

Dissertation zur Erlangung des Doktorgrades vorgelegt am Fachbereich Mathematik der Freien Universität Berlin 1998  
von Michael Goedert

the similarity between geometric  
objects in higher dimensions

verteidigt am 18. Dezember 1998

betreut von Helmut Alt  
begutachtet von Helmut Alt    Günter Ziegler

# On the complexity of measuring the similarity between geometric objects in higher dimensions

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