



Institut für Volkswirtschaftslehre

Universität Augsburg

Volkswirtschaftliche Diskussionsreihe

**Agent-Based Modelling: A Methodology
for Neo-Schumpeterian Economics**

Andreas Pyka, Giorgio Fagiolo

Beitrag Nr. 272, Februar 2005

Agent-Based Modelling: A Methodology for Neo-Schumpeterian Economics

Andreas Pyka

Economics Department
University of Augsburg, Augsburg (Germany)

Giorgio Fagiolo

Faculty of Economics
University of Verona, Verona (Italy)
and
Laboratory of Economics and Management
Sant'Anna School of Advanced Studies, Pisa (Italy)

February 2005

JEL: B52, O30

Key-words: Simulation, Neo-Schumpeterian Economics, Agents

Abstract: Modellers have had to wrestle with an unavoidable trade-off between the demand of a general theoretical approach and the descriptive accuracy required to model a particular phenomenon. A new class of simulation models has shown to be well adapted to this challenge, basically by shifting outwards this trade-off: So-called agent-based models (ABMs henceforth) are increasingly used for the modelling of socio-economic developments. Our paper deals with the new requirements for modelling entailed by the necessity to focus on qualitative developments, pattern formation, etc. which is generally highlighted within Neo-Schumpeterian Economics and the possibilities given by ABMs.

1. Introduction

The tremendous development of an easy access to computational power within the last 30 years has led to the widespread use of numerical approaches in almost all scientific disciplines. Nevertheless, while the engineering sciences focused on the applied use of simulation techniques from the very beginning, in the social sciences most of the early examples of numerical approaches were purely theoretical.

There are two reasons for this. First, since the middle of the 20th century, starting with economics, equilibrium-oriented analytical techniques flourished and were developed to a highly sophisticated level. This led to the widely shared view that within the elegant and formal framework of linear analysis offered by neoclassical economics, the social sciences could reach a level of accuracy and stringency not previously thought to be possible.

Second, within the same period, new phenomena of structural change exerted a strong influence on the social and economic realms. Despite the mainstream neoclassical successes in shifting the social sciences to a strong mathematical foundation, an increasing dissatisfaction with this approach emerged. For example, by the 1970s the benchmark of atomistic competition in neoclassical economics had already been replaced by the idea of monopolistic and oligopolistic structures under the heading of workable competition (e.g. Scherer and Ross, 1990). A similar development emphasising positive feedback effects and increasing returns to scale caused by innovation led to the attribute "new" in macroeconomic growth theory in the 1980s (Romer, 1990).

In addition to these stepwise renewals of mainstream methodology, an increasingly larger group claimed that the general toolbox of economic theory, emphasising rational behaviour and equilibrium, were no longer suitable for the analysis of complex social and economic changes. In a speech at the *International Conference on Complex Systems* organised by the New England Complex Systems Institute in 2000, Kenneth Arrow stated that until the 1980s the "sea of truth" in economics laid in simplicity, whereas since then it has become recognised that "the sea of truth lies in complexity". Adequate tools have therefore to include the heterogeneous composition of agents (see, e.g., Kirman (1989;1997b) and Saviotti (1996)), the possibility of multilevel feedback effects or interactions (Kirman, 1997a; Cantner and Pyka, 1998; Fagiolo, 1998) and a realistic representation of dynamic processes in historical time (Arthur, 1988; Marengo and Willinger, 1997). These requirements are congruent with

the possibilities offered by simulation approaches. It is not surprising that within economics the first numerical exercises were within evolutionary economics, where phenomena of qualitative change and development are at the front of the research programme.

The first generation of simulation models were highly stylised and did not focus on empirical phenomena. Instead, they were designed to analyse the logic of dynamic economic and social processes, exploring the possibilities of complex systems behaviour.

However, since the end of the 1990s, more and more specific simulation models aiming at particular empirically observed phenomena have been developed focusing on the interaction of heterogeneous actors responsible for qualitative change and development processes. Modellers have had to wrestle with an unavoidable trade-off between the demand of a general theoretical approach and the descriptive accuracy required to model a particular phenomenon. A new class of simulation models has shown to be well adapted to this challenge, basically by shifting outwards this trade-off: So-called agent-based models (ABMs henceforth) are increasingly used for the modelling of socio-economic developments – see, e.g., Gilbert and Troitzsch (1999).

Our paper deals with the new requirements for modelling entailed by the necessity to focus on qualitative developments, pattern formation, etc. which is generally highlighted within Neo-Schumpeterian Economics and the possibilities given by ABMs.

The paper is organized as follows. In Section 2 we examine in more detail the basic motivations underlying the emergence of the ABM paradigm and we sketch its main underpinnings. Section 3 presents the building blocks of an ABM and briefly discusses the extent to which ABMs can be employed to deal with empirical phenomena. Finally, Section 4 concludes and flags open problems in the ABM research agenda.

2. Micro-Macro Systems, Mainstream Models and Agent-Based Approaches

Generally speaking, ABMs deal with the study of socio-economic systems that can be properly conceptualized by means of a set of “micro-macro” relationships. In such systems, the micro level typically contains heterogeneous basic entities, the additional decomposition of which does not help in explaining the phenomena under study (e.g. firms, consumers, workers). Repeated interactions among these entities over time induce ceaselessly changing

microeconomic patterns (e.g. production and consumption levels). These micro patterns, once aggregated over the relevant set of micro entities, generate a macro dynamics for the aggregate variable of interest (e.g. GNP). The goal of ABMs is to properly describe such complicated systems and to analyze their properties. More precisely, agent-based formalizations depict decentralized economies as complex systems and try to infer their aggregate properties – in a bottom-up perspective – from interactions and behaviours of micro entities.

2.1. Mainstream Models: A Brief Critical Discussion

As briefly discussed in the introduction, the need for ABM approaches has been mostly driven by an increasing dissatisfaction with how “mainstream” theorists model “micro-macro” relationships. The classic reference here is the class of so-called “micro-founded macroeconomic models” (Sargent, 1987), which became the yardstick for any representation of dynamic decentralized economies composed of agents autonomously undertaking courses of actions and decisions over time (cf. Dosi and Orsenigo (1994) for a critical discussion).

As well known, these models take a pragmatic and positivist perspective and solve the trade-off between analytical solvability and descriptive accuracy in favour of the first one. Indeed, many over-simplifying assumptions – often considered as “free goods” – are employed in order to derive sharp, analytical conclusions. For example, the interaction structure (i.e. the assumptions over the set of channels connecting agents and conveying information at each point in time among them) is either of a degenerate type – agents do not interact at all, as happens in models where a “general equilibrium” microfoundation is assumed – or it can be traced back to a “complete” network – agents interact with anyone else, as happens in micro-macro models based on a game-theoretic microfoundation. No room for intermediate and more complicated interaction patterns is left on the ground.

In a similar vein, any heterogeneity across agents (concerning e.g. agents’ properties such as endowments, wealth, etc., and, more generally, behavioural rules, competencies, learning, etc.) is abolished – and, whenever introduced in the model (think to e.g. the standard general equilibrium framework), its role is not even addressed (Kirman, 1989).

Moreover, agents are typically assumed to be hyper-rational entities, holding rational (sometimes even technological) expectations and possessing no computational bounds. This is

of course at odds with any experimental (and casual, by the way) evidence and has crucial implications in the way aggregate properties and models' outcomes are interpreted (Dosi et al., 2005). In fact, the strong consistency requirements induced by hyper-rationality compress any sequence of decisions made over time by the agents into a single and coherent stream of decisions made once and for all in an irreversible manner. These models can generate only equilibrium outcomes, and *only* equilibrium observations can be observed in reality.

These examples show that in general the dissatisfaction towards mainstream approaches can be traced back to the way in which the latter deals with some key ingredients of the modelling process as a whole, namely: (i) assumptions and modelling design; (ii) analysis of the properties of the model; (iii) generation of testable implications; (iv) model validation and rejection. Let us briefly summarize here the debate about these four issues.

Assumptions and modelling design

As argued above, mainstream models of dynamic decentralized economies employ assumptions as a “free good”. They are considered functional to the construction of an analytically solvable model. Sometimes, a feedback process going from model solutions back to assumptions is employed, and the latter are modified to the extent they are able to allow for analytical solutions, no matter whether they still preserve some economic interpretations or not. This can generate awkward and pathological situations, where the answer to the question “What is the economic intuition behind this result?” is “The third derivative of the utility function is negative”.

More rigorously, we know from the impressive work in cognitive psychology and experimental economics (Kagel and Roth, 1995; Plott and Smith, 1998) that the classes of assumptions discussed above have almost no links with empirically observed patterns of micro behaviours and interactions, and that, in the case of agents' rationality, interactions, heterogeneity, etc. if any link is present, the assumptions are often *against* the evidence!

This attitude becomes even more manifest as far as innovation and uncertainty modelling is concerned. Given the restrictions imposed e.g. by the hyper-rationality paradigm, it is well known that mainstream models are not able to deal with structural innovation endogenously and imperfectly introduced in the system by the agents. These innovative behaviours – which are typically driven by persistent mistakes and trial-and-error learning processes which must cope with a truly uncertain environment (Dosi et al., 2005) – cannot be accounted for, almost

by construction. However, as we know from a huge empirical literature on innovation and technological change, innovative behaviours in presence of true uncertainty are at the core of any growth and development process – see, among others, Freeman (1982, 1994), Rosenberg (1982, 1994), David (1975), Dosi (1988), Nelson (1995), Lundvall (1993), Granstrand (1994), Stoneman (1995), and fair parts of Dosi et al. (1988) and Foray and Freeman (1992).

Analysis of the properties of the model

In turn, the need for sharp, analytical implications – coupled with the nature of the mathematical toolbox employed in the analysis, cf. Sargent (1987) – has generated a class of models characterized by an often-excessive commitment to equilibrium analysis. As already suggested, any macro property of the system has to be conceptualized as an equilibrium one. In turn, if one assumes that the model is an adequate one to address empirical problems, any real-world observation has to be interpreted as happening in equilibrium. In this way, aggregate behaviour is nothing else than a straightforward and tautological implication of the micro-level: after all, individual behaviours must be coherent in equilibrium over time and agents are all the same (see however Kirman (1992) and Forni and Lippi (1997) for a discussion on the risks of interpreting aggregate outcomes in mainstream microfounded macro models).

Generation of testable implications

A large part of mainstream micro-macro formalizations we are discussing here must be interpreted as toy models which provide a theoretical ground where some basic socio-economic principles and causal chains can be better spelled out. These models do not deliver any testable implications and therefore cannot be taken to the data in order to validate them.

However, the belief of many scholars is that this critique applies more generally to the entire class of micro-macro formalizations. Whenever the model is built in order to explain or reproduce some “stylized fact” or observable property, it is maintained, there is the feeling that the number of over-simplifying, *ad-hoc* assumptions required to get analytically solvable implications in line with the observed phenomena enormously increases with the number of facts that one would like to explain simultaneously. Examples here range from labor market dynamics, to growth and development, consumption and demand, etc.: in all these cases one often finds many simple models each addressing a separate fact in isolation, rather than more

robust models explaining together many related facts (see e.g. the discussion in Fagiolo et. al (2004a)).

In brief, mainstream models often show a limited attention to empirical validation and joint reproduction of stylized facts. This is true not only as far as macro-dynamics is concerned, but also, more dramatically, at the micro level: consider for instance the lack of micro-macro models that jointly replicate *micro stylized facts* such as firm size and growth distributions across sectors and *macro stylized facts* concerning statistical properties of aggregated growth time-series.

Model Validation and Rejection

Mainly as a consequence of the points discussed above, mainstream micro-macro models lack a serious procedure of model development, with obsolete and weakly performing models replaced by better ones. Model performance is related here to the ability of a model in replicating and explaining (possibly many) stylized facts and observable phenomena.

The common practice is instead that of retaining as much as possible the analytical apparatus (and, with it, the main philosophical building blocks) informing mainstream, neoclassical micro-founded macro models. Optimization, forward-looking rational expectations, equilibrium, representative individuals, etc. continue to form the core of these formalizations despite their often-limited explicative and interpretative capabilities.

A crucial question then must be asked. As Richard Day puts it in his plenary talk at the 11th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics (2003): “Can one do good science with assumptions that are clearly at odds with any empirical evidence about micro behaviour?”

This question may in our view perfectly synthesise the debate that we have been trying to sketch so far. More to the point: There seems to be a sort of pathological pessimism of neoclassical economics with respect to many ingredients which empirical evidence points at as *the* crucial ones to understand micro-macro relationships. Our preferred example is once again innovation. Without a minimum willingness to cope with true uncertainty, innovation processes cannot be analysed. In traditional economic modelling approaches, this minimum willingness is not reproducible: economic agents always prefer “risky” to “uncertain” situations.

2.2. Agent-Based Modelling: An Alternative Modelling Methodology

In the last twenty years, an alternative modelling strategy has been pursued by an increasingly large number of scholars, often sharing an interdisciplinary perspective drawing stimuli, inspirations and ideas from disciplines such as biology, physics, sociology, history, computer-science, etc.

Methodologically, this alternative strategy – which we have labelled agent-based modelling – is rooted in the use of numerical techniques and simulation analysis, which are regarded as major tools in developing and analyzing this class of models (Kwásnicki, 1998; Aruka, 2001). Although simulation analysis comes in various flavours, most of them reflect Boulding's call that we need to develop ‘mathematics which is suitable to social systems, which the sort of 18th-century mathematics which we use is not’ (Boulding, 1991).

In a nutshell, this approach consists of a decentralized collection of agents acting autonomously in various contexts (see Section 3 below for a more detailed description). The massively parallel and local interactions can give rise to path dependency, dynamic returns and feedbacks between the two. In such an environment, global phenomena as the development and diffusion of technologies, the emergence of networks, herd-behaviour, etc. – which cause the transformation of the observed system – can be modelled adequately. This modelling approach focuses on depicting the agents, their relationships and the processes governing the transformation.

Broadly speaking, the application of an *agent based modelling approach* offers two major advantages. The first advantage of ABMs is their capability to show how collective phenomena came about and how the interaction of the autonomous and heterogeneous agents leads to the genesis of these phenomena. Furthermore, agent-based modelling aims at the isolation of critical behaviour in order to identify agents that more than others drive the collective result of the system. It also endeavours to single out points of time where the system exhibits qualitative rather than sheer quantitative change (Tsfatsion, 2001b). In this light, it becomes clear why agent-based modelling conforms to the principles of Neo-Schumpeterian Economics (Lane, 1993a and 1993b). It is “the” modelling approach to be pursued in evolutionary settings.

The second advantage of ABM, which is complementary to the first one, is a more normative one. Agent-based models are not only used to get a deeper understanding of the inherent

forces that drive a system and influence the characteristics of a system. Agent based modellers use their models as computational laboratories to explore various institutional arrangements, various potential paths of development so as to assist and guide e.g. firms, policy makers, etc., in their particular decision context.

ABM thus uses methods and insights from diverse disciplines such as complexity sciences, cognitive science and computer science in its attempt to model the bottom-up emergence of phenomena and the top down influence of the collective phenomena on individual behaviour (Tesfatsion, 2002).

The recent developments in new programming techniques (as object-oriented programming) and, in particular, the advent of powerful tools of computation such as evolutionary computation (for a summary of the use of evolutionary computation and genetic programming in particular see Ebersberger (2002)) opens up the opportunity for economists to model economic systems on a more realistic (complex) basis (Tesfatsion, 2001b).

Before describing in more detail the structure of ABMs, a remark is in order. In the last years, many classes of formalizations that basically share the same philosophical and methodological underpinnings (e.g. focus on agents, heterogeneity, bounded-rationality, non-trivial interaction structures, true uncertainty, etc.), but have been labelled in different ways, have emerged in both theoretical and applied literature. Evolutionary economics, agent-based computational economics, neo-Schumpeterian models and history-friendly models, to cite only a few of them, have all been addressing similar questions on the grounds of similar approaches. In what follows, we will try to discuss the common features of these classes of models – rather than the distinctive ones. In fact, we prefer to consider them as complementary approaches, rather than “competing brands”. Of course, one has to be aware that, from a descriptive perspective, the dimensions along which these classes of models differ might also characterize in a positive way their richness and their ultimate goals. For example, evolutionary models typically stress the selection-dimension of market mechanisms, while the “Agent-based Computational Economics” (ACE) models mainly focus on both the tool used to build and analyze them (e.g. object-oriented programming) and on the open-ended, evolving nature of individual behavioural rules.

3. Agent-Based Models in Economics

3.1. Building Blocks

Irrespective of the particular label under which different classes of ABMs have become known among economics scholars, they all share a common set of qualitative assumptions that reflect their underlying modelling philosophy. In what follows, we will try to discuss the most important ones. Our goal is, first, to define the boundaries of an admittedly huge class of models (or a “meta-model”) and, second, to single out some relevant sub-classes of models that can be considered instances of that meta-model, but only feature a subset of building blocks.

Bottom-up Philosophy. Any satisfactorily account of a decentralized economy must be addressed in a bottom-up perspective (Tsfatsion, 2002). Aggregate properties must be viewed as the outcome of micro dynamics involving basic entities (agents). This approach might be contrasted with the typical *top-down* nature of all mainstream micro-macro models, where the bottom level is typically compressed into the behaviour of a representative individual.

The Evolving Complex System (ECS) Approach. Agents live in complex systems evolving through time (Kirman, 1998). Therefore, aggregate properties are seen to emerge out of repeated interactions among simple entities, rather than from consistency requirements carried through by rationality and equilibrium assumptions made by the modeller.

Heterogeneity. Agents are (or might be) heterogeneous in almost all their characteristics. The latter range from endowments and other agents’ properties, all the way to behavioural rules, competencies, and rationality and computational skills.

Bounded Rationality. The environment where agents are thought to live is too complex for any notion of “hyper-rationality” to be viable (Dosi et al., 1996). Therefore, one might at most impute to the agents some local (both in space and time) and partial principles of rationality, e.g. a myopic optimization rule. More generally, agents are assumed to behave as boundedly rational entities with adaptive expectations. Contrary to mainstream “neoclassical” models – where, *strictu sensu*, learning cannot take place as the agents already know everything they need to – here many forms of uncertainty can be postulated (e.g. substantive vs. procedural, risky environment vs. true uncertainty). Consequently, different regimes of individual and

collective learning can be modelled. For example: learning on “fixed menus” vs. learning in open-ended (endogenously evolving) spaces, learning on the spaces of actions/strategies, learning on the representations of the world, learning on the space of performances, learning on the space of preferences (Dosi et al., 1996).

“True” dynamics. Partly as a consequence of adaptive expectations (i.e. agents observe the past and form expectations about the future on the basis of the past), ABMs are characterized by a true, non-reversible, dynamics: the state of the system path-dependently evolves and cannot be considered as a coherent *whole* as happens in mainstream models (Marengo and Willinger, 1997).

Direct (Endogenous) Interactions. Agents interact directly: The decisions undertaken today by any agent directly depend – through adaptive expectations – on past choices made by subgroups of other agents in the population (Fagiolo, 1997). These subgroups are typically those who are the “closest ones” in some socio-economic spaces (i.e. the “neighbours” or the “relevant ones”). In turn, these interaction structures may endogenously change over time, as agents can strategically decide whom to interact with on the basis of expected payoffs. All that, together with heterogeneity and bounded rationality, may of course entail non-trivial aggregation processes, non-linearities and, sometimes, the emergence of structurally new objects.

Endogenous and persistent novelty. Socio-economic systems are inherently non-stationary. Agents face “true uncertainty”, as they are only partly able to form expectations e.g. on technological outcomes. Agents can endogenously introduce, through their decisions, structural changes in technological spaces, which typically become open-ended.

Selection-based market mechanisms. Agents are typically selected against – over many different dimensions – by market mechanisms (Nelson and Winter, 1982). This generates, e.g. in industry dynamics, additional turbulence in the system due to the entry-exit process of firms.

3.2. The Basic Structure of ABMs

Let us now turn to a more formal description of the basic structure of an ABM. As we did for its methodological building blocks, we will list here a very broad set of ingredients. We will then briefly comment on the flexibility of this description and we will provide some examples

of existing classes of models that are in the spirit of ABM and share an increasingly larger set of building blocks.

- a. *Time*: We typically model a system evolving in discrete time steps, i.e. $t = 1, 2, \dots$
- b. *Agents (or Actors)*: The system is populated by a set of agents $I_t = \{1, 2, \dots, N_t\}$. In many examples, but not necessarily, a constant population size is assumed ($N_t = N$).
- c. *Micro States (or Actions)*: Each agent $i \in I_t$ is characterized by a vector of L microeconomic states (or micro-variables) $\underline{x}_{i,t} = (x_{i,t}^1, \dots, x_{i,t}^L)$. These variables are fast ones, which can be endogenously modified by agents' decisions (e.g. firm's output, consumption levels, etc.)
- d. *Micro-Parameters*: Each agent $i \in I_t$ is also characterized by a vector of H microeconomic parameters $\underline{\theta}_i = (\theta_i^1, \dots, \theta_i^H)$. Micro-parameters are slow-variables, i.e. quantities that cannot be endogenously modified by the agents within the time-scale of the dynamic process. Therefore, $\underline{\theta}_i$ typically contains information about behavioural and technological characteristics of agent i (e.g. endowments, firms' factors productivity, workers' reservation wages, consumption elasticities, etc.)
- e. *Macro-Parameters*: The system as a whole is instead characterized by a vector of M time-independent macro-parameters $\underline{\Theta} = (\Theta_1, \dots, \Theta_M)$ governing the overall technological and institutional setup. Once again, $\underline{\Theta}$ are slow-variables and cannot be modified by the agents. Examples of $\underline{\Theta}$ parameters are the level of opportunities in a technological environment, the strength of unions in wage-bargaining, etc.
- f. *Interaction Structures*: At each t , the way in which information is channelled among agents is governed by a (directed and possibly weighted) graph G_t containing all directed links ij_t currently in place (i.e. open) from agent i to agent j . The existence of a directed link ij_t means that agent i , when updates his micro-variables $\underline{x}_{i,t}$, is affected by the choices made in the past by agent j (i.e. past j 's micro variables).
- g. *Micro Decision Rules*: Each agent is endowed with a set of decision rules $\mathfrak{R}_{i,t} = \{R_{i,t}^b(\cdot | \cdot), b=1, \dots, B\}$, mapping observable variables (e.g. past micro variables of relevant agents, micro and macro parameters, etc.) into next-period micro-variables

$\underline{x}_{i,t+1}$. Examples of such decision rules are: production functions, innovation rules, consumers' demand, etc.

- h. *Aggregate variables*: By aggregation (e.g. average, sum, etc.) of micro-variables, one obtains a vector of K macro-variables $\underline{X}_t = (X_t^1, \dots, X_t^K)$ which contain all macro information relevant to the analysis of the system. Examples are: GDP, aggregate demand, unemployment, etc. Moreover, \underline{X}_t can appear as arguments of $R_{i,t}^b$ as well: this is a source of feedbacks from the macro level to the micro level.

Notice that, on the basis of these broad ingredients, one can conceive a huge class of applications. Indeed, the flexibility of the agent-based approach – together with the easiness to implement in a modular way alternative assumptions through computer programming – allows one to envisage a large spectrum of models. For example, micro decision rules can fall in the wide range whose extremes are represented by (deterministic or stochastic) best-replies rules (as in evolutionary games), routines (as employed e.g. in evolutionary and Neo-Schumpeterian models, see also below) and by algorithmic, complicated, if-then rules, accounting for a large number of conditions and non-linear feedbacks (as happens in artificial-intelligence applications such as neural networks, genetic programming, etc.). Similarly, expectations can have the form of simple myopic rules (i.e. “tomorrow will be like today”) or can employ in more intelligent ways large amounts of information coming from the past (as happens in econometric-based prediction models). Different interactions structures can also be experimented. This allows one to answer questions related to whether the properties of the networks where agents are placed (i.e. regular vs. asymmetric, small-worlds, hierarchic relations, competitive vs. cooperative interaction patterns, bilateral vs. multilateral links, etc.) affect the aggregate properties of the system. Finally, one can compare systems where micro decision-rules and networks are static, with others where agents can endogenously and strategically act over their own rules and interaction links (i.e. additional, agent-specific, meta-rules are assumed that govern how $\mathfrak{R}_{i,t}$ and G_t endogenously change).

As one can see, many classes of well known, recently appeared models can be traced back to the meta-model presented above. Examples range from evolutionary games (Vega-Redondo, 1996), (local) interaction models (Fagiolo, 1998), endogenous network-formation models (Fagiolo et al., 2004b, Pyka et al., 2004), and Polya-urn schemes (Arthur, 1994); to more microfounded models such as industry-dynamics models in the Nelson and Winter's spirit, evolutionary growth models (Silverberg and Verspagen, 1994; Fagiolo and Dosi, 2003) and

ACE models of market dynamics (Epstein and Axtell, 1996; Tesfatsion, 2001a, Grebel, Pyka and Hanusch, 2003).

3.3. The Outcomes of ABMs and their Analysis

Let us consider now how a system modelled as in Section 3.2 evolves through time. At each point in time, the agents according to their decision rules update micro-variables.

A particular *updating scheme* (i.e. a rule that governs how many – and who – are allowed to update their micro-variables at time t) is typically assumed. This scheme will have an *asynchronous* nature if only a subset of all agents (at one extreme, only *one of them*) is allowed to reconsider the state of their micro-variables. Conversely, we will postulate a *parallel* updating scheme if *all* agents will have the opportunity to update their micro-variables. Notice that this is a crucial assumption as far as asymmetry of information is concerned: the more the updating scheme will be *asynchronous*, the more agents will tend to act over different information sets (see also Page, 1997).

Suppose some choice for initial conditions about variables and parameters, both at the micro and at the macro level has been done. Then, the dynamics of $\mathbf{x}_t = (\underline{x}_{1,t}, \dots, \underline{x}_{N,t})$ induced at the micro-level by individual updating will entail at the macro level, simply by aggregation, a dynamics over the set of macro-variables $\underline{X}_t = (X_t^1, \dots, X_t^K)$.

The stochastic components possibly present in decision rules, expectations, and interactions will in turn imply that the dynamics of micro and macro variables can be described by some (Markovian) stochastic process parameterised by the micro-parameters matrix $\Theta = (\underline{\theta}_1, \dots, \underline{\theta}_N)$ and the macro-parameter vector $\underline{\Theta}$ (given initial conditions \mathbf{x}_0 and \underline{X}_0).

Non-linearities which are typically induced by decision rules, expectations, and interactions networks and feedbacks, may imply that it is hard to analytically derive laws of motion, kernel distributions, time- t probability distributions, etc. for the stochastic processes governing $\{\mathbf{x}_t\}$ and $\{\underline{X}_t\}$ – and *a fortiori* the two jointly.

This implies that the researcher must often resort to computer simulations in order to analyze the behaviour of the system he/she has modelled along the lines sketched in the general framework of Section 3.2. Two remarks are in order. First, in some simple cases such systems allow for analytical solutions of some kind. For example, some evolutionary games model

(Vega-Redondo, 1996) allows for analytical solutions as far as equilibria and the size of their basin of attraction are concerned. Needless to say, the more one injects into the model assumptions sharing the philosophy of building blocks discussed in Section 3.1, the less tractable turns out to be the model, and the more one needs to resort to computer-simulations.

Second, we employ here the term “computer simulation” in a very broad sense. As we briefly noticed in the introduction, one might indeed think to an entire range of simulation analyses. At one extreme, one might employ simulation-like exercises to find numerical solutions of dynamical problems that have some closed-form representation, e.g. in terms of systems of (partial) differential equations (see Judd, 1998; Amman et al., 1996). Similarly, simulation techniques might be used to address the study of the properties of some particular test statistics or estimator in econometrics (Gourieroux and Monfort, 1996). At the other extreme, one might employ simulations in a more constructive way either to reproduce algorithmically the rules entailed by some complicated dynamic game (cf. for example Fagiolo, 2005), or to “grow” a society “from the bottom up”, in the spirit of object-oriented programming (cf. Epstein and Axtell, 1996; Tesfatsion, 2001a)

When studying the outcomes of ABMs, the researcher often faces the problem that the economy he/she is modelling is by definition out-of-equilibrium (Fisher, 1985). The focus is seldom on static equilibria or steady-state paths. Rather, the researcher must more often look for long-run statistical equilibria (cf. e.g. Foley, 1994) and emergent/transient (statistical) properties of aggregate dynamics (Lane, 1993a,b). Such an exploration is by definition very complicated and it is made even more difficult by the fact that the researcher does not even know in advance whether the stochastic process described by its AGM is ergodic or not – and, if it somehow converges, how much time will take for the behaviour to become sufficiently stable.

Suppose for a moment that the modeller knows – e.g. from a preliminary simulation study or from some ex-ante knowledge coming from the particular structure of the ABM under study – that the dynamic behaviour of the system becomes sufficiently stable after some time horizon T^* for (almost all) points of the parameter space.

Then a possible procedure that can be implemented to study the output of the ABM runs as the one synthetically depicted in Figure 1 (see Fagiolo and Dosi (2003) for an example of such a procedure). Given some choice for initial conditions, micro and macro parameters (i.e.

θ , Θ , \mathbf{x}_0 and \underline{X}_0), assume to run our system until it relaxes to some stable behaviour (i.e. for at least $T > T^*$ time steps). Suppose we are interested in a set $S = \{s_1, s_2, \dots\}$ of statistics to be computed on the simulated data $\{\mathbf{x}_t, t=1, \dots, T\}$ and $\{\underline{X}_t, t=1, \dots, T\}$. For example, one of the micro variables might be individual firm's output and the correspondent macro variable could then be GNP. In such a case, one could be interested in an aggregate statistics s_j like the average rate of growth of the economy over the T time-steps (e.g. quarters). For any given run $m=1, 2, \dots, M$, the program will output a value for s_j . Given the stochastic nature of the process, each run – and thus each value of s_j – will be different from the other ones. Therefore, after having produced M independent runs, one has a distribution for s_j containing M observations, which can be summarized by computing e.g. its mean $E(s_j)$, its variance $V(s_j)$, etc.

[Figure 1 about here]

Recall, however, that moments will depend on the choice for θ , Θ , \mathbf{x}_0 and \underline{X}_0 . By exploring a sufficiently large number of points in the space where initial conditions and parameters are allowed to vary, and by computing $E(s_j)$, $V(s_j)$, etc. at each point, one might get a quite deep understanding of the behaviour of the system. Consider again the output example introduced above. For instance, one may simply plot $E(s_j)$, that is the Monte-Carlo mean of the economy's average growth rates, against some important macro parameters such as the level of aggregate propensity to invest in R&D. This might allow one to understand whether the overall performance of the economy increases in the model with that propensity. Moreover, non-parametric statistical tests may be conducted to check if $E(s_j)$ significantly differs in two extreme cases, e.g. high vs. low propensity to invest in R&D.

3.4. Model Selection and Empirical Validation in ABMs

From the foregoing discussion clearly emerges that in agent-based modelling – as in many other modelling endeavours – one often faces a trade-off between *descriptive accuracy* and *explanatory power* of the model. The more one tries to inject into the model “realist” assumptions as agents' heterogeneity, open-ended evolution, endogenous interactions, structural innovation, boundedly-rational behaviours, etc., the more the system becomes complicated to study and the less clear the causal relations going from assumptions to implications are. ABM researchers are well aware of this problem and have been trying to develop an effective strategy to select “bad” from “good” models (see e.g. Edmonds and Moss (2004), Frenken (2005), Werker and Brenner (2004)).

A first set of strategies – which is typically applied in the early stages of model building – concerns the process of assumption selection. For example, one can judge a model on the basis of the “realism” of its assumptions (Mäki, 1994), where an assumption has a higher degree of realism if it is supported by some robust experimental evidence. Alternatively, one can try to solve the trade-off between *descriptive capability* and *explanatory power* either by beginning with the most simple model and complicate it step-by-step (i.e. the so-called KISS strategy, an acronym standing for “Keep it simple, stupid!”) or by starting with the most descriptive model and simplify it as much as possible (i.e. the so-called KIDS strategy, “Keep it descriptive, stupid!”). A third, alternative strategy prescribes instead to start with an existing model and successively complicate it with incremental additions (this strategy might be labelled TAPAS, which stands for ‘Take A Previous model and Add Something’). In all these procedures, the crucial variable that should be able to discriminate the point at which any procedure should stop would determine the *explanatory power* of the model.

This is where a second set of strategies – which typically applies *ex-post*, once a sufficiently small number of “satisficing” models has been developed – enters the picture. This second set of strategies is indeed based on how good the model is at replicating reality. Notice that the very structure of ABMs naturally allows one to take the model to the data and validate it against observed real-world observations. Indeed, an ABM model can be thought to provide a family of data generation processes (DGPs), which we think real-world observations being a realization of.

Many approaches to empirical validation (and selection) of ABMs can be in principle taken, and the debate is very open here. For example, one might select among ABMs (and within different parameter setups of the same ABM) with respect to the number of stylized facts each of them is able jointly to replicate. A typical procedure to be followed starts with asking whether a particular model is able jointly to replicate some set of stylized facts for a given parameterisation (a sort of “exercise in plausibility”); then explore what happens when the parameter setup changes; finally, examine if some meaningful causal explanation can be derived out of that step-by-step analysis. Alternatively, one can first select among parameters by calibrating the model (e.g. by directly estimate parameters, when possible, with micro or macro data) and then judge to which extent the calibrated model is able to reproduce the stylised facts of interest.

It must be noticed, however, that the issue whether an ABM should deliver quantitative implications (and must be then judged on the grounds of its fit to real-world data) is still open. Some scholars advocate, for instance, that an ABM should be used as a “research” tool addressing qualitative issues only: in this view, ABMs are viewed as laboratories where some simple theories, often in the form of causal relationships, can be tested (Valente, 2004). As empirical validation is no longer required, there is no need for either calibration exercises, or Monte-Carlo types of explorations of the space of parameters. Indeed, in a way or in the other, both types of exercises aim at selecting a preferred world (i.e. a particular DGP among the family postulated by the ABM description) and can be considered as a strategy to indirectly maximize the likelihood of a particular DGP. In other words, among the space of all parameters and initial conditions, one tries to select the one that it is assumed to generate – with the highest probability – the “unique” observation that we have in reality. If one instead employs ABMs as generators of qualitative and causal implications only, also a low-probability event generated by the model is important (and often crucial) to understand some casual mechanism going on in the real world. After all, the world where we live could have well been the outcome of a small probability event.

3.5. Designing Agents in ABMs: Beyond the Basic Framework

In ABMs, agents are considered as being the major driving force of evolution. As such, we regard them as the reason for the manifestation of qualitative developments going on in the system. Being the crucial component of the system, their description can go well beyond the introductory one presented above.

To begin with, one can think to implement a multi-agent approach, which assumes that agents populating the model can be divided into various categories, according to their initial endowments (i.e. availability of capital, entrepreneurial attitude, technological competencies etc.).

Accordingly, a central issue is the general design of the agents. Agents might be represented as a piece of code that has the standard attributes of intelligent agents (Wooldridge and Jennings, 1995) that is: (i) *Autonomy*, which means that agents operate without other agents having direct control of their actions and internal states. This is a necessary condition for implementing heterogeneity; (ii) *Social ability*, i.e. agents are able to interact with other agents not only in terms of competition but also in terms of cooperation. This includes the

possibility to model agents that show various forms of interaction blended from competition and cooperation; (iii) *Reactivity*, agents are able to perceive their environment and respond to it; (iv) *Proactivity*, which enables the agents to take initiatives. This means that they are not only adapting to changing circumstances, rather are they engaged in goal-directed behaviour.

The above points indicate that the actors in the simulation are able not only to adapt their behaviour to a given set of circumstances but they are also in a Neo-Schumpeterian sense able to learn from their own experience and to modify their behaviour creatively so as to change the circumstances themselves.

In turn, decision rules (often in the form of routines) allow actors to manipulate the reality. It is not only the endowment with resources that shapes the nature of the actors, it is their individual routines that make up a large part of the actors heterogeneity. Nelson and Winter (1982) relate routines to satisficing behaviour and bounded rationality of actors. Routinized behaviour causes some stickiness and some inertia of the system that results in some stability of the system – stability, at least to a certain degree. Furthermore, routines are not only focused on internal procedures of the actors, but they also govern external relationships with actors of the same basic group and with actors of other groups.

Finally, ABMs' micro parameters often take the form of initial endowments. Access to material and immaterial resources, their availability together with the individual experiences, make up the endowment of the agents. They combine the different components in order to realize their goals. Accordingly, endowments are the crucial assets of agents in accomplishing their tasks. Agents are typically heterogeneous in their sets of endowments. It is obvious that autonomy of agents can only be achieved with the notion of personal and individual endowment of certain factors. It is the idea of individual property rights on production factors or income that enables us to model actors acting on with their sets of endowments.

4. Conclusions

In this paper we attempted to provide an introduction to ABMs in economics. We have begun with a discussion of the main motivations that, in the last years, led many scholars to supplement “mainstream” (neoclassical) treatments of microfounded models of macrodynamics with alternative approaches, rooted in “more realist” assumptions as heterogeneity, interactions, bounded rationality, endogenous novelty, etc.. After presenting the building blocks shared by this class of models, we suggested a meta structure common to (almost all)

ABMs employed in economics. Finally, we examined some standard strategies used to analyze the outcomes of ABMs.

In our view, the attempt to model the aggregate dynamics of decentralized economies on the basis of a more detailed (and more realist) microfoundation such as the one postulated by ABMs is the primary requirement to pursue one of the most prominent challenges in social sciences today, namely the analysis of qualitative change. Our discussion suggests that ABMs are offering an adequate framework for this, overcoming the severe restrictions which orthodox economic approaches are confronted with. By emphasizing the role of true uncertainty and irreversibility, one is able to model qualitative development as an endogenous process driven by the agents and their interactions.

Agent based models allow for an explicit consideration of these characteristic features, and therefore can be considered as “the” modelling tool for the analysis of qualitative development and transformation processes. In a way, agent based models can be considered a systemic approach, allowing the consideration and integration of different social “realities” which makes them an extremely valuable tool for the analysis of social processes which can be generally considered as multifaceted phenomena.

The field of agent-based modelling is – especially in economics, but more generally in social sciences – very far from its maturity. Many issues, especially methodological ones, are still debated, both on the model development side and on the model analysis side. Here, as a way of conclusion, we will try to briefly mention some of the most crucial ones.

First, on the model building and development side, one faces a huge heterogeneity in the way agents and their behavioural and interaction rules are assumed and implemented. In the relevant literature, one often deals with many structurally different ABMs addressing very similar issues. This practice – which is ultimately caused by the flexibility of programming languages and their heterogeneity – can certainly turn out to be a plus, because it might favour a better understanding of the deep causes of a given phenomenon. However, it can also generate in the long-run an inherent impossibility to compare different models and to pursue a coherent procedure of model improvement (with old and obsolete models replaced by higher-performance ones).

Second, and relatedly, an agent-based modeller confronting the KISS vs. KIDS problem will often end up with an over-parameterised model. In order to limit as much as possible all critiques regarding the robustness of results to different parameterisations and initial conditions, an exhaustive exploration of both parameters' and initial conditions' sets is required. But, even when the ABM has been thoroughly analyzed e.g. along the lines suggested in Section 3.3, some further problems arise. To begin with, how can one know for sure that the system is ergodic? And, even if it is ergodic, how can one be sure to have correctly estimated the relaxation time of our stochastic process? And, even more importantly, how can one be sure not to have neglected some truly “emergent” properties? After all, any truly emergent property should not be totally comprehensible on the grounds of the “alphabet”, “syntax” and “grammar” which we employed to describe existing entities.

Third, even when the foregoing critiques have been carefully considered, an agent-based modeller should be aware of the fact that all his/her results could be heavily affected by the particular sets of behavioural and interaction rules that he/she has assumed. Those rules are often kept fixed across time. This can be justified by the observation that the rules themselves typically change slower than the variables which they act upon (e.g. micro and macro variables). However, in the evolutionary spirit informing ABMs, a necessary step would be that of modelling the rules themselves as endogenously changing objects. An example here concerns learning and decision rules, which can be endogenously modified by the agents along the process.

Finally, on the normative side, it must be noticed that so far ABMs have almost exclusively addressed the issue of replication of stylized facts. However, ABMs can and should be employed to address policy issues as well. A need for increasingly normative-oriented ABMs delivering policy implications and out-of-sample predictions is nowadays strongly felt in the community. Thanks to the flexibility and the power of agent-based approaches, it is easy to conceive frameworks where policy experiments are carried out to evaluate the effectiveness of different policy measures (e.g., anti-trust policies), for a range of different institutional setups and behavioural rules.

References

- Amman, H., Kendrick, D. and Rust, J. (1996), *Handbook of Computational Economics*, Elsevier-North Holland, Amsterdam, The Netherlands.
- Arthur, W. B. (1988), "Competing Technologies: an Overview", in: Dosi, G. et al. (eds.), *Technical Change and Economic Theory*, Francis Pinter, London.
- Arthur, W.B. (1994), *Increasing returns and path-dependency in economics*, University of Michigan Press, Ann Arbor.
- Aruka, Y. (2001), *Evolutionary Controversies in Economics: A New Transdisciplinary Approach*, Springer, Tokyo and Berlin.
- Boulding, K. E. (1991), "What is evolutionary economics?", *Journal of Evolutionary Economics*, 1: 9-17.
- Cantner, U. and Pyka, A. (1998), "Technological Evolution - an Analysis within the knowledge-based Approach", *Structural Change and Economic Dynamics*, 9: 85-108.
- David, P.A. (1975), *Technical choice, Innovation and Economic Growth*, Cambridge University Press, Cambridge.
- Dosi, G. (1988), "Sources, Procedures and Microeconomic Effects of Innovation", *Journal of Economic Literature*, 26: 126-171.
- Dosi, G. and Orsenigo, L. (1994), "Macrodynamics and microfoundations: an evolutionary perspective", in Granstrand, O. (ed.), *The economics of technology*, North Holland, Amsterdam.
- Dosi, G., Freeman, C., Nelson, R.R., Silverberg, G. and Soete, L. (1988) (eds.), *Technical Change and Economic Theory*, Francis Pinter, London.
- Dosi, G., Marengo, L. and Fagiolo, G. (2005), "Learning in Evolutionary Environment", in Dopfer, K. (ed.), *Evolutionary Principles of Economics*, Cambridge, Cambridge University Press.
- Ebersberger, B. (2002), *Genetische Programmierung: Ein Instrument zur empirischen Fundierung ökonomischer Modelle*, forthcoming.
- Edmonds, B. and Moss, S. (2004), "From KISS to KIDS: an 'anti-simplistic' modelling approach", Centre for Policy Modelling, Manchester Metropolitan University, mimeo.
- Epstein, J.M. and Axtell, R. (1996), *Growing Artificial Societies: Social Science from the Bottom-Up*, Washington D.C., MIT Press and Brooking Press.
- Fagiolo, G. (1998), "Spatial interactions in dynamic decentralized economies: a review", in Cohendet, P., Llerena, P., Stahn, H. and Umbhauer, G. (Eds.), *The Economics of Networks. Interaction and Behaviours*, Springer Verlag, Berlin - Heidelberg.
- Fagiolo, G. (2005), "Endogenous Neighborhood Formation in a Local Coordination Model with Negative Network Externalities", *Journal of Economic Dynamics and Control*, 29: 297-319.
- Fagiolo, G. and Dosi, G. (2003), "Exploitation, Exploration and Innovation in a Model of Endogenous Growth with Locally Interacting Agents", *Structural Change and Economic Dynamics*, 14: 237-273.

- Fagiolo, G., Dosi, G. and Gabriele, R. (2004a), "Matching, Bargaining, and Wage Setting in an Evolutionary Model of Labor Market and Output Dynamics", *Advances in Complex Systems*, 14: 237-273.
- Fagiolo, G., Marengo, L. and Valente, M. (2004b), "Endogenous Networks in Random Population Games", *Mathematical Population Studies*, 11: 121-147.
- Fisher, F.M. (1985), *Disequilibrium Foundations of Equilibrium Economics*, Cambridge University Press, Cambridge.
- Foley, D.K. (1994), "A statistical equilibrium theory of markets", *Journal of Economic Theory*, 62: 321-345.
- Foray, D. and Freeman, C. (1992), *Technology and the wealth of nations*, OECD, Paris.
- Forni, M. and Lippi, M. (1997), *Aggregation and the Microfoundations of Dynamic Macroeconomics*, Clarendon Press, Oxford.
- Freeman, C. (1982), *The Economics of Industrial Innovation*, Francis Pinter, London.
- Freeman, C. (1994), "The economics of technical change", *Cambridge Journal of Economics*, 18: 463-514.
- Frenken, K. (2005), "History, state and prospects of evolutionary models of technical change: a review with special emphasis on complexity theory", Utrecht University, The Netherlands, mimeo.
- Gilbert, N. and Troitzsch, K. (1999), *Simulation for the Social Scientist*, Open University Press, Milton Keynes.
- Gourieroux, C. and Monfort, A. (1996), *Simulation-based econometric models*, Oxford University Press, Oxford.
- Granstrand, O. (Ed.) (1994), *The economics of technology*, North Holland, Amsterdam.
- Grebel, T., Pyka, A. and Hanusch H. (2003), "An Evolutionary Approach to the Theory of the Entrepreneur", *Industry and Innovation*, Vol. 10, Dec. 2003, 493-514.
- Judd, K. (1998), *Numerical Methods in Economics*, MIT Press, Cambridge, Massachusetts.
- Kagel, J.H. and Roth, A.E. (1995) (Eds.), *The Handbook of Experimental Economics*, Vol. 1, Princeton University Press, Princeton, N.J.
- Kirman, A.P. (1989), "The intrinsic limits of modern economic theory: the emperor has no clothes", *Economic Journal*, 99: 126-139.
- Kirman, A.P. (1992), "Whom or what does the representative individual represent ?", *Journal of Economic Perspectives*, 6: 117-136.
- Kirman, A.P. (1997a), "The Economy as an Interactive System", in Arthur, W.B., Durlauf, S.N. and Lane, D. (eds.), *The Economy as an Evolving Complex System II*, Santa Fe Institute, Santa Fe and Reading, MA, Addison-Wesley.
- Kirman, A.P. (1997b), "The Economy as an Evolving Network", *Journal of Evolutionary Economics*, 7: 339-353.

- Kwásnicki, W. (1998), "Simulation methodology in evolutionary economics", in F. Schweitzer and G. Silverberg (eds), *Evolution und Selbstorganisation in der Ökonomie*, Vol. 9 of *Selbstorganisation: Jahrbuch für Komplexität in den Natur-, Sozial- und Geisteswissenschaften*, Duncker&Humblot, Berlin.
- Lane, D. (1993a), "Artificial worlds and economics, part I", *Journal of Evolutionary Economics*, 3: 89-107.
- Lane, D. (1993b), "Artificial worlds and economics, part II", *Journal of Evolutionary Economics*, 3: 177-197.
- Lundvall, B.A. (1993), *National systems of innovation*, Francis Pinter, London.
- Mäki, U. (1994), "Reorienting the assumptions issue", in R. Backhouse (ed.), *New Directions in Economic Methodology*, Routledge, London and New York.
- Marengo, L. and Willinger, M. (1997), "Alternative Methodologies for Modeling Evolutionary Dynamics: Introduction", *Journal of Evolutionary Economics*, 7: 331-338
- Nelson, R. R. and Winter, S. G. (1982), *An Evolutionary Theory of Economic Change*, Cambridge University Press, Cambridge, MA.
- Nelson, R.R. (1995), "Recent Evolutionary Theorizing About Economic Change", *Journal of Economic Literature*, 33: 48-90.
- Page, S.E. (1997), "On Incentives and Updating in Agent Based Models", *Computational Economics*, 10: 67-87.
- Plott, C.R. and Smith, V.L. (1998) (eds.), *Handbook of Experimental Economics Results*, North Holland, Amsterdam/New York.
- Pyka, A., Ahrweiler, P. and Gilbert, N. (2004), „Simulating knowledge dynamics in innovation networks (SKIN)“, , Institut für Volkswirtschaftslehre der Universität Augsburg, Volkswirtschaftliche Diskussionsreihe, #267, 2004.
- Romer, P. M. (1990), "Endogenous Technological Change", *Journal of Political Economy*, 98: 77-102.
- Rosenberg, N. (1982), *Inside the Blackbox*, Cambridge University Press, Cambridge.
- Rosenberg, N. (1994), *Exploring the Black Box: Technology, Economics and History*, Cambridge University Press, Cambridge.
- Sargent, T.J. (1987), *Dynamic macroeconomic theory*, Harvard University Press, Cambridge.
- Saviotti, P. P. (1996), *Technological Evolution, Variety and the Economy*, Edward Elgar, Aldershot.
- Scherer, F.M. and Ross, D. (1990), *Industrial Market Structure and Economic Performance*, Houghton and Mifflin Company, Boston.
- Schumpeter, J. A. (1912), *Theorie der wirtschaftlichen Entwicklung*, Duncker & Humblot, Berlin.
- Silverberg, G. and Verspagen, B. (1994), "Collective learning, innovation and growth in a boundedly rational, evolutionary world", *Journal of Evolutionary Economics*, 4: 207-226.

- Stoneman, P. (1995) (ed.), *Handbook of the Economics of Innovation and Technological Change*, Blackwell, Oxford.
- Tesfatsion, L. (2001a), "Structure, Behavior, and Market Power in an Evolutionary Labor Market with Adaptive Search", *Journal of Economic Dynamics and Control*, 25: 419-457.
- Tesfatsion, L. (2001b), "Agent-based modelling of evolutionary economic systems", *IEEE Transactions on Evolutionary Computation*, 5: 1-6.
- Tesfatsion, L. (2002), "Agent-based Computational Economics: Growing Economies from the Bottom Up", Ames, Iowa, Working Paper, No.1, Iowa State University, Dept. of Economics.
- Valente, M. (2004), "Qualitative Simulation Modelling", Faculty of Economics, University of L'Aquila, L'Aquila (Italy), mimeo.
- Vega-Redondo, F. (1996), *Evolution, Games, and Economic Behavior*, Oxford University Press, Oxford.
- Werker, C. and Brenner, T. (2004), "Empirical Calibration of Simulation Models", Papers on Economics and Evolution # 0410, Max Planck Institute for Research into Economic Systems, Jena.
- Wooldridge, M. and Jennings, N.R. (1995), "Intelligent agents: theory and practice", *Knowledge Engineering Review*, 10: 115-152.

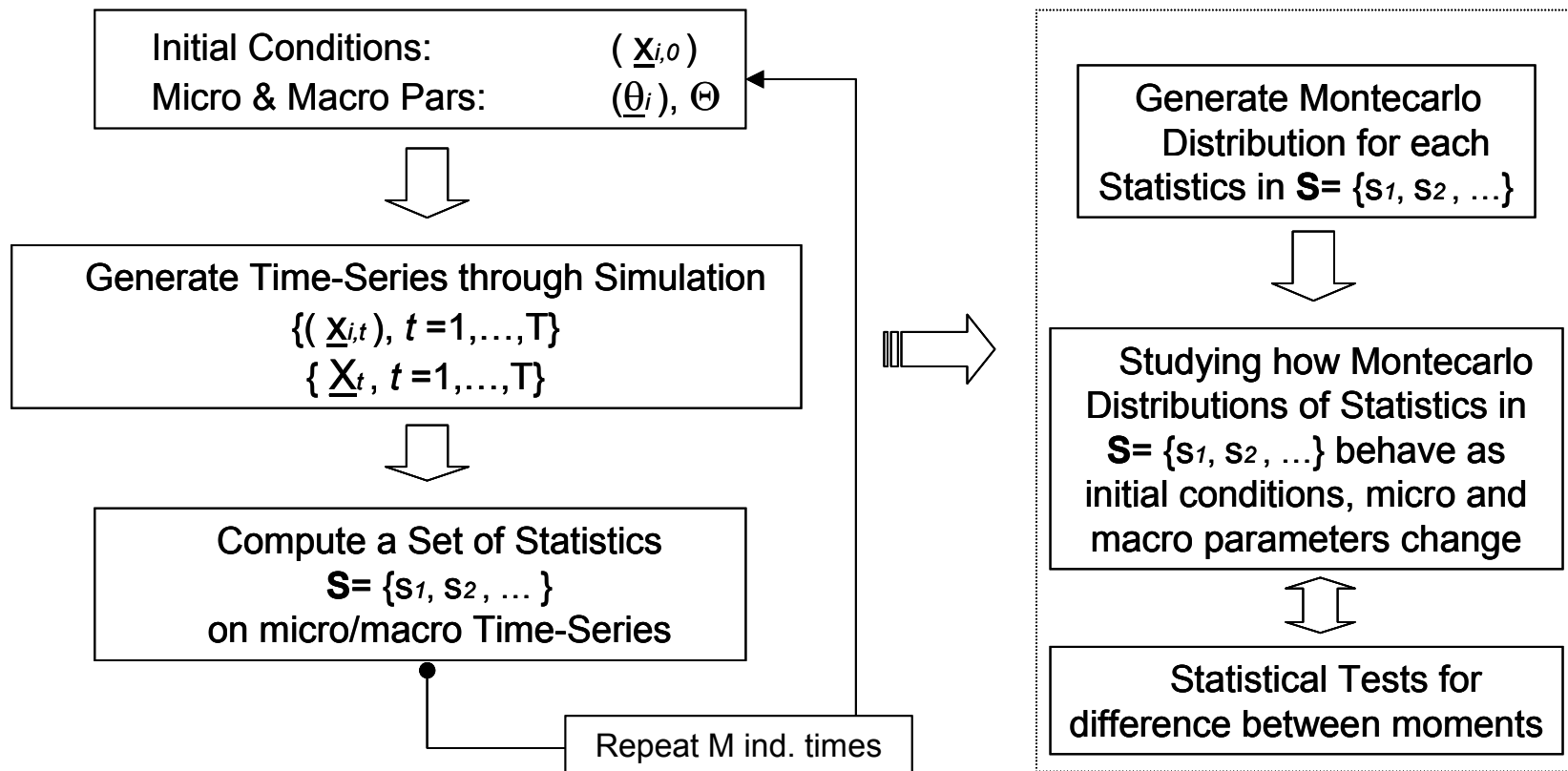


Figure 1: Statistical Analysis of Agent-Based Models. A Suggested Monte-Carlo Approach.