Chapter 1

Introduction

1.1 Overview

In pattern recognition and quality control the comparison of geometric objects is an often considered problem. In order to quantify the similarity between geometric objects a natural approach is considering them as elements of some metric space and evaluating their degree of similarity by simply computing the distance between them.

The geometric objects we consider in this thesis are curves, surfaces or analogues in higher dimensions usually seen as equivalence classes of parameterized curves, parameterized surfaces and so on. We assume the objects to be described by a finite structure. That means for curves that they consist of finitely many line segments and for surfaces that they consist of triangles and so on.

A natural metric defining the similarity between them is the Fréchet distance, first described in [Fré06]. Especially in the calculus of variations this is the standard metric considered, see [Ewi85] for example. Algorithms for piecewise affine curves already have been investigated by [God91b], [Nat91], [God91a], [AG92], [AG95]. The given algorithms for computing the distance between such curves have had a polynomial runtime of low degree. In higher dimensions it is not known whether there exists even a polynomial time algorithm. However, among other things, in this thesis it will be shown that the problem is NP-hard for higher dimensions.[*] And there is some evidence for the fact that the problem of calculating the frechet distance will be undecidable for dimension $\geqslant 4$.[**]

In search for a more efficient way to calculate the distance between those objects we investigate the well known Hausdorff metric as well. For this metric we give polynomial time algorithms for any dimension. This metric is, however, less appropriate for this type of objects. Instead, the Hausdorff metric is actually a metric for point sets of some metric space, thus some information about the structure of the objects will be lost. We investigate the relationship between the Hausdorff and the Fréchet metrics in chapter 4 on page 57.

[*] That is on chapter 3 on pages 19–56. As a byproduct of this research we also prove some NP-hardness result in graph drawing which will be considered in chapter 5 on page 61.

[**] We will discuss this on section 3.13 on page 55.

1.2 How to read this thesis

The chapters can be read more or less independently, chapter 4, however, uses the definition of the metrics mentioned there.*

Chapter 5 is an exception, in the sense that in the proofs to be found there we need concepts and notions from the proofs of chapter 3. Thus if you want to read chapter 4 solely you have to read the pages 21–25 from chapter 3 first.

Furthermore there is an appendix consisting only of three pages showing drawings which are already contained in chapter 3 and which therefore are actually needless. It is usefull, however, to have these drawings handy while reading some proofs of chapter 3. And they are reprinted in the appendix just to make it painless to cut them out of the book.**

Finally there is an index, but you should not overestimate the value of this index. Actually in this index are contained only and exactly these terms which are printed in some unique boldface font in the text like this ***useless example of an index entry***. In general these terms are terms which are defined just around the place there are printed boldface and which should be kept in memory (and sometimes only for technical reasons in the proofs) a little bit longer. May this help to skip back the pages.

1.3 Credits

This should be the place for some personal remarks in a more informal style about the history of the project and the persons involved and I would like to do this here. Nevertheless it is likely for me to have forgotten some people even if I should have to mention them. So treat this section as a preliminary thing.

First of all, I have to thank Helmut Alt for the obvious things. But doing this that way would be an understatement. In fact this thesis has never been written without his encouragement even and especially in seemingly hopeless situations. And, by the way, it was always a calming experience to have someone to talk about the scientific matters of a thesis. Chapter 3 is, in particular, a partial answer I found 1992 to a question Helmut posed 1991 concerning generalizing things for higher dimensions which turned out to be the main topic of this thesis later on. Chapter 2 is the natural outgrowth of this.

Next I would like to thank Otfried Cheong (né Schwarzkopf) for citing the right thing, Klaus Kriegel for translating a paper from Russian into German, Annamaria Amenta for proof reading a paper of mine (and, of course our foreign language secretary, Susanne Schöttker for being just that), an unknown number of people for writing software I have used for typesetting this thesis (in fact I intentionally only used free software on a free operating system for doing that and I hope that there will be an opportunity for publishing this thesis through the internet as well) and finally I would like to thank Monika for her patience.

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*obviously to be found in sections 2.1 and 3.2, i.e. on pages 5 and 19–20

**Yes, dear reader, you are encouraged to modify this thesis according to your personal needs.