy. The MIT Press, Cambridge MA.

Allen, R.C. 1983. Collective invention. Journal of Economic Behavior and Organization 4 (1) 1–24.

Bagozzi, R.P., U.M. Dholakia. 2006. Open source software user communities: a study of participation in Linux user groups. Management Science 52 (7) 1099–1115.

Bessen, J. 2006. Open source software: free provision of complex public goods. J. Bitzer, P.J.H. Schröder, eds. The Economics of Open Source Software Development, Elsevier, Amsterdam, The Netherlands, 57–81.

Bezroukov, N. 1999a. Open source software development as a special type of academic research (critique of Vulgar Raymondism). First Monday 4 (10) at: http://firstmonday.org/issues/issue4_10/bezroukov/index.html

Bezroukov, N. 1999b. A second look at the cathedral and Bazaar by Nikolai Bezroukov. First Monday 4 (12) at: http://firstmonday.org/issues/issue4_12/bezroukov/index.html

Bitzer, J., J.H.P Schröder. 2005. Bug-fixing and code-writing: The private provision of open source software. Information Economics and Policy 17 389–406.

Bonaccorsi, A., S. Giannangeli, C. Rossi. 2006. Entry strategies under competing standards: hybrid business models in the open source software industry. Management Science 52 1085-1098.

Bonaccorsi, A., C. Rossi. 2006. Comparing motivations of individual programmers and firms to take part in the open source movement: from community to business. Knowledge, Technology and Policy, 18 (4) 40 - 64.

Bonaccorsi, A., C. Rossi. 2003. Why open source software can succeed. Research Policy 32 (7) 1243–1258.

Carraro, C., D., Siniscalco, D., 2003. Science versus profit in research. Journal of the European Economics Association 1 (2-3) 576–590.

Casadesus-Masanell, R., P. Ghemawat, P. 2006. Dynamic mixed duopoly. Management Science 52 (7) 1072–1084.

Cohendet, P., F. Creplet, O. Dupouët. 2001. Communities of practice and epistemic communities: a renewed approach of organizational learning within the firm. Paper presented at the Workshop on Economics and Heterogeneous Interacting Agents, June 2001 Marseille, France.

Dahlander, L. 2007. Penguin in a new suit: A tale of how de novo entrants emerged to harness free and open source software communities. Industrial and Corporate Change 16 (5) 913–943.

Dahlander, L., M. G. Magnusson. 2005. Relationships between open source software companies and communities: Observations from Nordic firms. Research Policy 34 481–493.

Dahlander, L., M.W. Wallin. 2006. A man on the inside: unlocking communities as complementary assets. Research Policy 35 1243–1259

Dalle, J.-M., P.A. David. 2003. The allocation of software development resources in 'open source' production mode. SIEPR Discussion Paper #02-27, Stanford University at http://opensource.mit.edu/papers/dalledavid.pdf

Dalle, J.-M., P.A. David. 2007. Simulating code growth in Libre (open source) mode. E. Brousseau, N. Curien, eds. Internet and Digital Economics. Cambridge University Press, New York, 391–422.

Dasgupta, P., P.A. David. 1987. Information disclosure and the economics of science and technology. G. Feiwel, ed. Arrow and the Ascendant of Economic Theory. New York University Press, New York, 519–542.

Dasgupta, P., P.A. David. 1994. Towards a new economy of science. Research Policy 23 (5)

487–521.

David, P.A., 1991. Reputation and agency in the historical emergence of institutions of 'open science.' Center for Economic Policy Research Publication # 261, Stanford University.

David, P.A., S. Arora, W.E. Steinmueller. 2001. Economic organization and viability of f/oss software: a proposal to the national science foundation. SIEPR, Stanford University, 22 January, at http://www-siepr.stanford.edu/programs/OpenSoftware_David/NSF-CISE-OpSourFinal.htm

David, P.A, D. Foray. 2003. Economic fundaments of the knowledge society. Policy Futures in Education 1 (1) 20–49.

David, P.A., J.S. Shapiro. 2008. Community-based production of open source software: what do we know about the developers who participate? Information Economics and Policy 20 (4) 364–398.

Economides, N., E. Katsamakas. 2006. Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. Management Science 52 (7), 1057–1071.

Edwards, K. 2001. Epistemic communities, situated learning and open source software development. Working paper, Department of Manufacturing Engineering and Management, Technical University of Denmark, at http://opensource.mit.edu/papers/kasperedwards-ec.pdf

Elliott, M.S., W. Scacchi. 2003. Free software: a case study of software development in a virtual organizational culture. Working paper, Institute for Software Research, University of California– Irvine, April.

Fosfuri, A., M.S. Giarratana, A. Luzzi. 2008. The penguin has entered the building: the commercialization of open source software products. Organization Science 19(2) 292-305.

Franke, N., S. Shah. 2003. How communities support innovative activities: An exploration of

assistance and sharing among end-users. Research Policy 32 (1) 157–178.

Gambardella, A., B. Hall. 2006. Proprietary vs public domain licensing of software and research products. Research Policy 35 (6) 875–892.

Ghosh, R.A., R. Glott, B. Kriege, G. Robles. 2002. Free/libre and open source software. Part iv: Survey of developers. International Institute of Infonomics, Berlecom Research GmbH, at http://www.infonomics.nl/free and open source/report/Final4.pdf.

Giuri, P., G. Rocchetti, S. Torrisi. 2002. Open source software: from open science to new marketing models an enquiry into the economics and management of open source software. Working paper, Laboratory of Economics and Management, Sant'Anna School of Advanced Studies (LEM), Pisa, Italy, at: http://www.lem.sssup.it/WPLem/files/2002-23.pdf.

Gonzalez-Barahona, J.M., G. Robles, R. Andradas-Izquierdo, R.A. Ghosh, R.A. 2008. Geographic origin of libre software developers. Information Economics and Policy 20 356–363.

Granovetter, M.1978. Threshold models of collective behavior. American Journal of Sociology 83 (6) 1420-1443

Green, E.L. 1999. Economics of open source software. At http://badtux.org/home/eric/editorial/economics.php

Henkel J., 2006. Selective revealing in open innovation processes: The case of embedded Linux. Research Policy 35, 953–969

Jeppesen, L.B., L. Frederiksen, L., 2006. Why do users contribute to firm-hosted user communities? The case of computer-controlled music instruments. Organization Science 17 (1) 45–63.

Johnson, J.P., 2006. Collaboration, peer review and open source software. Information Economics and Policy 18 477–497.

Kelty, C.M. 2001. Free software/free science. First Monday 6 (12) at

http://firstmonday.org/issues/issue6_12/kelty/index.html

Lakhani, K.R., R. G. Wolf. 2005. Why hackers do what they do: understanding motivations and effort in free/open source software projects. J. Feller, B. Fitzgerald, S. Hissam, K.R. Lakhani, eds. Perspectives on Free and Open Source Software. MIT Press, Cambridge MA.

Lakhani, K.R., R. Wolf, J. Bates, C. Di Bona. 2002. Hacker survey (release 0.73), the Boston Consulting Group. Presented at the O'Reilly Open Source Software Conference, 24 July, San Diego, CA. Available at http://www.osdn.com/bcg.

Lanzi, D., 2009. Competition and open source with perfect software compatibility. Information Economics and Policy 21 (3) 192-200.

Lave, J., E. Wenger 1991. Situated Learning: Legitimate Peripheral Participation. Cambridge University Press, Cambridge UK.

Lee, G.K., R.E. Cole. 2003. From a firm-based to a community-based model of knowledge creation: the case of the Linux Kernel development. Organization Science 14 (6) 633–649.

Lerner, J., J. Tirole. 2002. Some simple economics of open source. Journal of Industrial Economics 50 (2) 197–234.

Lin, Y., 2003a. Culture, creativity, and innovation: practices of locally-crafted software. Presented at the joint seminar SATSU and HCI group at computer science and psychology departments on 20 May, University of York, UK. Available at http://www.york.lug.org.uk/talks/yuwei_satsu/satsu_paper.html (accessed May 5th, 2005).

Lin, Y., 2003b. The institutionalisation of hacking practices. Ubiquity 4 (4) at:

http://www.acm.org/ubiquity/views/y_lin_1.html

Lin, Y., 2004a. Contextualising knowledge–making in Linux user groups. First Monday 9 (11) at:

http://firstmonday.org/issues/issue9_11/lin/index.html

Lin, Y., 2004b. Epistemologically multiple actor-centered systems: or, EMACS at work! Ubiquity 5 (1) at: http://www.acm.org/ubiquity/views/v5i1_lin.html

Luo, J., 2002. Why people are willing to share knowledge: an open source account. Queen's KBE Centre for Knowledge-Based Enterprises, Doctoral Consortium 2002 Papers, June.

Mateos-Garcia, J., W.E. Steinmueller 2008. The institutions of open source software: examining the Debian community. Information Economics and Policy 20 333–344.

Muller, P., 2006. Reputation, trust and the dynamics of leadership in communities of practice. Journal of Management and Governance 10 381–400.

Mustonen, M., 2003. Copyleft—the economics of Linux and other open source software. Information Economics and Policy 15 99–121.

Nuvolari, A., 2004. Collective invention during the British Industrial Revolution: The case of the Cornish pumping engine. Cambridge Journal of Economics 28 (3) 347–363.

O'Neil, M. 2009. Cyberchiefs: autonomy and authority in online tribes, Pluto Press.

O'Mahony, S., F. Ferraro. 2007. The emergence of governance in an open source community. Academy of Management Journal 50 (5) 1079–1106.

Raymond, E., 1998. Homesteading the noosphere. First Monday 3 (10) at http://www.firstmonday.dk/issues/issue3_10/raymond/#d12

Roberts, J.A., I. Hann, S.A. Slaughter. 2006. Understanding the motivations, participation, and performance of open source software developers: a longitudinal study of the apache projects. Management Science 52 (7) 984–999.

Sadowski, B.M., G. Sadowski-Rasters, G. Duysters. 2008. Transition of governance in a mature open software source community: evidence from the Debian case. Information Economics and Policy 20 323–332.

Shah, S.K. 2006. Motivation, governance, and the viability of hybrid forms in open source software development. Management Science 52 (7) 1000–1014.

Torvalds, L., A. Cox. 2003. Open letter on software patents from Linux developers, 21 September 2003, http://www.effi.org/patentit/patents_torvalds_cox.html

von Hippel, E. 1988. The Sources of Innovation. Oxford University Press, New York.

von Hippel, E. 2001. Innovation by user communities: learning from open source software. Sloan Management Review 42 (4) 82–86.

von Hippel, E., G. von Krogh. 2003. Open source software and the 'private-collective' innovation model: Issues for Organization Science. Organization Science 14 (2) 209–223.

Wenger, E., 1998. Community of Practice: Learning Meaning and Identity. Cambridge University Press, Cambridge UK.

Williams, S., 2002. Free as in Freedom: Richard Stallman's Crusade for Free Software. O'Reilly, Sebastopol CA.

Appendix

Proof of Proposition 1

By use of the implicit function theorem, we get:

$$\frac{d\hat{x}^T}{dn} = -\frac{\left\{\dfrac{\partial^2 \Pr^T}{\partial x^T \partial X_{-i}^T}\dfrac{\partial \hat{X}_{-i}^T}{\partial n} + \dfrac{\partial^2 \Pr^T}{\partial x^T \partial X^C}\dfrac{\partial \hat{X}^C}{\partial n}\right\} R^T}{\dfrac{\partial^2 \Pr^T}{\left(\partial x^T\right)^2} R^T - \dfrac{\partial^2 c^T}{\left(\partial x^T\right)^2}} < 0$$

$$\frac{d\hat{x}^C}{dn} = -\frac{\left\{\dfrac{\partial^2 \Pr^C}{\partial x^C \partial X^T}\dfrac{\partial \hat{X}^T}{\partial n} + \dfrac{\partial^2 \Pr^C}{\partial x^C \partial X^C}\dfrac{\partial \hat{X}^C}{\partial n}\right\} k^C + \dfrac{\partial e}{\partial n}\dfrac{\partial Y}{\partial x_i^C} + e(n)\left\{\dfrac{\partial^2 Y}{\partial x^C \partial X^T}\dfrac{\partial \hat{X}^T}{\partial n} + \dfrac{\partial^2 Y}{\partial x^C \partial X^C}\dfrac{\partial \hat{X}^C}{\partial n}\right\}}{\dfrac{\partial^2 \Pr^C}{\left(\partial x_i^C\right)^2} k^C - \dfrac{\partial^2 c^C}{\left(\partial x_i^C\right)^2} + e(n)\dfrac{\partial^2 Y}{\left(\partial x_i^C\right)^2}} > 0$$

whose signs are direct consequences of the assumptions made in the paper.

Proof of Proposition 2

The sign of (12) comes directly from the assumptions made in the paper. On Equation (13) notice that

$$\left\{\frac{\partial \Pr^C}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial \Pr^C}{\partial X^T}\frac{dX^T}{dn}\right\} k^C \quad \text{and} \quad \frac{\partial e}{\partial n}\left[\frac{\partial Y}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial Y}{\partial X^T}\frac{dX^T}{dn}\right] \quad \text{are negative for the assumptions}$$

made in the paper, while is $-\dfrac{\partial C}{\partial n}$ is positive. The overall sign is then ambiguous. If

$$\left\{\frac{\partial \Pr^C}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial \Pr^C}{\partial X^T}\frac{dX^T}{dn}\right\} k^C + \frac{\partial e}{\partial n}\left[\frac{\partial Y}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial Y}{\partial X^T}\frac{dX^T}{dn}\right] < -\frac{\partial C}{\partial n}$$

for any *n*, then it is always $\dfrac{d\Pi_i^C(n)}{dn} > 0.$ If

$$\left\{\frac{\partial \Pr^C}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial \Pr^C}{\partial X^T}\frac{dX^T}{dn}\right\} k^C + \frac{\partial e}{\partial n}\left[\frac{\partial Y}{\partial X_{-i}^C}\frac{dX_{-i}^C}{dn} + \frac{\partial Y}{\partial X^T}\frac{dX^T}{dn}\right] > -\frac{\partial C}{\partial n}$$

for any *n,* then it is always $\dfrac{d\Pi_i^C(n)}{dn} < 0$. Suppose now that there are some values $\tilde{n}$ for which

$\dfrac{d\Pi_i^C(\tilde{n})}{dn} = 0$. Since we assumed $\dfrac{d^2\Pi_i^C(n)}{dn^2} < 0$, $\tilde{n}$ is unique, and it is the global maximizer of

$\Pi_i^C(n)$ in the relevant interval. Consequently, $\Pi_i^C(n)$ is increasing in *n* until $\tilde{n}$, and then decreasing.

Proof of Proposition 3

Consider Scenario I. If $\Pi_i^T(0) > \Pi_i^C(0)$ and $\Pi_i^T(N) > \Pi_i^C(N)$, with $\Pi_i^T(n) < \Pi_i^C(n)$ for some

values of *n*, then $\Pi_i^C(n)$ must be first increasing and then decreasing in *n*. Consequently

$\Pi_i^T(n)$ crosses $\Pi_i^C(n)$ twice, in $n_1* \in (0, N)$ and $n_2^* \in (0, N)$, with $n_1^* < n_2^*$. Since

$\Pi_i^T(0) > \Pi_i^C(0)$ n_1* is stable ($\Pi_i^T(n)$ "cuts" $\Pi_i^C(n)$ from below), while $\Pi_i^C(n)$ cuts

$\Pi_i^T(n)$ from above in n_2^*, and then n_2^* is unstable. Since n_2^* is unstable, also *n=N* is a stable

equilibrium.

Consider Scenario II. If $\Pi_i^T(0) < \Pi_i^C(0)$ and $\Pi_i^T(N) > \Pi_i^C(N)$, then $\Pi_i^T(n)$ and $\Pi_i^C(n)$ cross

only once given our assumption. Since $\Pi_i^T(0) < \Pi_i^C(0)$, in $n^* \in (0, N)$ where

$\Pi_i^T(n) = \Pi_i^C(n)$ $\Pi_i^C(n)$ cuts $\Pi_i^T(n)$ from above, and then the equilibrium in unstable.

Consequently $n^* = 0$ and $n^* = N$ are stable equilibria.

Consider finally Scenario III. If $\Pi_i^T(0) > \Pi_i^C(0)$ and $\Pi_i^T(N) < \Pi_i^C(N)$, $\Pi_i^T(n)$ and $\Pi_i^C(n)$, which

are continuous, must cross at least once. Given $\dfrac{d^2\Pi_i^C(n)}{dn^2} < 0$ and $\dfrac{d^2\Pi_i^T(n)}{dn^2} > 0$, the value

$n^* \in (0, N)$ where $\Pi_i^T(n) = \Pi_i^C(n)$ must be unique. Since $\Pi_i^T(0) > \Pi_i^C(0)$, then

$\Pi_i^T(n)$ "cuts" $\Pi_i^C(n)$ from above, which guarantees stability.

Figure 1

(Scenario I)

(Scenario II)

(Scenario III)