Economic Robustness of Scheduling Algorithms for Distributed Storage Systems

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1 Introduction

The integration of non-conventional supply and demand resources will be a crucial challenge in the upcoming modernization and restructuring of the power grids. On the supply side, increasing the shares of renewable and distributed energy resources (DER) is a trend in most industrialized countries. On the demand side, grid-integrated electric vehicles, storage devices, and demand side management (DSM) applications begin to change the traditionally known load profiles.

While the supply side has been flexible and responsive in the traditional grid, a higher share of renewable and DER, i.e., intermittent and non-dispatchable resources will reduce the supply side's degree of flexibility. The classical way of maintaining the supply side's flexibility is an increase of control capacity through additional capacity from fossil power plants, e.g., fast-starting gas power plants. Alternatively, this article will investigate how distributed storage systems (DSS) on the consumer side can compensate this flexibility reduction.

A prerequisite for a distributed, load-responsive DSM are flexible electricity prices that provide an economic incentive for load-shifting. We assume that flexible prices appropriately reflect the demand-supply-situation on the market. Traditionally, flexible prices only apply to producers and large industrial consumers. This is about to change by aiming at "smarter", i.e., more responsive consumers in the future. In Germany, according legal changes are already passed. End consumers will have access to load- and time-dependent electricity tariffs from 2011 onwards (Federal Law Gazette, 2008).

Within a context of uncertain price and load forecasts, the article will analyze the robustness of economic results in dependence on the scheduling algorithms in the DSS. We build our analysis on the basic DSS model in Ahlert and van Dinther (2009) aiming at arbitrage accommodation. Their storage application optimally achieves up to 17% savings on the annual electricity costs. The optimal operations base on an optimal charge-discharge-schedule (CDS) that requires ex-ante knowledge of load and price data. In reality, these data are not ex-ante available. Section

3.4 defines 9 simulation scenarios with different forecast accuracy levels. The simulations run these scenarios using different scheduling algorithms and benchmark their results against the optimal outcome.

In Section 2, the article will give an overview about related work on storage models. In Section 3, we will describe our simulation model and its parameters. Section 4 presents the results of the simulation model and Section 5 gives a conclusion and an outlook to further research.

2 Related Work

Storage optimization models aiming at arbitrage accommodation[1] can be distinguished into four groups with respect to their solution approaches and applied algorithms. The difference that we want to emphasize in this paper is that the first three groups do not take forecast uncertainties into account when optimizing storage schedules, while the last group does so.

The first group contains models using a (static) statistical analysis to assess the economics of DSS. E.g., Nieuwenhout et al. (2006, pp. 10-12) analyze two settings for load shifting at the end consumer level. The second group is models building on (multi-pass) dynamic programming (DP) methods to determine the optimal CDS for the storage system. Examples are Lee and Chen (1995, pp. 562-568) as well as Maly and Kwan (1995, pp. 453-458) that present analyses with a particular focus on the Taiwanese tariff system, which provides combined incentives for time-of-use (TOU) optimization and peak load reduction. The third group contains papers that present linear optimization (LO) models determining the optimal CDS. E.g., Wu et al. (2002) and Bathurst and Strbac (2003, pp. 1-8) analyze how storage systems can foster the integration of renewable (intermittent) resources from an economic perspective. Exarchakos et al. (2008, pp. 62-76), Graves et al. (1999, pp. 46-56), Sioshansi et al. (2009, pp. 269-277) analyze arbitrage opportunities on wholesale markets, and Ahlert and van Dinther (2009) analyze arbitrage accommodation based on TOU at the end consumer level.

All of the aforementioned models assess the economics of DSS on ex-ante given data and do not incorporate the uncertainty induced through forecast errors. The forth group builds on simulations that incorporate data streams underlying probabilistic distributions. Barton and Infield (2004, pp. 441-448) investigate how time-shifting the supply from wind farms can be used to maximize revenues. The uncertainty in their model stems from the distribution of wind speeds, which are an essential input factor for the wind farm's electricity production. Walawalkar et al. (2007, pp. 2558-2568) analyze the economics of energy storage systems in New York. They use probabilistic input data streams for revenues and charging costs in order to analyze the net present value of two defined storage systems. In the fol-

[1] This includes storage models that optimize their schedules according to time-of-use tariffs.

lowing, this article will present a simulation model that incorporates uncertainty through imperfect price and load forecasts into the economic evaluation of a DSS model.

3 Simulation Model and Scheduling Algorithms

The simulation model consists of three main steps for each day of the simulation period: (1) Generating a forecast, (2) determining an arbitrage-maximizing CDS, and (3) executing the CDS.

Step 1 generates a forecast of the hourly load volumes and electricity prices for the next 48 hours. Step 2 determines an arbitrage-maximizing CDS based on the previously generated forecasts. In Step 3, the analysis model tries to operate the battery storage based on this schedule whenever technically possible, i.e., the schedule is kept as long as it does not violate any technical constraints of the given storage system.

The total costs of the simulated period are calculated based on the same model as in Ahlert and van Dinther (2009, pp. 3-4), which minimizes the total costs of electricity supply, DSS installation and operation:

$$\min \rightarrow K^{fixed} + \sum_{t=1}^{T} K_t^{\text{var}} \tag{1}$$

with K^{fixed} being a constant representing the capital and maintenance costs as well as annual depreciations and K_t^{var} being the variable operation costs that result from the executed CDS (see Section 3.3) including costs for charging and using (depreciating) the storage device. The simulation model within this paper builds on the same assumptions, specifications, and input datasets as the basic model.[2]

3.1 Forecast Generation

The forecast generation module derives an artificial forecast $\mathrm{F_t}$ $(1 \le t \le T^{FP})$ for a forecast period of length T^{FP} from the actual data $\mathrm{A_t}$. The Mean Absolute Percentage Error (MAPE) $\bar{x}_t$ of forecast data points F_t^n (forecast value in simulation n $(1 \le n \le N)$)[3] to an actual data point $\mathrm{A_t}$ is defined as

[2] The price curve reflects the price distribution of market prices from the European Energy Exchange (EEX) (EEX, 2009) and is transformed to a weighted average price of 0.20 EUR/kWh. The price curve is modeled as an exogenous variable, i.e., it does not change dynamically (DSS are price takers). The load data distribution corresponds to the VDEW standard household profiles (VDEW, 2009) and an annual load of 2000 kWh. For the technical parameter specification of the lead acid-based battery storage system, see Ahlert and van Dinther (2009, p. 3).

[3] The forecast generation module involves random numbers (Monte Carlo simulation). Each simulation run is therefore repeated up to 500 times to average out the statistical fluctuations.

$$\bar{x}_{\mathrm{t}} = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \frac{\left|F_t^n - A_t\right|}{A_t} \tag{2}$$

The forecast module allows specifying forecast error levels at the beginning and the end of the forecast horizon. Moreover, the degree of forecast error autocorrelation can be specified with a Durbin-Watson-Test (DWT) value (a DWT value of 0 indicates a perfect positive autocorrelation, whereas a value of 2 indicates no autocorrelation of the forecast errors). A detailed description of the stochastic process is presented in Ahlert and Block (2010).

3.2 Schedule Determination

Determining an arbitrage-maximizing CDS is subject to various papers (see Section 2). As an alternative to LO algorithms and the DP approaches, we define a heuristic scheduling algorithm. Results of the LO model by Ahlert and van Dinther (2009, pp. 5-7) and the heuristic algorithm in this paper are benchmarked in Section 4. The main idea of the heuristic algorithm is to quickly generate a robust schedule, i.e., being less vulnerable to fluctuations of the forecast accuracy and the degree of forecast error autocorrelation.

In the **first phase**, the algorithm determines price limits for charging and discharging. I.e., the storage system considers charging as soon as the forecast price falls below the charge limit price; analogously it considers discharging when the forecast price exceeds the discharge limit price. The parameters used for the price limit determination function are:

- $L_t(price.fo, load.fo)$: List of Price-Load-Tuples (forecasts) for each timeslot t in the forecast period $1 \le t \le T$. Accessing the price is defined as $L_{t'}^{price.fo}$ (load analogously).
- $\vec{L}_{t'}(price.fo, load.fo)$: Sorted list (by ascending forecast prices), i.e., $\vec{L}_{t'}^{price.fo} \le \vec{L}_{t''}^{price.fo} \quad \forall t' < t'' \quad \left(1 \le t', t'' \le T^{FP}\right)$
- κ : Price per storage capacity unit [EUR/kWh].
- γ : Number of expected nominal (full) charge cycles over the lifetime of the storage device [#].
- $\Delta = \kappa \cdot \gamma^{-1}$: Required price difference between charge and discharge timeslots to cover variable costs of storage usage [EUR/kWh].
- $p^{\mathrm{charge}} = \vec{L}_i^{price.fo}$: Charge price limit [EUR/kWh], i is the charge index for list $\vec{L}$
- $p^{\mathrm{discharge}} = \vec{L}_j^{price.fo}$: Discharge price limit [EUR/kWh], j is the discharge index for list $\vec{L}$
- η^{out} : is the output efficiency of the storage system [%]

Determining the price limits follows two rules:

- **Rule 1:** The difference between the highest charging price and the lowest discharging price must always be greater or equal than the required price difference Δ. This ensures that each charge-discharge-cycle covers the variable costs of storage usage, regardless of the order in which the prices occur.
- **Rule 2:** The expected charge volume and the aggregated volume of the expected discharge timeslots are always balanced. I.e., the aggregated volume of the expected discharge timeslots never exceeds the expected charge volume and the expected charge volume never exceeds the expected discharge volume by more than the volume of one nominal charge timeslots.

Formally, the price limit determination function is defined as

$$\min \rightarrow j - i \tag{3}$$

aiming at a maximization of the economically beneficial charge and discharge volumes. The constraints are Rule 1 (eq. (4) and Rule 2 (eq. (5)). Equation 6 ensures that values of i and j do not cross and remain in valid ranges.

$$\vec{L}_j^{price.fo} - \vec{L}_i^{price.fo} \geq \Delta \tag{4}$$

$$i \cdot \frac{C}{v} - \frac{1}{\eta^{out}} \sum_{t'=j}^{T^{FP}} \vec{L}_{t'}^{load.fo} < \frac{C}{v} \tag{5}$$

$$1 \leq \mathrm{i} \leq \mathrm{T}^{\mathrm{FP}} - 1, \quad 2 \leq j \leq T^{FP}, \quad i < j \tag{6}$$

In the **second phase**, the algorithm determines the CDS. Additionally to the simplest variant *Heuristic 1*, variants *Heuristic 2* and *Heuristic 3* base the decision whether to actually charge or discharge the DSS not only on the previously determined price limits, but also on the expected alternatives in the succeeding timeslots (based on the forecast). In both cases, the rationale behind the additional conditions is to find local minima for charging and discharging in the forecast data (refinement of Rule 1). Additionally to the parameters defined for the Phase 1, Phase 2 uses the following parameters:

- p_t^{fo}: Price forecast for timeslot t [EUR/kWh].
- ℓ_t^{fo}: Load forecast for timeslot t [EUR/kWh].
- $q_t^{out.fo}$: Maximal discharge volume in timeslot t according to the load forecast [kWh].
- ξ_t: State of charge (SOC), i.e., stored electricity in timeslot t [kWh].
- $\hat{\delta}$: Maximal depths of discharge (DOD) [%].
- C: Maximal storage capacity [kWh].
- v: Maximal charging speed, i.e., required timeslots for a nominal charge cycle [#].

The variant *Heuristic 1* marks timeslots for charging as soon as the expected price falls below the charge price limit (discharging analogously).

$$\text{Condition 1}: \qquad p_t^{fo} \le p^{\text{charge}} \qquad \forall t \tag{7}$$

$$\text{Condition 2}: \qquad p_t^{fo} \ge p^{\text{discharge}} \qquad \forall t \tag{8}$$

The variant *Heuristic 2* additionally performs a refinement of the charge condition. A timeslot is only marked for charging, if the volume that could be charged in later timeslots with relatively lower prices (until the next discharge cycle) does not exceed the expected discharge volume of the next discharge cycle.

$$\text{Condition 3}: \qquad \frac{C}{v} \cdot \left|T_1'\right| < C - \xi_t \tag{9}$$

with $$T_1' = \left\{t' \mid p_{t'}^{fo} < p_t^{fo} : t' > t \land \neg\exists t'' \le t' : p_{t''}^{fo} \ge p_t^{\text{discharge}}\right\} \tag{10}$$

Heuristic 3 extends *Heuristic 2* by an additional discharge condition. A timeslot is only marked for discharging, if the expected discharge volume of later timeslots with relatively higher prices does not exceed the available volume in the DSS until the next charge cycle.

$$\text{Condition 4}: \qquad \frac{1}{\eta^{out}} \cdot \sum_{t' \in T_2'} q_{t'}^{out.fo} < \xi_t - \left(1 - \hat{\delta}\right) \cdot C \tag{11}$$

with $$T_2' = \left\{t' \mid p_{t'}^{fo} > p_t^{fo} : t' > t \land \neg\exists t'' \le t' : p_{t''}^{fo} \le p_t^{\text{charge}}\right\} \tag{12}$$

3.3 Schedule Execution

The previous step of the simulation model determined a charge schedule φ_t $1 \le t \le T^{FP}$ and a discharge schedule λ_t based on the price and load forecasts. Due to deviations between the forecast and the actual data, this schedule might violate the (technical) constraints of the storage device. In particular, the constraints are the lower and upper bound of the SOC, i.e., the realized DOD and the maximal storage capacity C. The schedule execution algorithm applies the actual load and price data (ℓ_t, p_t) instead of the load forecast and tries to execute the CDS as complete as possible within these constraints. If necessary, $\varphi_{t'}$ and $\lambda_{t'}$ are updated according to the technical constraints within the execution period $1 \le t' \le T^{EP}$. The variable costs $K_{t'}^{\text{var}}$ of timeslot t' within eq. (1) that result from the updated (executed) CDS equal to

$$K_{t'}^{\text{var}} = \varphi_{t'} \cdot p_{t'} \cdot q^{in} + \lambda_{t'}\left(q_{t'}^{out}\left(\frac{\kappa}{\gamma} - p_{t'}\right)\right) \qquad \forall t' \tag{13}$$

3.4 Simulation Scenarios

The set of scenarios (Figure 1) combines three basic accuracy scenarios for both price and load forecasts to 9 scenarios in total. Each scenario has three parameters: (1) MAPE at the beginning of the first day of the forecast period (lower error bound), (2) MAPE at the end of the last day of the forecast period (upper error bound), and (3) a specific degree of autocorrelation of relative forecast errors (indicated with the DWT value).

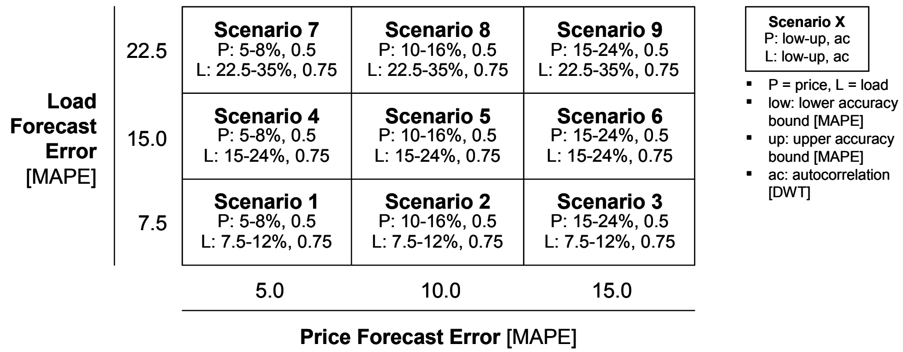

Figure 1: Simulation Scenarios

For the price forecast, the lower accuracy bound value is 5% in the first case. This value reflects a realistic value of MAPE reported in literature.[4] For the more conservative price forecast scenarios, these values are 2 respectively 3 times higher. The default value for autocorrelation of price forecast errors is 0.5 (DWT value).

Load forecast scenarios are defined analogously, just that the lower error bound in the first load forecast scenario is 7.5% MAPE[5]. The default value for autocorrelation of load forecast errors is 0.75 (DWT value).

4 Results

As explained in Section 3.2, *Heuristic 1* defines the simplest and most myopic algorithm, whereas *Heuristic 2* includes a refinement of the charge condition and *Heuristic 3* includes an additional refinement of the discharge condition. The question is to what extent these refinements impact the robustness of the heuristic algorithm towards variations of the forecast accuracy and how the heuristic scheduling algorithms perform in comparison with the linear optimization algorithm. All results

[4]Cuaresma et al. (2004) report a MAPE of 4% for day-ahead hourly price forecasts; Nogales et al. (2002) report 3-5% [MAPE], Li and Wang (2006) report 3.5-5.15% [MAPE].

[5]Pahasa and Theera-Umpon (2008) report a MAPE of 5.5-6.7% for day-ahead hourly load forecasting of small consumption units (substation level); Worawit and Wanchai (2002) report 7.3% MAPE; Espinoza et al. (2005) report 4.3% MAPE.

are benchmarked against the optimal result, which is obtained through running an optimal schedule on the actual data.

4.1 Comparison of Heuristic Scheduling Algorithms

The variation of the lower price forecast error bound reveals that all variants lead to approximately equal deviations from the optimum for a MAPE above 35%. All variants result in a deviation of more than 60% from the optimal benchmark value. The results are different for lower price forecast errors. Schedules determined with *Heuristic 1* lead to significantly higher deviations from the optimum (40-60%) than *Heuristic 2* and *3* (15-60%). When comparing the second and the third variant, there is hardly any difference visible in Figure 2 (the numeric results reveal that *Heuristic 3* is marginally superior up to a MAPE of 15%).

The implication of these results is that the refinement of the charge schedule determination leads to a result improvement for forecasts with errors below 40% MAPE, whereas the refined discharge schedule determination only has a marginal effect for forecasts with up to 15% MAPE. Hence, determining the charge price limit correctly is more important than determining the discharge price limit. For real-world applications, only *Heuristic 2* and *Heuristic 3* are relevant variants, since forecasts with a MAPE of more than 40% lead to unacceptable high deviations from the optimum anyway.

4.2 Heuristic vs. Linear Optimization Algorithm

To compare the performance of the heuristic and the linear optimization algorithm, 4 simulation setups have been defined and run for each of the 9 scenarios. The first two simulation setups use the linear optimization algorithm for determining the CDS. Setup 3 and 4 use the heuristic algorithm. Moreover, Setup 1 and 3 use price forecasts with autocorrelated forecast errors, whereas the scheduling algorithms in Setup 2 and 4 operate on forecast without autocorrelated forecast errors.

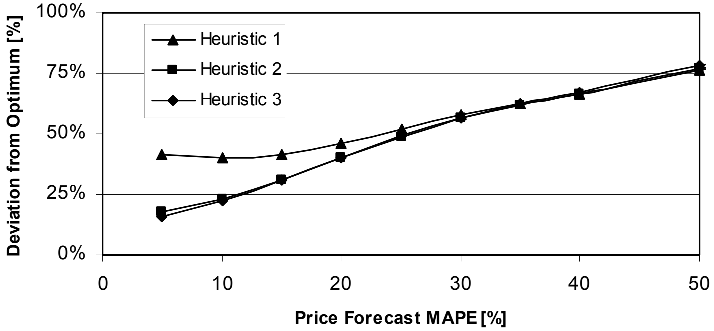

Figure 2: Impact of forecast errors on the variants of the heuristic algorithm

As already stated in Ahlert and Block (2010, p. 7), autocorrelation of price forecast errors improves the results when using a linear optimization algorithm for determining the CDS. If forecast errors are not autocorrelated, results deteriorate significantly. The results in Figure 3 confirm this observation, when comparing the results of Setup 1 and Setup 2 for each scenario. The reason behind this observation is the systematic over- or underestimation of prices in consecutive timeslots. The structural condition supports the linear optimization algorithm, which selects the timeslots with the relatively highest (lowest) prices for discharging (charging).

The aim of the heuristic algorithm is to deliver a robust alternative scheduling procedure for both setups (3 and 4) and all scenarios. The results in Figure 3 reveal that the algorithm achieves its objective only for Setup 4, but not for Setup 3. I.e., in comparison to the linear optimization algorithm, the heuristic cannot deal with autocorrelated forecast errors. A comparison of the results for Setup 3 and 4 reveals that the heuristic scheduling algorithm deals much better with uncorrelated than with autocorrelated price forecast errors in each of the 9 scenarios. When comparing the results of Setup 2 and 4 (no autocorrelation of price forecast errors), the heuristic algorithm clearly delivers better results. For Setups 1 and 3 the contrary is the case.

A key finding of the analysis among the 9 scenarios is that the heuristic algorithm delivers better, i.e., more robust results than the linear optimization algorithm for increasing price forecast errors (lower increase of the deviation from the optimum when increasing the price forecast error and/or the load forecast error - comparison of Setups 1 and 4).

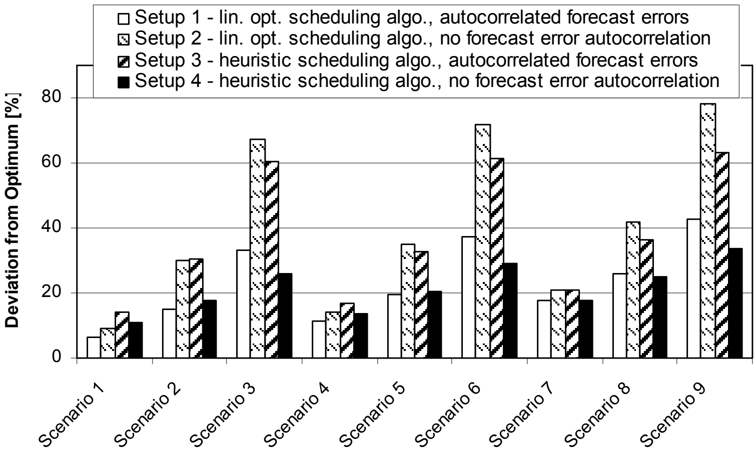

Figure 3: Results of scheduling algorithms in combination with forecast error auto correlation variations

The result overview for all scenarios also reveals that primarily the price forecast error sets the level for the achievable savings. For all setups, the average results of Scenarios 1, 4, 7 (price forecast with 5% MAPE, results deviate in a range of 7-

18% from the optimum) are distinct from the average results of Scenarios 2, 5, 8 (10% MAPE, 15-28% deviation from the optimum) and Scenarios 3, 6, 9 (15% MAPE, 26-42% deviation from the optimum). Extreme load forecast errors have a significantly lower impact than the price forecast errors.

5 Conclusions

The article analyzed the economic robustness of scheduling algorithms for DSS under the condition of forecast errors. Nine scenarios defining different levels of forecast errors for price and load forecasts are analyzed using Monte Carlo simulations. As an alternative to existing algorithms building on a linear optimization model, this article presents a heuristic scheduling algorithm, which is benchmarked against an optimal value and compared with the performance of a scheduling algorithm using a LO model. The optimal value is obtained through running with LO model with actual data only.

Firstly, the simulations reveal that the choice of the schedule determination algorithm depends on the autocorrelation of forecast errors within the available price forecast data. If the price forecast errors are autocorrelated, a LO algorithm will deliver best results. Otherwise the heuristic algorithm delivers superior results. Secondly, the results reveal that both algorithms deliver fairly robust results when varying the level of forecast accuracy. Variations of the load forecast accuracy only have a minor impact on the realized savings, whereas variations of the price forecast accuracy influence the results significantly. Still, even for very conservative scenarios assuming price forecast with a MAPE of 15% for day-ahead hourly forecasts in combination with 22.5% MAPE of load forecasts, both algorithms deviate less than 45% from the optimal result. Available forecast methods in practice achieve around 5% (price) respectively 7.5% (load) MAPE. Thus, the DSS scheduling algorithms show a high robustness against accuracy variations of the schedule.

Since this paper has shown the practical feasibility to economically operate DSS under forecast uncertainty, further research will have to address the question, how many DSS installations can operate profitably in a given grid. The analysis in this paper assumed that a DSS is a price taker. If a large number DSS would operate on the same or similar schedules, this would certainly have an effect on the market price in peak and off-peak hours (reduction of spreads). I.e., there must be a grid-specific limit to the number of DSS than can be run economically.

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